

Spectral properties and conditioning of IGA Galerkin and Collocation approximations of acoustic wave problems

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ABSTRACT

In recent years, several works have focused on the Isogeometric (IGA) discretization of acoustic and elastic waves [1], [2]. In our previous works, we have studied the stability and convergence properties of IGA Galerkin [3] and Collocation [4] approximations of the acoustic wave equation with proper absorbing boundary conditions using Newmark's time-advancing scheme. Since both the IGA Galerkin and Collocation mass matrices are not diagonal, the solution of the linear system at each time step is a crucial point, regardless of whether the Newmark scheme is explicit or implicit. Therefore, the main difference between explicit and implicit IGA Newmark schemes is related to the stability bounds for the time step Δt , which are only partially based on proven results, due to the lack of theoretical estimates regarding eigenvalues and conditioning of the IGA mass and stiffness matrices. An extensive numerical comparison between the conditioning of the Spectral Element Method and NURBS-based IGA Galerkin applied to the Poisson problem has been presented in [5].

In this presentation, we illustrate a detailed experimental study of the behavior of the eigenvalues and condition numbers of the mass and stiffness IGA collocation matrices for acoustic wave problems in the reference domain, varying the polynomial degree p , mesh size h , regularity k , time step Δt and parameter β of the Newmark scheme. We also present some results on the sparsity of the matrices and the eigenvalue distribution with respect to the degrees of freedom d.o.f. and the number of nonzero entries nz . Our results show that the condition numbers of the IGA collocation matrices related to acoustic waves problems with absorbing boundary conditions satisfy the same estimates that hold for the Poisson problem with Dirichlet boundary conditions discretized with IGA Galerkin, and in some cases better bounds hold for the collocation matrices. Finally, we mention some preliminary numerical results on the application of an additive overlapping Schwarz preconditioner to both IGA Galerkin and Collocation approximations, testing its performance with GMRES or preconditioned conjugate gradients iterative methods.

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