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Extending Dialectical Classical Logic Argumentation with Unrestricted Rebut and Occam Razor Defeats

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Abstract. Dialectical Classical Logic Argumentation (*D-Cl-Arg*) formalises maxi-consistent non-monotonic reasoning under the practical assumption that agents have bounded resources for classical inference, and that agents do not typically check arguments' premises for subset minimality and consistency. However, *D-Cl-Arg* still satisfies all rationality postulates. Moreover *D-Cl-Arg* accommodates uses of argument characteristic of dialectical practice. This paper extends *D-Cl-Arg* to accommodate further dialectical uses of argument; in particular unrestricted rebuts on the deductively derived conclusions of arguments, and Occam Razor defeats that dialectically demonstrate that an argument makes use of redundant premises. We show that all rationality postulates are still satisfied, while relaxing constraints on preference relations that were previously required to prove rationality.

Keywords: Classical Logic, Argumentation, Non-monotonic Reasoning, Resource Bounds

1. Introduction

Argumentation, Non-monotonic Reasoning and Dialogue. A Dung framework (DF) [1] consists of defeats amongst arguments constructed from a belief base $\mathcal{B}$. The conclusions of credulously (cr) or sceptically (sc) justified arguments under a semantics s, yield argumentation defined credulous, respectively sceptical, *nm* consequences (denoted $\mid\!\sim_s^{cr}$ respectively $\mid\!\sim_s^{sc}$). These can be shown to equate with the consequences ($\mid\!\sim_{\mathcal{L}}$) of various non-monotonic logics $\mathcal{L}$, defined directly over $\mathcal{B}$:

$$\mathcal{B} \mid\!\sim_s^{cr/sc} \alpha \underset{\text{soundness}}{\overset{\text{completeness}}{\rightleftarrows}} \mathcal{B} \mid\!\sim_{\mathcal{L}}^{cr/sc} \alpha \tag{1}$$

For example, given a totally ordered base $(\mathcal{B}, \leq)$ of classical formulae, standard approaches to classical logic argumentation [2,3,4] define arguments (Δ, α) such that: **C1**) $\Delta \subseteq \mathcal{B}, \Delta \vdash_{Cl} \alpha$; **C2**) Δ is consistent ; **C3**) no $\Delta' \subset \Delta$ classically entails α. Then $X = (\Delta, \alpha)$ attacks $Y = (\Gamma, \beta)$ if α negates a premise in Γ. One then identifies the extensions (under various semantics) of the DF relating arguments by *defeats* (the attacks that succeed given a strict preference relation $\prec$ over arguments, defined on the basis of $\leq$). The conclusions of arguments in stable extensions ($\mid\!\sim_{stable}^{cr}$) are then shown

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[5,4,6] to equate with the credulous Preferred Subtheories consequences [7] (the classical consequences of the preferred maximal consistent subsets of $(\mathcal{B}, \leq)$).

One can then aim at dialogical generalisations of argumentation (e.g., [8,9,10]) in which agents exchange locutions that conform to protocols, so that the following result obtains upon evaluation of the resultant *graph of locutions*:

$$\text{the status of a communicated claim } \alpha \text{ is 'winning' iff } \mathcal{B} \mid\!\sim_s^{cr/sc} \alpha \quad (2)$$

where $\mathcal{B}$ on the *rhs* of Eq. 2 are the beliefs *incrementally defined* by the declarative contents of exchanged locutions (rather than by a given static belief base $\mathcal{B}$ as in single agent reasoning). Substituting the *rhs* of Eq. 1 for the *rhs* of Eq. 2, one thereby aims at dialogical formalisations of *distributed nm* reasoning [2]

Practical accounts of individual and distributed *nm* reasoning require argumentative formalisations that are fully rational under resource bounds (D1) and accommodate dialectical uses of arguments witnessed in real world debate and discussion (D2). Regarding (D1), standard approaches to classical logic argumentation (*Cl-Arg*) [2,3,4] tacitly assume 'logical omniscience' in the sense that *all* arguments defined by $\mathcal{B}$ are assumed to instantiate a DF (**C1**). This is clearly not feasible for agents with limited resources, given the undecidability of first order classical logic, and that even in the propositional case, deciding whether $\Delta \vdash_{Cl} \alpha$ is in general co-NP complete, and therefore most likely intractable. Moreover, the intractability of deductive closure is further exacerbated by the consistency (**C2**) and subset-minimality (**C3**) checks on arguments' premises; the latter being a problem in the second level of the polynomial hierarchy [12]. However, **C1**, **C2** and **C3**, and so called 'reasonable' preference relations, are required to show that argument evaluation yields rational outcomes (i.e., satisfy the consistency, closure [13] and non-contamination [14] postulates).

Regarding D2), enforcing **C2** and **C3** does not reflect real-world dialectical practice. Firstly, the inconsistency of arguments' premises is typically demonstrated dialectically – "you've contradicted yourself !" – as illustrated in the classic Socratic move [15]. Also, checking every argument for subset minimality, which enforces relevance of an argument's premises w.r.t. the conclusion, is not what one expects of agents in practice.

Dialectical Classical Logic Argumentation [16,17,18] aims at satisfying desiderata D1 and D2. Addressing D1), *Dialectical Cl-Arg* does not enforce logical omniscience (**C1**); only minimal assumptions are made as to the resources available for constructing arguments. **C2** and **C3** are also not enforced. Regarding D2), *Dialectical Cl-Arg* (and *Dialectical* ASPIC+ [18]), adopt a distinction ubiquitous in dialectical practice; the *epistemic* distinction between premises that an agent commits to, and premises that can be supposed; in particular premises supposed in virtue of their commitment by a (possibly imaginary in the case of single agent reasoning) interlocutor, thus anticipating dialogical formalisations of *nm* reasoning. Moreover, if arguments commit to inconsistent premises, they can be defeated by an argument that dialectically demonstrates the inconsistency; "supposing only the premises you've committed to, you've contradicted yourself !". Then, despite dropping **C1**, **C2** and **C3**, satisfaction of the consistency and closure postulates is shown, and non-contamination is satisfied if one either deploys proof theories that exclude use of syntactically disjoint premises (e.g., [19,20]), or else one enforces that arguments are not strengthened by inclusion of syntactically disjoint premises.

[2] Thus enabling joint human-AI reasoning. Indeed, human-AI dialogue can ensure that AI reasoning and decision making is aligned with human values [11].

Contributions. This paper extends *Dialectical Cl-Arg* (*D-ClArg*) with features that further render the approach more suitable for real-world application (specifically w.r.t. D2).

Firstly, consistency is violated if *Cl-Arg* allows 'unrestricted rebut' attacks that target the conclusions of strict inferences [13]. Hence, $X = (\Delta, \beta)$ and $Y = (\Gamma, \neg\beta)$ do not attack each other. But then to ensure that no extension contains both X and Y one has to assume sufficient resources to construct all arguments under 'contraposition' [13] (an assumption that holds given logical omniscience). However, in practice, it seems natural to move X as an unrestricted rebut on Y. In *D-ClArg*, one is *not* required to assume construction of all arguments under contraposition. Rather, X and Y can be excluded from any extension E, by a defeating argument that dialectically demonstrates that E commits to the inconsistent $\Delta \cup \Gamma$. However, the practical rationale for allowing unrestricted rebuts remains. Hence, this paper's first contribution is to *extend D-ClArg to accommodate unrestricted rebuts, and show that all rationality postulates are still satisfied.*

Secondly, satisfaction of non-contamination requires that *D-ClArg* arguments are not strengthened when adding syntactically disjoint premises. However, this compromises the kinds of preference relations that are allowed. Hence, this paper also extends *D-ClArg* so that use of irrelevant premises by an argument is demonstrated dialectically, by a defeating argument that makes additional use of suppositions to reveal that the same conclusion can be obtained without committing to irrelevant premises; a dialectical move we might expect in principled dialogue. This paper's second contribution is to then *extend D-ClArg to accommodate* (what for obvious reasons we call) *Occam Razor defeats, and show that all rationality postulates are still satisfied.*

Organisation of Paper. In Section 2 we review *Dialectical Cl-Arg* [16,17] and its satisfaction of rationality postulates under resource bounds. Section 3 then extends *Dialectical Cl-Arg* to accommodate unrestricted rebuts and Occam Razor defeats, and we show that so long as preference relations are 'logically rational' all rationality postulates are satisfied. We conclude in Section 4.

2. Dialectical Classical Logic Argumentation

A Dung framework is instantiated *by some subset* of the classical logic arguments defined by a finite set of classical (propositional or first order) formulae $\mathcal{B}$, with attacks defined in the usual way. Note that arguments are *not* checked for premise consistency and subset minimality. (In what follows, if ϕ is of the form $\neg\alpha$ then $-\phi = \alpha$, else $-\phi = \neg\phi$, and given an argument $X = (\Delta, \alpha)$, $\texttt{Conc}(X) = \alpha, \texttt{Prem}(X) = \Delta\}$). Then, the *epistemic* distinction between committed and supposed premises is only adopted when evaluating the acceptability of arguments.

Definition 1. *Let* $DF_{\mathcal{B}} = (\mathcal{A}_{\mathcal{B}}, \mathcal{C}_{\mathcal{B}})$*, where* $\mathcal{A}_{\mathcal{B}} \subseteq \{(\Delta, \alpha) \mid \Delta \subseteq \mathcal{B}, \Delta \vdash_{\mathsf{CL}} \alpha\}$ *and* $\mathcal{C}_{\mathcal{B}} = \{(X,Y) | X, Y \in \mathcal{A}_{\mathcal{B}}, \texttt{Conc}(X) = \phi, -\phi \in \texttt{Prem}(Y)\}$.

Definition 2. *Let* $X = (\Delta, \alpha)$*. Then* $\mathbb{X} = (\Sigma, \Gamma, \alpha)$ *is an epistemic variant of* X *iff* $\texttt{Prem}(X) = \Delta = \Sigma \cup \Gamma$ *and* $\Sigma \cap \Gamma = \emptyset$*. We also say that* $\mathbb{X}$ *is a dialectical argument.*

- $\texttt{Conc}(\mathbb{X}) = \texttt{Conc}(X)$ *and* $\texttt{Com}(\mathbb{X})$ *denotes the committed premises (commitments)* Σ *of* $\mathbb{X}$*,* $\texttt{Sup}(\mathbb{X})$ *the supposed premises (suppositions)* Γ *of* $\mathbb{X}$*.*

- $\|X\|$ *denotes the set of all epistemic variants of* X, $\|E\|$ *denotes* $\bigcup_{X \in E} \|X\|$. *Let* $\mathcal{E}$ *be any set of dialectical arguments. Then* $\mathtt{Com}(\mathcal{E})$, $\mathtt{Sup}(\mathcal{E})$ *and* $\mathtt{Conc}(\mathcal{E})$ *respectively denote* $\bigcup_{\mathbb{X} \in \mathcal{E}} \mathtt{Com}(\mathbb{X})$, $\bigcup_{\mathbb{X} \in \mathcal{E}} \mathtt{Sup}(\mathbb{X})$ *and* $\bigcup_{\mathbb{X} \in \mathcal{E}} \mathtt{Conc}(\mathbb{X})$.

We assume a strict preference relation $\prec$ over $\mathcal{A}_{\mathcal{B}}$ (which may or may not be defined by an ordering over $\mathcal{B}$). Then, when establishing whether $\mathbb{X} = (\Sigma, \Pi, \alpha)$ is acceptable w.r.t. a set $\mathcal{E}$ of dialectical arguments, it is only the committed premises Σ that can be targeted. Furthermore, an attack by $\mathbb{Y} = (\Delta, \Gamma, \alpha)$ on $\beta \in \Sigma$ is contingent on the suppositions Γ of $\mathbb{Y}$ being commitments in $\mathbb{X}$ *and* $\mathcal{E}$, i.e., $\Gamma \subseteq \mathtt{Com}(\mathcal{E} \cup \{\mathbb{X}\})$. Intuitively:

> "*Given that I commit to* Δ *and supposing for the sake of argument your commitments* Γ *in* $\mathcal{E}$ *and* $\mathbb{X}$*, I can construct an argument* $\mathbb{Y}$ *that challenges your premise* $\beta \in \Sigma$*.*"

Such an attack succeeds as a defeat only if $(\{\beta\}, \beta)$ is not strictly preferred to Y. An argument of the form $\mathbb{Y} = (\Delta, \Gamma, \bot)$ can challenge $\mathbb{X}$ by arguing that the premises Γ committed in $\mathcal{E} \cup \{\mathbb{X}\}$, together with Δ, are inconsistent. $\mathbb{X}$ should only then be targeted if at least one of $\mathbb{X}$'s committed premises β is in Γ and so is 'culpable' in contributing to the inconsistency. Again, $\mathbb{Y}$ defeats $\mathbb{X}$ if $Y \not\prec (\{\beta\}, \beta)$. However, if $\Delta = \emptyset$ then $\mathbb{Y}$ dialectically demonstrates a commitment to inconsistent premises Γ in $\mathcal{E} \cup \{\mathbb{X}\}$. To prefer that one commits to inconsistent premises is clearly incoherent. Therefore such an attack succeeds as a defeat independently of preferences. Finally, $\mathbb{Z} \in \mathcal{E}$ can defend $\mathbb{X}$ by defeating $\mathbb{Y}$, while supposing any of $\mathbb{Y}$'s commitments, i.e., $\mathtt{Sup}(\mathbb{Z}) \subseteq \mathtt{Com}(\mathbb{Y})$[3].

Definition 3. *Let* $DF_{\mathcal{B}} = (\mathcal{A}_{\mathcal{B}}, \mathcal{C}_{\mathcal{B}})$*, and* $\prec$ *a strict partial ordering over* $\mathcal{A}_{\mathcal{B}}$*. Let* $\mathcal{E} \subseteq \|\mathcal{A}_{\mathcal{B}}\|$, $\mathbb{Y} = (\Delta, \Gamma, \phi) \in \|Y\|$, $\mathbb{X} = (\Pi, \Sigma, \psi) \in \|X\|$, $X, Y \in \mathcal{A}_{\mathcal{B}}$*.*

1. *if* $\phi \neq \bot$*, then* $\mathbb{Y}$ *defeats* $\mathbb{X}$ *w.r.t.* $\mathcal{E}$*, denoted* $\mathbb{Y} \Rightarrow_{\mathcal{E}} \mathbb{X}$*, iff:*
   (a) $(Y, X) \in \mathcal{C}_{\mathcal{B}}$ *on* $X' = (\{-\phi\}, -\phi)$, $-\phi \in \mathtt{Com}(\mathbb{X})$ *and* $Y \not\prec X'$*;*
   (b) $\Gamma \subseteq \mathtt{Com}(\mathcal{E} \cup \{\mathbb{X}\})$*.*
2. *if* $\phi = \bot$*, then* $\mathbb{Y}$ *defeats* $\mathbb{X}$ *w.r.t.* $\mathcal{E}$*, denoted* $\mathbb{Y} \Rightarrow_{\mathcal{E}} \mathbb{X}$*, iff:*
   (a) $\Gamma \cap \mathtt{Com}(\mathbb{X}) \neq \emptyset$ *and* $\Gamma \subseteq \mathtt{Com}(\mathcal{E} \cup \{\mathbb{X}\})$*;*
   (b) *either* $\Delta = \emptyset$ *or* $\forall \beta \in \Gamma \cap \mathtt{Com}(\mathbb{X})$, $Y \not\prec (\{\beta\}, \beta)$*.*

   In both cases we say $\mathbb{Y}$ *defeats* $\mathbb{X}$ *on* $\mathbb{X}' = (\{\beta\}, \emptyset, \beta)$ *or* $\mathbb{Y}$ *defeats* $\mathbb{X}$ *on* β *(where in case 2,* $\beta \in \Gamma \cap \Pi$*).*

Definition 4. *Let* $\mathbb{X} \in \|\mathcal{A}_{\mathcal{B}}\|$*. Then* $\mathbb{X}$ *is **acceptable** w.r.t.* $\mathcal{E} \subseteq \|\mathcal{A}_{\mathcal{B}}\|$ *iff* $\forall \mathbb{Y} \in \|\mathcal{A}_{\mathcal{B}}\|$ *s.t.* $\mathbb{Y} \Rightarrow_{\mathcal{E}} \mathbb{X}$, $\exists \mathbb{Z} \in \mathcal{E}$ *s.t.* $\mathbb{Z} \Rightarrow_{\{\mathbb{Y}\}} \mathbb{Y}$*.*

• $\mathcal{E} \subseteq \|\mathcal{A}_{\mathcal{B}}\|$ *is **conflict free** iff* $\neg\exists \mathbb{X}, \mathbb{Y} \in \mathcal{E}$ *s.t.* $\mathbb{Y} \Rightarrow_{\mathcal{E}} \mathbb{X}$*. Let* $\mathcal{E}$ *be conflict free. Then* $\mathcal{E}$ *is: a dialectical **admissible** extension iff* $\forall \mathbb{X} \in \mathcal{E}$, $\mathbb{X}$ *is acceptable w.r.t.* $\mathcal{E}$*; a dialectical **complete** extension iff* $\mathcal{E}$ *is dialectical admissible and* $\forall \mathbb{X} \in \|\mathcal{A}_{\mathcal{B}}\|$, $\mathbb{X}$ *is acceptable w.r.t.* $\mathcal{E}$ *implies* $\mathbb{X} \in \mathcal{E}$*; the dialectical **grounded** extension iff* $\mathcal{E}$ *is the minimal under set inclusion dialectical complete extension; a dialectical **preferred** extension iff* $\mathcal{E}$ *is a maximal under set inclusion dialectical complete extension; a dialectical **stable** extension iff* $\forall \mathbb{Y} \in \|\mathcal{A}_{\mathcal{B}}\| \setminus \mathcal{E}$, $\exists \mathbb{X} \in \mathcal{E}$ *s.t.* $\mathbb{X} \Rightarrow_{\{\mathbb{Y}\}} \mathbb{Y}$*.*

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[3]If $\mathbb{Y}$ challenges the acceptability of $\mathbb{X}$ w.r.t. $\mathcal{E}$, it is not required that $\mathbb{Y}$ itself be acceptable w.r.t. some set of dialectical arguments. Hence $\mathbb{Z}$'s defeat on $\mathbb{Y}$ only supposes the committed premises of $\mathbb{Y}$.

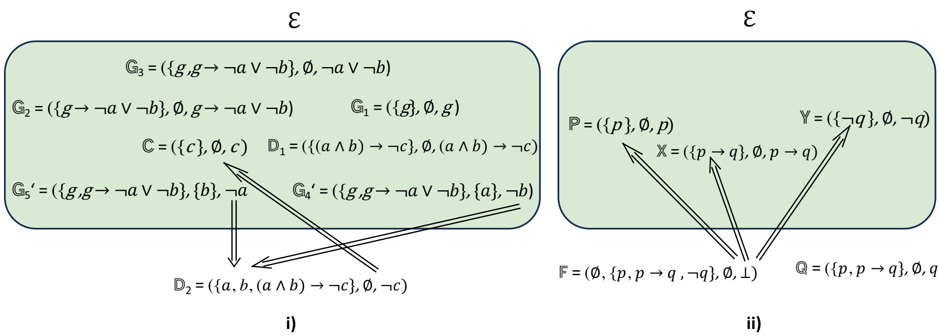

Figure 1. i) shows the dialectical grounded extension for Eg.1. ii) shows how defeats from unassailable falsum arguments preserve consistency.

Remark 1. *Notice that dialectical arguments of the form* $\mathbb{X} = (\emptyset, \Gamma, \phi)$ *cannot be defeated, given the empty commitments, and so any such* $\mathbb{X}$ *is acceptable w.r.t. any set of dialectical arguments, and hence is said to be '*unassailable*'.*

Example 1. *Let* $\mathcal{B} = \{a, b, c, (a \wedge b) \rightarrow \neg c, g, g \rightarrow \neg a \vee \neg b\}$ *where* a, b, c *respectively denote 'attend conference A,B,C' and* g *denotes 'the conference budget = £2000'. Suppose the total ordering* $\leq$ *over* $\mathcal{B}$ *(with* $<$ *and* $\approx$ *defined in the usual way):*

$$c < a \approx b \approx (a \wedge b) \rightarrow \neg c < g \approx g \rightarrow \neg a \vee \neg b$$

Let $\mathcal{A}_{\mathcal{B}}$ *be the arguments:*

$A = (\{a\}, a)$	$G_1 = (\{g\}, g)$
$B = (\{b\}, b)$	$G_2 = (\{g \rightarrow \neg a \vee \neg b\}, g \rightarrow \neg a \vee \neg b)$
$C = (\{c\}, c)$	$G_3 = (\{g, g \rightarrow \neg a \vee \neg b\}, \neg a \vee \neg b)$
$D_1 = (\{(a \wedge b) \rightarrow \neg c\}, (a \wedge b) \rightarrow \neg c)$	$G_4 = (\{g, a, g \rightarrow \neg a \vee \neg b\}, \neg b)$
$D_2 = (\{a, b, (a \wedge b) \rightarrow \neg c\}, \neg c)$	$G_5 = (\{g, b, g \rightarrow \neg a \vee \neg b\}, \neg a)$
$H = (\{a, b, g \rightarrow \neg a \vee \neg b\}, \neg g)$	

We have attacks (G_4, B)*,* (G_5, A)*,* (D_2, C) *and* (H, G_1)*, and* $G_4 \not\prec B, G_5 \not\prec A, D_2 \not\prec C, H \prec G_1$ *given the* Elitist *preference relation [4] defined by the ordering over* $\mathcal{B}$*:*

$$(\Gamma, \phi) \prec (\Delta, \theta) \text{ iff } \exists \alpha \in \Gamma \text{ such that } \forall \beta \in \Delta, \alpha < \beta \qquad (\text{Eli})^4$$

As shown in Fig. 1, $\mathbb{D}_2 \Rightarrow_{\mathcal{E}} \mathbb{C}$ *given* $D_2 \not\prec C$*, and* $\mathbb{D}_2 \in \|D_2\|$*,* $\mathbb{C} \in \|C\|$*, and (trivially)* $\texttt{Sup}(\mathbb{D}_2) = \emptyset \subseteq \texttt{Com}(\mathcal{E} \cup \mathbb{C})$*. Note also the epistemic variants*

$$\mathbb{G}_4' = (\{g, g \rightarrow \neg a \vee \neg b\}, \{a\}, \neg b) \in \|G_4\|, \mathbb{G}_5' = (\{g, g \rightarrow \neg a \vee \neg b\}, \{b\}, \neg a) \in \|G_5\|$$

that are undefeated (and hence in $\mathcal{E}$*). Both* $\mathbb{G}_4'$ *and* $\mathbb{G}_5'$ *defeat* $\mathbb{D}_2$*, given that* $\texttt{Sup}(\mathbb{G}_4') = \{a\} \subseteq \texttt{Com}(\mathbb{D}_2)$ *and* $\texttt{Sup}(\mathbb{G}_5') = \{b\} \subseteq \texttt{Com}(\mathbb{D}_2)$*.*

Epistemic variants (i.e., dialectical arguments) are only deployed when determining acceptability, w.r.t. sets of dialectical arguments. Then, only the conclusions of *unconditional* dialectical arguments, that commit to *all* their premises, identify the conclusions supported by dialectical extensions. Hence, we identify a DF's s extensions ($s \in$ {admissible, complete, grounded, preferred, stable}) by reference to the *unconditional* arguments in the DF's *dialectical* s extensions.

[4]Notice that $\forall \Gamma'$: $(\Gamma, \phi) \prec (\{\theta\}, \theta)$ implies $(\Gamma \cup \Gamma', \phi) \prec (\{\theta\}, \theta)$.

Definition 5. *Let $\mathcal{E}$ be a dialectical s extension of $(\mathcal{A}, \mathcal{C})$. Then $E = \{(\Delta, \alpha) | (\Delta, \emptyset, \alpha) \in \mathcal{E}\}$ is an s extension of $(\mathcal{A}, \mathcal{C})$.*

In Example 1, $\mathcal{E}$ is the *dialectical* grounded extension shown in Fig. 1i, and so the grounded extension is $\{G_1, G_2, G_3, D_1, C\}$. Intuitively, since the budget g precludes attendance at both conferences a and b (and neither A or B are in the grounded extension since the ordering over $\mathcal{B}$ does not decide between the two), then attendance at c is justified since only if one attends *both* a and b is attendance at c precluded. However, in standard approaches to *Cl-Arg*, (e.g., the *ASPIC*$^+$ formalisation of *Cl-Arg* [4]) C is *not* in the grounded extension E. This is because G_4 or G_5 are required to defend C against D_2. But neither A or B, and so neither G_4 or G_5, are in E. Moreover, although G_3 is in E, G_3 does not defeat D_2 on a *and* b since attacks can only target single premises (targeting multiple premises results in violation of consistency [3]). However, Dialectical *Cl-Arg*'s use of suppositions means that $\mathbb{G}'_4$ and $\mathbb{G}'_5$ (each of which only commit to g and $g \rightarrow \neg a \vee \neg b$) defend $\mathbb{C}$, since $\mathbb{G}'_4$ and $\mathbb{G}'_5$ need not commit to attendance at a, respectively b. Intuitively, $\mathbb{G}'_4$ ($\mathbb{G}'_5$) argues that given the budget precludes attendance at a *and* b, and supposing $\mathbb{D}_2$'s commitment to attending b (a), one cannot attend a (b).

Consider Fig. 1ii. As shown in [4], ensuring no complete extension E contains $P = (\{p\}, p), X = (\{p \rightarrow q\}, p \rightarrow q), Y = (\{\neg q\}, \neg q)$, resources must suffice to construct $R = (\{p, \neg q\}, \neg(p \rightarrow q))$ and $Z = (\{\neg q, p \rightarrow q\}, \neg p)$ (i.e., arguments are 'closed under contraposition'). Also, one must assume a preference relation such that either $Z \not\prec P$, $R \not\prec X$ or $Q \not\prec Y$ ($Q = (\{p, p \rightarrow q\}, q)$). This then means that one of the attacks from Z to P or R to X or Q to Y succeeds as a defeat. Hence, defending against one of these defeats requires some argument in E that defeats Z or R or Q. But then such an argument would defeat P or X or Y and thus E would not be conflict free, contradicting E is complete. However, suppose $Q \prec Y$ and resources do not suffice to construct R or Z. Then one cannot exclude E being a complete extension.

On the other hand Dialectical *Cl-Arg* only requires that if resources suffice to recognise the mutual inconsistency of a set of premises, by stint of constructing two arguments from these premises with conflicting conclusions, and resources suffice to combine the premises of these two arguments to obtain an argument concluding $\perp$, then no *admissible* extension can contain arguments committing to mutually inconsistent premises (or indeed arguments with conflicting conclusions). Thus, given that construction of Q and Y signals the inconsistency of $\{p, p \rightarrow \neg q, q\}$, it suffices to combine their premises to construct $F = (\{p, p \rightarrow \neg q, q\}, \perp)$. Then, $\mathcal{E}$ in Fig. 1ii cannot be *dialectical* admissible (hence $\{P, X, Y\}$ cannot be admissible), since the unassailable epistemic variant $\mathbb{F}$ of F dialectically demonstrates that $\mathbb{P}, \mathbb{X}, \mathbb{Y}$ commit to inconsistent premises.

The non-contamination postulates [14] state that adding syntactically disjoint premises to $\mathcal{B}$ should not invalidate any of $\mathcal{B}$'s argumentation defined consequences. We illustrate what is meant by 'contamination':

Exp-C Suppose $\mathcal{B} = \{s\}$. Hence $\mathcal{B} \mid\!\sim^{sc}_{gd} s$ ('*gd*' is short for 'grounded'). If upon adding $\{p, \neg p\}$ to $\mathcal{B}$ one licenses construction of $X = (\{p, \neg p\}, \neg s)$ which then defeats $(\{s\}, s)$, then $\mathcal{B} \cup \{p, \neg p\} \not\mid\!\sim^{sc}_{gd} s$.

Red-C Suppose $\mathcal{B} = \{s, \neg s\}$ and $(\{\neg s\}, \neg s) \prec (\{s\}, s)$, so that $\mathcal{B} \mid\!\sim^{sc}_{gd} s$. If upon adding $\{r\}$ to $\mathcal{B}$, one licences construction of $Y = (\{r, \neg s\}, \neg s)$, and $(\{r, \neg s\}, \neg s) \not\prec (\{s\}, s)$, then $\mathcal{B} \cup \{r\} \not\mid\!\sim^{sc}_{gd} s$.

The subset minimality, respectively consistency, checks on *Cl-Arg* arguments preclude construction of X, respectively Y. Hence non-contamination is satisfied. However, for the aforementioned reasons, Dialectical *Cl-Arg* does not enforce these premise checks. Rather if the 'explosively contaminating' $\mathbb{X} = (\{p, \neg p\}, \emptyset, \neg s)$ defeats $(\{s\}, \emptyset, s)$, the latter is defended by $\mathbb{F} = (\emptyset, \{p, \neg p\}, \bot)$ defeating $\mathbb{X}$, where the unassailable $\mathbb{F}$ is in the grounded extension. Moreover, [16] shows that non-contamination is satisfied if one deploys proof theories (e.g, [19,20]) for classical logic that do not generate 'redundantly contaminated' arguments that incorporate syntactically disjoint irrelevant premises (e.g., $Y = (\{r, \neg s\}, \neg s)$). If one's proof theory does generate such arguments, then non-contamination is satisfied if: a) adding syntactically disjoint redundant premises (e.g., r) does not strengthen arguments (i.e., $(\{r, \neg s\}, \neg s) \prec (\{s\}, s)$), and b) given an argument for α with syntactically disjoint subsets of premises Δ and Γ (denoted $\Delta \| \Gamma$), resources suffice to construct an argument concluding $\bot$ from Δ or α from Γ ([16, Proposition 30] shows that: $\Delta \cup \Gamma \vdash_{Cl} \alpha$ and $\Delta \| \Gamma \cup \{\alpha\}$ implies $\Delta \vdash_{Cl} \bot$ or $\Gamma \vdash_{Cl} \alpha$).

Finally, observe that [16] assumes that preference relations are *dialectically coherent*. The idea is that $(\Delta, -\phi) \prec (\{\phi\}, \phi)$ can be interpreted as:

Given that it is rationally incoherent to commit to the inconsistent $\Delta \cup \{\phi\}$*, one would retain a commitment to* ϕ *in preference to retaining a commitment to all premises in* Δ.

Thus, $\prec$ is assumed to satisfy

$$\forall(\Delta, \bot) \in \mathcal{A} : \exists \alpha \in \Delta \text{ s.t. } (\Delta, \bot) \not\prec (\{\alpha\}, \alpha) \qquad \textbf{\textit{(DCPref)}}$$

since if $\forall \alpha \in \Delta$, $(\Delta, \bot) \prec (\{\alpha\}, \alpha)$, this would entail preferentially committing to an inconsistent set of beliefs Δ, which is clearly *logically* incoherent.

In summary, if arguments are not strengthened by adding syntactically disjoint premises, and $\prec$ satisfies *DCPref*, then full rationality is satisfied[5] by a DF $(\mathcal{A}_\mathcal{B}, \mathcal{C}_\mathcal{B})$ where $\mathcal{A}_\mathcal{B}$ is any subset of arguments defined by $\mathcal{B}$ that satisfy the following minimal assumptions as to the resources available for constructing arguments:

P1 $\alpha \in \mathcal{B}$ implies $(\{\alpha\}, \alpha) \in \mathcal{A}_\mathcal{B}$.
P2 (Δ, α) and $(\Gamma, -\alpha) \in \mathcal{A}_\mathcal{B}$ implies $(\Delta \cup \Gamma, \bot) \in \mathcal{A}_\mathcal{B}$.
P3 $(\Delta \cup \Gamma, \alpha) \in \mathcal{A}_\mathcal{B}$ and $\Delta \| \Gamma \cup \{\alpha\}$, implies $(\Delta, \bot) \in \mathcal{A}_\mathcal{B}$ or $(\Gamma, \alpha) \in \mathcal{A}_\mathcal{B}$.

3. Incorporating Unrestricted Rebut and Occam Razor Defeats

As discussed in [13], allowing unrestricted rebuts may result in violation of consistency[6]. Apart from this technical reason for prohibiting unrestricted rebuts, one may in principle want to preclude an *ideally rational* agent targeting the conclusion of a deductive inference. However, in practice it would seem quite natural to move $\mathbb{Y} = (\Delta, \Gamma, \phi)$ as an unrestricted rebut on $\mathbb{X} = (\Pi, \emptyset, -\phi)$:

[5] In [16, p. 27&28], Theorems 11 and 12 show mutual consistency of the conclusions and premises of arguments in admissible extensions, and Theorems 10 and 13 show sub-argument closure and closure under strict rules for complete extensions. Theorems 44 and 46 show satisfaction of non-contamination [16, p. 39&40].

[6] For an example illustrating violation by *Cl-Arg*, suppose A1, A2 and A3 unrestrictedly rebut A5, A6 and A4 respectively, in [4, Fig4a]. It is easily seen that $\{A1, A2, A3\}$ is admissible and so consistency is violated.

"*Given your commitment to* Π *and hence* $-\phi$*, and supposing your commitment to premises* Γ*, I have an argument for* ϕ*, although I am uncertain as to which of your premises* $\beta \in \Pi$ *should be challenged.*"

An unrestricted rebut succeeds so long as some premise in the attacked argument is not strictly preferred to the attacking argument. For example, if $\exists \beta \in \Pi$ s.t. $Y = (\Delta \cup \Gamma, \phi) \not\prec (\{\beta\}, \beta)$. To see why, observe that $\Delta \cup \Gamma \cup \Pi \vdash \bot$, and so given the rationale for *DCPref*, then only if $\forall \beta \in \Pi$, $Y \prec (\{\beta\}, \beta)$, is one definitively committed to the premises Π in X (and hence the conclusion -ϕ) in preference to retaining a commitment to all premises in $\Delta \cup \Gamma$. We therefore propose that attacks $\mathcal{C}_{\mathcal{B}}$ in Definition 1 include $\{(Y, X) | \texttt{Conc}(Y) = -\texttt{Conc}(X)\}$, and define the notions of conflict free and acceptability that accommodate unrestricted rebut attacks that succeed as defeats:

Definition 6. *Let* $\mathbb{Y} = (\Delta, \Gamma, \phi)$*,* $\mathbb{X} = (\Pi, \emptyset, -\phi)$*.* $\mathbb{Y}$ *unrestricted rebut defeats ('u-defeats')* $\mathbb{X}$ *w.r.t.* $\mathcal{E}$*, denoted* $\mathbb{Y} \Rightarrow^{u}_{\mathcal{E}} \mathbb{X}$*, iff* $\Gamma \subseteq \texttt{Com}(\mathcal{E} \cup \{\mathbb{X}\})$ *and* $\exists \beta \in \Pi$*:* $Y \not\prec (\{\beta\}, \beta)$*. When writing* $\mathbb{Y} \Rightarrow_{\mathcal{E}(u)} \mathbb{X}$ *this denotes that* $\mathbb{Y}$ *defeats or unrestricted rebut defeats* $\mathbb{X}$*, whereas* $\mathbb{Y} \Rightarrow_{\mathcal{E}} \mathbb{X}$ *denotes that* $\mathbb{Y}$ *defeats* $\mathbb{X}$ *as defined in Def. 3.*

Allowing unrestricted rebuts does not compromise satisfaction of the consistency postulates. [16] shows consistency for dialectical *admissible* (and not just complete) extensions, by showing that no such extension $\mathcal{E}$ contains unconditional [7] arguments that:

- conclude $\bot$, or arguments concluding ϕ and $-\phi$ ([16, Theorem 11]);
- commit to inconsistent premises Δ such that $\exists(\Delta, \bot) \in \mathcal{A}$[8] ([16, Theorem 12]).

Proofs of Theorems 11 and 12 in [16] rely only on P2 and the fact that defeats by the unassailable (hence undefendable) $(\emptyset, \Theta, \bot)$ preclude a dialectical admissible $\mathcal{E}$ from containing arguments $\mathcal{E}'$ s.t. $\texttt{Com}(\mathcal{E}') = \Theta$ (e.g., $\mathcal{E}' = \{(\Delta, \emptyset, \phi), (\Pi, \emptyset, -\phi)\}$ $(\Theta = \Delta \cup \Pi)$). It is straightforward to then see that these proofs are unaffected by u-defeats.

However, the remaining results for Dialectical *Cl-Arg* (*Sub-argument closure*, *Closure under Strict Rules* and the non-contamination postulates *Non-interference* and *Crash Resistance*) require that we show equivalence of complete extensions. In what follows, we prefix 'acceptable', 'conflict free', 'admissible' and 'complete' by u, to refer to acceptability and extensions that are defined assuming *u*nrestricted rebuts.

Lemma 1. *Let* $\mathcal{E}$ *be* u*-complete,* $\mathbb{X}$ *any dialectical argument such that* $\texttt{Com}(\mathbb{X}) \subseteq \texttt{Com}(\mathcal{E})$*, and suppose* $\neg\exists \mathbb{Y}$ *s.t.* $\mathbb{Y} \Rightarrow^{u}_{\mathcal{E}} \mathbb{X}$*. Then* $\mathbb{X} \in \mathcal{E}$*.*
Proof. **We show** $\mathbb{X}$ **is** u**-acceptable w.r.t.** $\mathcal{E}$: Suppose $\mathbb{Y} = (\Delta, \Gamma, \phi) \Rightarrow_{\mathcal{E}} \mathbb{X}$ on $X' = (\{\alpha\}, \alpha)$. Since $\alpha \in \texttt{Com}(\mathbb{X}) \subseteq \texttt{Com}(\mathcal{E})$, then $\Gamma \subseteq \texttt{Com}(\mathcal{E})$ and $\exists \mathbb{X}'' \in \mathcal{E}$ s.t. $\mathbb{Y} \Rightarrow_{\mathcal{E}} \mathbb{X}''$ on X'. Since $\mathcal{E}$ is u-complete $\exists \mathbb{Z} \in \mathcal{E}$ s.t. $\mathbb{Z} \Rightarrow^{(u)}_{\mathbb{Y}} \mathbb{Y}$. Hence $\mathbb{X}$ is u-acceptable w.r.t. $\mathbb{E}$.
Suppose for contradiction that $\mathcal{E} \cup \{\mathbb{X}\}$ **is not** u**-conflict free**. There are four cases:

1. $\mathbb{W} \in \mathcal{E}, \mathbb{Z} \in \mathcal{E}$, $\mathbb{W} \neq \mathbb{X}, \mathbb{Z} \neq \mathbb{X}$, and $\mathbb{W} \Rightarrow_{\mathcal{E} \cup \{\mathbb{X}\}^{(u)}} \mathbb{Z}$. Since $\texttt{Com}(\mathbb{X}) \subseteq \texttt{Com}(\mathcal{E})$, $\mathbb{W} \Rightarrow_{\mathcal{E}(u)} \mathbb{Z}$, contradicting $\mathcal{E}$ is u-complete and hence u-conflict free.
2. $\mathbb{W} \in \mathcal{E}$ and $\mathbb{W} \Rightarrow_{\mathcal{E} \cup \{\mathbb{X}\}} \mathbb{X}$. Since $\texttt{Com}(\mathbb{X}) \subseteq \texttt{Com}(\mathcal{E})$, $\mathbb{W} \Rightarrow_{\mathcal{E}} \mathbb{X}$. Since $\mathbb{X}$ is u-acceptable w.r.t. $\mathcal{E}$, $\exists \mathbb{Z} \in \mathcal{E}$, $\mathbb{Z} \Rightarrow_{\mathbb{W}(u)} \mathbb{W}$. Hence $\mathbb{Z} \Rightarrow_{\mathcal{E}(u)} \mathbb{W}$, contradicting $\mathcal{E}$ is u-conflict free.

[7] That is to say, the arguments with empty suppositions that define extensions of a DF; recall Definition 5.
[8] I.e., such that resources suffice to construct $(\Delta, \bot)$, signifying recognition of the inconsistency of Δ.

3. $\mathbb{W} \in \mathcal{E}$ and $\mathbb{X} \Rightarrow_{\mathcal{E}\cup\{\mathbb{X}\}^{(u)}} \mathbb{W}$. Hence, since $\mathtt{Com}(\mathbb{X}) \cap \mathtt{Sup}(\mathbb{X}) = \emptyset$ and $\mathtt{Sup}(\mathbb{X}) \subseteq \mathtt{Com}(\mathcal{E} \cup \{\mathbb{X}\})$, then $\mathtt{Sup}(\mathbb{X}) \subseteq \mathtt{Com}(\mathcal{E})$. Hence $\mathbb{X} \Rightarrow_{\mathcal{E}^{(u)}} \mathbb{W}$. Since $\mathcal{E}$ is u-complete, $\exists \mathbb{Z} \in \mathcal{E}$, $\mathbb{Z} \Rightarrow_{\mathbb{X}} \mathbb{X}$. Hence $\mathbb{Z} \Rightarrow_{\mathcal{E}} \mathbb{X}$. Since $\mathbb{X}$ is u-acceptable w.r.t. $\mathcal{E}$, $\exists \mathbb{V} \in \mathcal{E}$, $\mathbb{V} \Rightarrow_{\mathbb{Z}^{(u)}} \mathbb{Z}$ and so $\mathbb{V} \Rightarrow_{\mathcal{E}^{(u)}} \mathbb{Z}$, contradicting $\mathcal{E}$ is u-conflict free.
4. $\mathbb{X} \Rightarrow_{\mathcal{E}\cup\{\mathbb{X}\}} \mathbb{X}$. Hence $\mathbb{X} \Rightarrow_{\mathcal{E}} \mathbb{X}$. Since $\mathbb{X}$ is u-acceptable w.r.t. $\mathcal{E}$, $\exists \mathbb{W} \in \mathcal{E}$, $\mathbb{W} \Rightarrow_{\mathbb{X}} \mathbb{X}$. Hence $\mathbb{W} \Rightarrow_{\mathcal{E}} \mathbb{X}$, and so $\exists \mathbb{Z} \in \mathcal{E}$, $\mathbb{Z} \Rightarrow_{\mathbb{W}^{(u)}} \mathbb{W}$ and so $\mathbb{Z} \Rightarrow_{\mathcal{E}^{(u)}} \mathbb{W}$, contradicting $\mathcal{E}$ is u-conflict free. □

Lemma 2. *Let $\mathcal{E}$ be u-complete. Then $\forall \mathbb{X}$ such that* $\mathtt{Com}(\mathbb{X}) \subseteq \mathtt{Com}(\mathcal{E})$*, $\mathbb{X} \in \mathcal{E}$.*

Proof. We show $\mathbb{X}$ is u-acceptable w.r.t. $\mathcal{E}$. There are two cases to consider: 1) $\neg\exists \mathbb{Y}$ s.t. $\mathbb{Y} \Rightarrow^u_E X$. Then by Lemma 1, $\mathbb{X} \in E$. 2) Suppose $\mathbb{Y} \Rightarrow^u_{\mathcal{E}} X$. Hence $\mathbb{Y} = (\Delta, \Gamma, \phi)$ and $\mathbb{X} = (\Pi, \emptyset, -\phi)$ and $\Gamma \cup \Pi \subseteq \mathtt{Com}(\mathcal{E})$. By P2 we have $\mathbb{Z}_1 = (\Delta, \Gamma \cup \Pi, \bot)$ and $\mathbb{Z}_2 = (\Gamma \cup \Pi, \Delta, \bot)$. Given that $\mathtt{Com}(\mathbb{Z}_2) \subseteq \mathtt{Com}(\mathcal{E})$ and $\mathbb{Z}_2$ cannot be targeted by an unrestricted rebut, then by Lemma 1, $\mathbb{Z}_2 \in E$. By *DCPref*, either

1. $\mathbb{Z}_2 \Rightarrow_{\mathbb{Z}_1} \mathbb{Z}_1$, and since $\mathtt{Com}(\mathbb{Z}_1) = \mathtt{Com}(\mathbb{Y})$, $\mathbb{Z}_2 \Rightarrow_{\mathbb{Y}} \mathbb{Y}$, or
2. $\mathbb{Z}_1 \Rightarrow_{\mathbb{Z}_2} \mathbb{Z}_2$ and so $\mathbb{Z}_1 \Rightarrow_{\mathcal{E}} \mathbb{Z}_2$. Since $\mathbb{Z}_2 \in \mathcal{E}$ and $\mathcal{E}$ is u-complete, $\exists \mathbb{X}' \in \mathcal{E}$ s.t. $\mathbb{X}' \Rightarrow_{\mathbb{Z}_1} \mathbb{Z}_1$, and so $\mathbb{X}' \Rightarrow_{\mathbb{Y}} \mathbb{Y}$.

$\mathcal{E} \cup \{\mathbb{X}\}$ is u-conflict free is shown in exactly the same way as in Lemma 1. □

Theorem 1. *$\mathcal{E}$ is a dialectical complete extension of $(\mathcal{A}, \mathcal{C})$ iff $\mathcal{E}$ is a dialectical u-complete extension of $(\mathcal{A}, \mathcal{C})$.*

Proof. **L** $\rightarrow$ **R**: Let $\mathcal{E}$ be complete. We show $\mathcal{E}$ is u-complete.

1) Let $\mathbb{X} = (\Pi, \emptyset, -\phi \in \mathcal{E}$. An additional u-defeat $\mathbb{Y} = (\Delta, \Gamma, \phi) \Rightarrow^u_{\mathcal{E}} \mathbb{X}$ does not invalidate u-acceptability of $\mathbb{X}$ w.r.t. $\mathcal{E}$, as we have $\mathbb{Z}_1 = (\Delta, \Gamma \cup \Pi, \bot)$, $\mathbb{Z}_2 = (\Gamma \cup \Pi, \Delta, \bot)$, and so (as shown in Lemma 2) there is an argument in $\mathcal{E}$ that defeats $\mathbb{Y}$.

2) Moreover, since $\mathcal{E}$ is conflict free, it suffices to show that no $\mathbb{Y} = (\Delta, \Gamma, \phi) \in \mathcal{E}$ can u-defeat $\mathbb{X} = (\Pi, \emptyset, -\phi) \in \mathcal{E}$, which follows given the un-defendable defeats $(\emptyset, \Delta \cup \Gamma \cup \Pi, \bot) \Rightarrow_{\mathcal{E}} \mathbb{X}(\mathbb{Y})$. Hence $\mathcal{E}$ is u-conflict free.

3) Finally, we need to show that $\forall \mathbb{Z} \notin \mathcal{E}$ (i.e.,$\forall \mathbb{Z}$ not acceptable w.r.t. $\mathcal{E}$), $\mathbb{Z}$ is not u-acceptable w.r.t. $\mathcal{E}$. Suppose otherwise. Then $\mathbb{X} = (\Pi, \Sigma, -\phi) \Rightarrow_{\mathcal{E}} \mathbb{Z}$ and $\exists \mathbb{Y} \in \mathcal{E}$ s.t. $\mathbb{Y} = (\Delta, \Gamma, \phi) \Rightarrow^u_{\mathbb{X}} \mathbb{X}$, and so $\Sigma = \emptyset$, $\Gamma \subseteq \Pi$ and $\Delta \cup \Pi \vdash_{Cl} \bot$. By P2 we have $\mathbb{Z}_1 = (\Delta, \Pi, \bot)$ and $\mathbb{Z}_2 = (\Pi, \Delta, \bot)$. Since $\mathtt{Com}(\mathbb{Z}_1) = \mathtt{Com}(\mathbb{Y})$ and $\mathbb{Y} \in \mathcal{E}$, then by [16, Lemma 9]), $\mathbb{Z}_1 \in \mathcal{E}$. By *(DCPref)*, either: a) $\mathbb{Z}_1 \Rightarrow_{\mathbb{Z}_2} \mathbb{Z}_2$ or b) $\mathbb{Z}_2 \Rightarrow_{\mathbb{Z}_1} \mathbb{Z}_1$. In case a), since $\mathtt{Com}(\mathbb{Z}_2) = \mathtt{Com}(\mathbb{X}) = \Pi$, $\mathbb{Z}_1 \Rightarrow_{\mathbb{X}} \mathbb{X}$. In case b), since $\mathtt{Com}(\mathbb{Z}_1) = \mathtt{Com}(\mathbb{Y})$ then $\mathbb{Z}_2 \Rightarrow_{\mathcal{E}} \mathbb{Y}$, and since $\mathbb{Y}$ is acceptable w.r.t. $\mathcal{E}$, $\exists \mathbb{W} \in \mathcal{E}$ s.t. $\mathbb{W} \Rightarrow_{\mathbb{Z}_2} \mathbb{Z}_2$ and so $\mathbb{W} \Rightarrow_{\mathbb{X}} \mathbb{X}$. Both cases contradict $\mathbb{Z}$ is not acceptable w.r.t. $\mathcal{E}$.

R $\rightarrow$ **L**: Let $\mathcal{E}$ be u-complete. We show $\mathcal{E}$ is complete. Trivially, if $\mathcal{E}$ is u-conflict free then $\mathcal{E}$ is conflict free. Suppose some $\mathbb{Z} \in \mathcal{E}$ that is u-acceptable w.r.t. $\mathcal{E}$ but not acceptable w.r.t. $\mathcal{E}$. Then $\mathbb{X} = (\Pi, \emptyset, -\phi) \Rightarrow_{\mathcal{E}} \mathbb{Z}$ and $\exists \mathbb{Y} \in \mathcal{E}$ s.t. $\mathbb{Y} = (\Delta, \Gamma, \phi) \Rightarrow^u_{\mathbb{X}} \mathbb{X}$. But then this contradicts $\mathbb{Z}$ is not acceptable w.r.t. $\mathcal{E}$, exactly as shown in L $\rightarrow$ R (3). Suppose some $\mathbb{X} = (\Pi, \Sigma, -\phi)$ that is *not* u-acceptable w.r.t. $\mathcal{E}$ but acceptable w.r.t. $\mathcal{E}$. By assumption of acceptability w.r.t. $\mathcal{E}$: $\forall \mathbb{Y}$ s.t. $\mathbb{Y} = (\Delta, \Gamma, \phi) \Rightarrow_{\mathcal{E}} \mathbb{X}$, $\exists \mathbb{Z} \in \mathcal{E}$ s.t. $\mathbb{Z} \Rightarrow_{\mathcal{E}} \mathbb{Y}$. Hence, if *not* u-acceptable: $\mathbb{Y} = (\Delta, \Gamma, \phi) \Rightarrow^u_{\mathcal{E}} \mathbb{X}$ and $\Sigma = \emptyset$ and $\neg\exists \mathbb{Z} \in \mathcal{E}$ s.t. $\mathbb{Z} \Rightarrow_{\mathcal{E}^{(u)}} \mathbb{Y}$. But then we have $\mathbb{Z}_1 = (\Delta, \Gamma \cup \Pi, \bot)$ and $\mathbb{Z}_2 = (\Gamma \cup \Pi, \Delta, \bot)$, and we can show as in

Lemma 2 that there is an argument in $\mathcal{E}$ that defeats $\mathbb{Y}$ on a committed premise in $\mathbb{Y}$. But then this contradicts $\mathbb{X}$ is *not* u-acceptable w.r.t. $\mathcal{E}$. □

We turn now to the introduction of Occam Razor defeats. Thus far, suppositions can only reference committed premises in arguments (and their containing extensions). However, a natural dialectical move would be to criticise an argument on the grounds that it uses irrelevant or unneeded premises (which, for example, may have been incorporated to add weight or authority to an argument). It may be that an argument's premises Π:

1) Cannot be partitioned into syntactically disjoint subsets, and all of Π could have been used in a proof, but not all are needed to derive the conclusion. For example, two applications of modus ponens deriving p from: $\Pi = \{p, p \to q, q \to p)\}$.
2) Cannot be partitioned into syntactically disjoint subsets, but all of Π cannot be used to derive the conclusion. For example, concluding q from $\Pi = \{p, p \to q, p \vee q\}$.
3) Can be partitioned into syntactically disjoint subsets, where one subset entails the argument's conclusion. For example, the earlier 'redundantly contaminated' argument concluding $\neg s$ from $\Pi = \{r, \neg s\}$.

One might therefore challenge such an argument by communicating an argument for the same conclusion that relies only on the needed premises:

"*Your argument $\mathbb{Y}$ concluding α from Π, makes use of unneeded premises, as evidenced by the $\mathbb{Y}'$ that concludes α from $\Pi' \subset \Pi$, and that excludes the unneeded premises*"

Notice that the success of such an attack as a defeat is not predicated on the epistemic partitioning of $\mathbb{Y}$'s premises Π into commitments and suppositions. The so called 'Occam Razor' defeat challenges the proof theoretic means used to conclude α, independently of the partitioning of Π. Moreover, when moving $\mathbb{Y}'$, one is not committing oneself to the premises Π' in the 'non-redundant' argument; they are supposed for the sake of argument, for the sake of critiquing the use of unneeded premises in $\mathbb{Y}$.

Given the intractability of checking subset minimality, we focus only on defeats that rely on distinguishing syntactically disjoint sets of premises (case 3)), so that in addition to u-defeats' (Definition 7), one can deploy Occam Razor defeats when determining the dialectical acceptability of arguments.

Definition 7. *Let $\mathbb{Y} \in \|(\Pi, \alpha)\|$, $\Pi = \Delta \cup \Gamma$, $\Delta \| \Gamma \cup \{\alpha\}$ and $\Gamma \vdash_{Cl} \alpha$. Then $\forall \mathcal{E}$: $\mathbb{Y}' = (\emptyset, \{\Gamma\}, \alpha)$ Occam Razor defeats ('o-defeats') $\mathbb{Y}$ w.r.t. $\mathcal{E}$, denoted $\mathbb{Y}' \Rightarrow^{o}_{\mathcal{E}} \mathbb{Y}$.*

Recall (Section 2) that for any $(\Delta \cup \Gamma, \alpha)$ s.t. $\Delta \| \Gamma \cup \{\alpha\}$, then by the properties of classical logic, there exists either an argument $(\Delta, \bot)$ or an argument (Γ, α), and P3 assumes that resources suffice to construct either of these two arguments. In the former case, the contaminating effect of an explosive argument $(\Delta, \emptyset, \phi)$ (where ϕ is any propositional or first order classical formula) is counteracted by a defeat from the unassailable $(\emptyset, \Delta, \bot)$. In the latter case, non-contamination is satisfied if one either: a) deploys a 'non-contaminating' proof theory (e.g., [19,20]), else; b) one needs to assume that adding syntactically disjoint premises Δ to (Γ, α) does not strengthen the argument.

Deploying o-defeats means that one can now accommodate proof theories that generate redundantly contaminated (*rc*) arguments such as $(\{r, \neg s\}, \neg s)$, *while only assuming preference relations that satisfy DCPref.* To see why, observe that the proofs of non-contamination in [16], which assume proof theories that *do not* generate *rc* arguments, would of course also immediately apply to any DF from which *rc* arguments have been removed, so negating the use of an *rc* argument to challenge the acceptability of arguments, or defend arguments against defeats that challenge their acceptability. *o*-defeats thus achieve the same effect, effectively negating the dialectical use of *rc* arguments, so that the non-contamination proofs that assume proof theories that do not generate *rc* arguments, also establish non-contamination for Dialectical *Cl-Arg* augmented to include unrestricted rebut and Occam razor defeats.

To elaborate, suppose the acceptability of some $\mathbb{X}$ w.r.t. $\mathcal{E}$ is challenged by a defeat from some $\mathbb{Z}$. Suppose the redundantly contaminated $\mathbb{Y} = (\Theta, \Sigma, \neg s) \in \mathcal{E}$ (e.g., where $\Theta \cup \Sigma$ is *any* partition of $\{r, \neg s\}$) defeats $\mathbb{Z}$ and so defends $\mathbb{X}$. Then the unassailable 'non-redundant counter-part' $\mathbb{Y}'$ of $\mathbb{Y}$ (e.g., $\mathbb{Y}' = (\emptyset, \{\neg s\}, \neg s)$) *o*-defeats $\mathbb{Y}$ [9]. Since $\mathbb{Y}'$ cannot be defeated on its empty commitments, and so is included in any conflict free complete $\mathcal{E}$, an argument such as $\mathbb{Y}$ can never be included in such an $\mathcal{E}$, and so used to defend against defeats that challenge the acceptability of arguments w.r.t. $\mathcal{E}$. Similarly, no such $\mathbb{Y}$ can successfully challenge the acceptability of some $\mathbb{Z}$ (such as $(\{s\}, \emptyset, s)$) w.r.t. some complete $\mathcal{E}$, since $\mathbb{Y}'$'s inclusion in $\mathcal{E}$ will defend $\mathbb{Z}$ by *o*-defeating $\mathbb{Y}$.

Finally, observe that recognising that the premises of an argument such as $\mathbb{Y}$ are syntactically disjoint, is tractable (c.f. the subset minimality check). It is straightforward to define a polynomial time algorithm to compute syntactic disjointedness.

4. Conclusions

Dialectical *Cl-Arg* extends Cl-Arg so as to accommodate uses of argument characteristic of dialectical practice. This enables formalisation of maxiconsistent non-monotonic reasoning, eschewing the need to check premise subset minimality and consistency, while making only minimal assumptions (P1, P2, P3) as to the resources available for classical inference. If preference relations are dialectically coherent, and arguments are not strengthened when adding syntactically disjoint premises, all rationality postulates are satisfied. One can thus formalise maxiconsistent non-monotonic reasoning employing resource bounded approximations of classical logic (as shown for Preferred Subtheories (*PS*) in [16,17]). In this paper we have extended Dialectical *Cl-Arg* to accommodate additional dialectical uses of argument – unrestricted rebut and Occam Razor defeats – while not only demonstrating that rationality is preserved, but also dropping the non-strengthening constraint on preference relations. The latter means that one can formalise maxiconsistent non-monotonic reasoning using argument preference relations that do not satisfy this constraint. For example, the so called *Democratic* preference ordering over arguments [21] – $(\Gamma, \phi) \prec (\{\beta\}, \beta)$ if $\forall \alpha \in \Gamma : \alpha < \beta$ – does not satisfy this constraint, in contrast with the *Elitist* preferences used in formalising *PS* consequence (recall Footnote 4). Future work will incorporate these new modes of defeat in Dialectical *ASPIC*$^+$'s[10] formalisation of rational resource-bounded non-monotonic reasoning [18]. Indeed, Dialectical *ASPIC*$^+$ requires (in the case that arguments make use of default/defeasible inference rules) that arguments are neither strengthened *or weakened* when adding syntactically disjoint premises/defeasible inference rules, so that employing Occam Razor defeats will avoid this restrictive constraint on allowable preference relations.

[9] $\mathbb{Y}$ could be *any* epistemic variant of $(\{r, \neg s\}, \neg s)$: $(\{r, \neg s\}, \emptyset, \neg s)$, $(\emptyset, \{r, \neg s\}, \neg s)$, $(\{\neg s\}, \{r\}, \neg s)$ or $(\{r\}, \{\neg s\}, \neg s)$. Satisfaction of P3 implies sufficient resources for constructing $Y' = (\{\neg s\}, \neg s)$.

[10] *ASPIC*$^+$ [22] formalises maxiconsistent *nm* logics as well as *nm* logics employing default inference rules.

Finally, observe that to the best of our knowledge, unrestricted rebut and Occam Razor defeats are not formalised in other approaches to *Cl-Arg*. However, [23] does show consistency for a restricted version of $ASPIC^+$ (that constructs arguments only from antecedent free strict and defeasible inference rules) that accommodates unrestricted rebuts, under only the grounded semantics, and assuming a total order over the defeasible inference rules.

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