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Planetesimal formation via fragmentation in self-gravitating protoplanetary discs

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ABSTRACT

An unsolved issue in the standard core accretion model for gaseous planet formation is how kilometre-sized planetesimals form from, initially, micron-sized dust grains. Solid growth beyond metre sizes can be difficult both because the sticking efficiency becomes very small, and because these particles should rapidly migrate into the central star. We consider here how metre-sized particles evolve in self-gravitating accretion discs using simulations in which the gravitational influence of the solid particles is also included. Metre-sized particles become strongly concentrated in the spiral structures present in the disc and, if the solid to gas density ratio is sufficiently high, can fragment because of their own self-gravity to form planetesimals directly. This result suggests that planetesimal formation may occur very early in the star formation process while discs are still massive enough to be self-gravitating. The dependence of this process on the surface density of the solids is also consistent with the observation that extrasolar planets are preferentially found around high-metallicity stars.

Key words: accretion, accretion discs – gravitation – instabilities – stars: formation – planetary systems: formation – planetary systems: protoplanetary discs.

1 INTRODUCTION

There are currently two models for the formation of gaseous, Jupiter-like planets. The most widely accepted is the core accretion model (Pollack et al. 1996) in which, initially, a core of rock/ice grows via the collisional accumulation of planetesimals (Safronov 1969) and then, once sufficiently massive, it accretes a gaseous envelope (Lissauer 1993; Pollack et al. 1996). The alternative scenario does not require the initial growth of a core and assumes that protoplanetary discs may become gravitationally unstable and that gas giant planets may subsequently form via direct gravitational collapse (Boss 1998, 2000; Mayer et al. 2004).

One aspect of the core accretion model that has yet to be satisfactorily resolved is how the kilometre-sized planetesimals that ultimately coagulate to form the planetary core grow from, initially, micron-sized dust grains (e.g. Safronov 1972; Weidenschilling & Cuzzi 1993). Core accretion models generally start with the assumption that these planetesimals have already formed. However, solid growth beyond metre sizes can be very difficult, because of two effects. On the one hand, the sticking efficiency of solids becomes relatively small in this size range (e.g. Supulver et al. 1997). On the other hand, it is well known that solid particles are influenced by gas drag and, for standard disc geometries, will generally lose angular momentum and migrate in towards the central star at a rate that depends on the particle size (Weidenschilling 1977). For a circumstellar disc with properties appropriate for planet formation, the maximum inward radial velocity will generally occur for particles with sizes between 1 cm and 1 m and may easily exceed 10^3 cm s^{-1} (Weidenschilling 1977). With such large inward radial velocities, it is possible that these objects may drift into the central star before becoming large enough to decouple from the disc gas, preventing the growth of the planetesimals required for the formation of the planetary cores.

The instability model, since it does not require the formation of a core, is unaffected by the above process. This scenario does, however, require that the disc be relatively massive, and that the disc be able to cool extremely efficiently (e.g. Gammie 2001; Rice et al. 2003). Although very few Class II protostars appear to have discs with the required mass (Beckwith & Sargent 1991), recent observations suggest that massive discs may be present during the Class 0 (Rodriguez et al. 2005) and Class I (Eisner et al. 2005) phases. Although this suggests that massive discs may be present at some stage during the star formation process, it is still not clear that such discs can cool sufficiently rapidly for the direct formation of gas giant planets.

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Even if gas giant planets cannot form via direct gravitational collapse in discs around Class 0 and Class I protostars, it is still likely that these discs will experience self-gravitating phases in which spiral structures will develop (Lin & Pringle 1987). The gas pressure on the inner edges of these spiral waves increases with radius, leading to super-Keplerian gas velocity. The drag force between the solid particles and the gas then results in the dust grains and small particles drifting towards the peaks of the spiral structures (Haghighipour & Boss 2003), rather than simply drifting towards the central star. Rice et al. (2004) have already shown, using numerical simulations, that for certain particles sizes, the local density could be enhanced by a factor of 10 or more. A similar effect would occur in the presence of rings (Durisen et al. 2005) or vortices (Godon & Livio 2000; Klahr & Bodenheimer 2003), and it has also been shown that density variations in the disc gas resulting from magnetorotational turbulence can also produce significant enhancements in particle densities (Johansen, Klahr & Henning 2006).

Particle density enhancements may aid planetesimal formation in two ways. First, the enhanced collision rate (Rice et al. 2004) could aid collisional grain growth; secondly, the densities achieved may be sufficient for planetesimal formation through direct gravitational collapse of the solid component of the disc (Goldreich & Ward 1973; Youdin & Shu 2002). It is this second possibility that we investigate in this paper. We extend the work of Rice et al. (2004) to include the gravitational influence of the solid particles. We consider three-dimensional, global simulations of self-gravitating accretion discs in which the gas is maintained in a state of marginal gravitational instability and is coupled to the solid particles via a drag force. We include the gravitational influence of both the gas and the solid particles self-consistently, and we consider different particle sizes and different initial gas to solid particle surface density ratios. We find that if the initial particle surface density is sufficiently high, a significant fraction of the solid component of the disc, which is strongly concentrated in the self-gravitating spiral waves, can become unstable and subsequently fragment into bound objects. We therefore conclude that what used to be considered the most difficult step in planetesimal formation (i.e. the growth beyond metre sizes) could actually be very rapid, thus leading directly to kilometre-sized objects.

2 NUMERICAL SIMULATIONS

The simulations performed here are very similar to those of Rice et al. (2004). We consider a system comprising a point mass, representing the central star, surrounded by two interpenetrating discs, a gas disc and a 'planetesimal' disc.

2.1 The gas disc

The three-dimensional gaseous disc is modelled using smoothed particle hydrodynamics (SPH), a Lagrangian hydrodynamics code (Benz 1990; Monaghan 1992), and is represented by 250 000 pseudo-particles. Each particle has a mass, an internal energy and a smoothing length that varies with time to ensure that the number of neighbours (SPH particles within two smoothing lengths) remains ∼50. These neighbouring particles are used to determine the gas density which, together with the internal energy, is used to determine the gas pressure. A tree is used to determine gravitational forces, and to determine the gas particle neighbours.

The simulation starts by considering only the gaseous disc which has a mass $M_{\rm disc} = 0.25\,{\rm M}_\odot$ and extends from 0.25 to 25 au around a 1-$\rm M_\odot$ central star. The initial surface density profile is $\Sigma_{\rm gas} \propto R^{-1}$, the initial temperature profile is $T \propto R^{-0.5}$, and the temperature is normalized such that the disc is initially gravitationally stable at all radii (Toomre 1964). The disc gas is allowed to heat up through both $P\,{\rm d}V$ work and viscous dissipation with the viscosity given by the standard SPH viscosity (Monaghan 1992). In the absence of cooling the gas has an adiabatic equation of state with $\gamma = 5/3$.

The disc gas is cooled with a radially dependent cooling time of $t_{\rm cool} = \beta\Omega^{-1}$, where Ω is the orbital angular frequency and β is a constant that determines the cooling rate relative to the angular frequency. In thermal equilibrium, the cooling time and the viscous α-parameter (Shakura & Sunyaev 1973), which characterizes angular momentum transport, are related through

$$\alpha = \frac{4}{9\gamma(\gamma-1)}\frac{1}{t_{\rm cool}\Omega}, \qquad (1)$$

where γ is the ratio of the specific heats. It has been shown (Rice, Lodato & Armitage 2005) that if the value of α required for thermal equilibrium is too large ($\alpha \gtrsim 0.06$), the disc will fragment rather than settle into a quasi-steady, long-lived state. We therefore use $\beta = 7.5$ which ensures that the related α-value is small enough for the disc, in the simulations presented here, to settle into a long-lived, quasi-steady, self-gravitating state (Gammie 2001; Rice et al. 2003; Lodato & Rice 2004, 2005; Rice et al. 2005). The 'planetesimal' disc is added to the simulation once this long-lived state has been achieved.

2.2 The 'planetesimal' disc

The 'planetesimal' disc is also modelled using SPH and is represented by 125 000 pseudo-particles. It is, however, assumed to be pressureless and hence the solid particles have no internal energy and experience only gravitational forces, and a drag force due to the difference between their velocity and the velocity of the disc gas (Weidenschilling 1977). As discussed in detail in Rice et al. (2004), the drag force depends on the particle size, the local gas density, the local gas velocity and the local gas sound speed. To determine the drag force each 'planetesimal' particle has a 'smoothing length' that is varied to ensure that the number of gas particle neighbours remains ∼50. These neighbouring particles are then used to calculate the gas density, velocity and internal energy at the location of every 'planetesimal' particle using the standard SPH formalism (Monaghan 1992). The exact value of the drag force is then determined by specifying the size of the solid particle. In each simulation, the 'planetesimal' disc is assumed to contain particles of a single size.

The primary difference between the work presented here and that presented by Rice et al. (2004) is that we include here the gravitational influence of the solid particles, rather than treating them as test particles. As with the gas particles, the gravitational force due to the 'planetesimal' particles is determined by including them in the tree. The details of the tree method can be found in Benz (1990), but essentially distant particles are grouped into nodes and their gravitational force is computed from the position of the node and its quadrupole correction. Very nearby particles are included in the neighbour list of the particle being considered, and the force due to these neighbouring particles is calculated directly. A direct gravitational force calculation that includes a 'planetesimal' particle is also softened. This is needed because 'planetesimal' particles do not feel any pressure forces and hence may undergo extremely close encounters with other particles. In most of the simulations we used a softening of 10^{-2} au, but repeated some simulations using 10^{-1} and 10^{-3} au.

The 'planetesimal' disc is only introduced once the gas disc has settled into a long-lived, quasi-steady, self-gravitating state. In this self-regulated state, the gravitational stability parameter $Q \sim 1$, and the disc temperature increases with radius from ~30 K in the inner disc to ~150 K in the outer disc. The disc temperature is therefore well below the dust sublimation temperature of ~1600 K and we would not expect grain evaporation to be an important process in the self-gravitating regions of the disc. It should, however, be noted that the magnitude of the temperature and the temperature profile are largely determined by our choice of disc mass and surface density profile. However, it would require a disc almost two orders of magnitude more massive than the one in these simulations for grain evaporation to be an important effect. A slightly more massive disc could, however, have a temperature that may be high enough to influence the condensation of ices.

The 'planetesimal' disc is initially located only in the mid-plane ($z = 0$) and extends from 2 to 20 au. The particles are distributed randomly in such a way as to give the initial surface density profile of $\Sigma_{\rm pl} \propto R^{-1}$, the same as that of the initial gas disc. An initial spiral structure is not introduced into the 'planetesimal' disc. Although the disc is initially located only in the mid-plane, it is given an isotropic velocity dispersion with a magnitude of 0.1 $c_{\rm s}$ where $c_{\rm s}$ is the local gas sound speed. This ensures that the planetesimal disc is, at the start of the simulation, gravitationally stable. The 'planetesimal' particles are free to move vertically allowing the disc to achieve a finite thickness, which turns out to be comparable to the gas disc thickness.

Once the 'planetesimals' have been added, the simulation is evolved for about an additional outer rotation period (~125 yr). We consider particle sizes of 150 and 1500 cm, and gas to dust surface density ratios of 100 and 1000.

3 SIMULATION RESULTS

As discussed above, the simulation initially considers only the gaseous disc component. This is evolved until it settles into a marginally stable, self-gravitating state (Lodato & Rice 2004). In this quasi-steady state, the instability produces spiral-like structures in the disc. This is illustrated in Fig. 1 which shows the surface density structure of the gaseous disc, once it has reached this long-lived, self-gravitating state. Although it is not clear that a protostellar disc could be self-gravitating at radii of a few au, it is quite likely that it plays a vital role in transporting angular momentum in the earliest phases of star formation (Lin & Pringle 1990). Bell & Lin (1994) also invoke self-gravity at ~1 au in their models of FU Orionis outbursts, and the presence of a dead zone (Gammie 1996; Armitage, Livio & Pringle 2001) could also allow a self-gravitating region to extend to small radii.

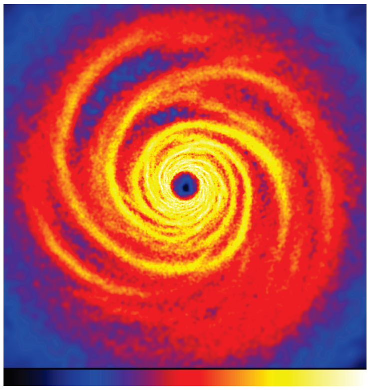

Figure 1. Surface density structure of a 0.25-M$_\odot$ disc around a 1 - M$_\odot$ star that has reached a long-lived, self-gravitating state. The scale shows the logarithm of the surface density, $\Sigma_{\rm gas}$, with the scale covering $2.4 < \log\ (\Sigma_{\rm gas}/{\rm g\,cm^{-2}}) < 4.4$.

Once the gas disc has evolved into this self-gravitating state, we introduce the 'planetesimal' disc as discussed in Section 2. For the gas disc that we consider here, the largest radial drift rates will occur for particles with sizes of ~100 cm (see Rice et al. 2004). It is particles of this size that we expect to become the most strongly concentrated in the spiral structures (Haghighipour & Boss 2003; Rice et al. 2004). Particles significantly larger, or significantly smaller, would not be as strongly influenced by the spiral structures.

We consider initially 150-cm particles with a surface density 100 times smaller than that of the gas disc. Fig. 2 shows the surface density structure of this disc ~80 yr after these particles are introduced. The colour scale differs from that in Fig. 1 because of the different initial surface densities. What is clear from Fig. 2 is that the spiral structures have fragmented into clumps. These clumps are bound and have densities significantly larger than that of the disc gas. We also find fragmentation throughout the planetesimal disc. At the time shown in Fig. 2, the innermost clump is at $r = 2.1$ and the outermost is at $r = 16.2$. It is likely that the outermost regions of the 'planetesimal' disc would also produce clumps if we were to run the simulation further.

The clump formation starts extremely quickly. Fig. 3 shows the fraction of solid particles that have a local density exceeding 6×10^{-8} g cm^{-3}. This threshold density exceeds the gas density everywhere in the disc and so ensures that these particles are all located in dense clumps. The first evidence of clump formation occurs ~20 yr after the solid particles are introduced into the simulation, and almost 15 per cent of the 'planetesimal' particles are located in dense clumps after ~80 yr. Although this gives some idea of the efficiency of the process, we were unable to continue the simulation much further because of the exceedingly small time-steps at which particles in dense clumps have to be advanced. We therefore cannot assess if the clump fraction levels off or continues to increase. The efficiency will also probably depend on the structures within the gas

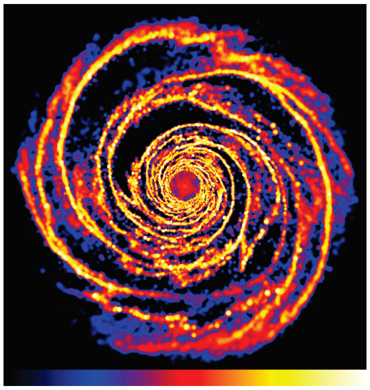

Figure 2. Surface density structure of the 'planetesimal' disc consisting of 150-cm particles ~80 yr after the 'planetesimal' disc is introduced into the simulation. The scale shows the logarithm of the surface density, $\Sigma_{\rm pl}$, with the scale covering $-0.6 < \log\ (\Sigma_{\rm pl}/{\rm g\,cm^{-2}}) < 2.4$. In this simulation, the spiral structures in the 'planetesimal' disc have fragmented producing bound clumps.

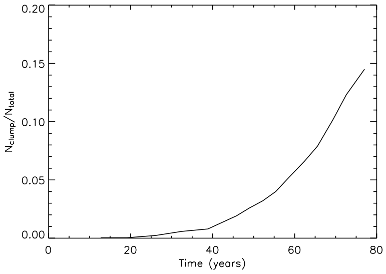

Figure 3. Fraction of 'planetesimal' particles located in clumps with densities exceeding 6×10^{-8} g cm^{-3}. The first sign of clump formation occurs about 20 yr after the solid particles are introduced into the simulation with about 15 per cent of the 'planetesimal' particles located in dense clumps after ~80 yr.

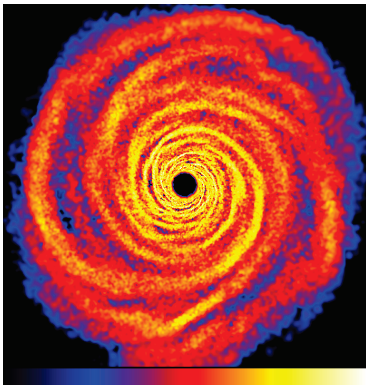

Figure 4. Surface density structure of the 'planetesimal' disc consisting of 1500-cm particles ~100 yr after the 'planetesimal' disc is introduced into the simulation. The scale shows the logarithm of the surface density, $\Sigma_{\rm pl}$, with the scale covering $-0.6 < \log{(\Sigma/{\rm g\,cm^{-2}})} < 2.4$.

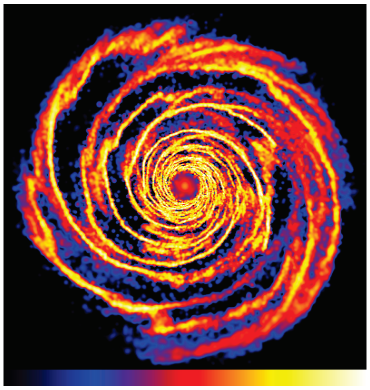

Figure 5. Surface density structure of the 'planetesimal' disc consisting of 150-cm particles ~80 yr after the 'planetesimal' disc is introduced into the simulation. The scale shows the logarithm of the surface density, $\Sigma_{\rm pl}$, with the scale covering $-1.6 < \log{(\Sigma_{\rm pl}/{\rm g\,cm^{-2}})} < 1.4$.

disc. A more massive disc that is likely to excite lower order modes will probably have a different efficiency from a lower mass disc that excites higher order modes (Laughlin & Różyczka 1996; Lodato & Rice 2004).

The above result shows that the particles that become strongly concentrated in the self-gravitating, gaseous, spiral structures may achieve densities that could lead to planetesimal formation through direct gravitational collapse. To test that this is not simply a numerical effect, we carried out some additional simulations. We repeated the above simulation, changing the gravitational softening for the solids to 10^{-1} and 10^{-3} au. To a certain extent, softening acts to effectively smooth out the mass distribution. If the softening is too large, this can act to suppress clumping. If it is too small, the 'graininess' of the mass distribution can exacerbate the gravitational dynamics, increasing the velocity dispersion of the particles. In our simulations, the particle separation at the beginning of clump formation was $\sim 10^{-2}$ au, comparable to our original softening of 10^{-2} au. Although clumps formed for all of the softenings considered, the maximum clump density was much lower when the largest softening was used, consistent with clump formation being suppressed for large softenings. For the smallest softening considered, clump formation was delayed relative to the larger softenings. This delay in fragmentation is, as mentioned earlier, a consequence of the increased velocity dispersion of the solid particles due to the softening not being large enough to effectively reduce the 'graininess' of the particle distribution. We also performed a simulation with 1500-cm particles, also with an initial surface density 100 times smaller than that of the disc gas, and another with 150-cm particles, but with an initial surface density 1000 times smaller than that of the disc gas. In both of these simulations the gravitational softening was 10^{-2} au.

Fig. 4 shows the surface density structure of the simulation with 1500-cm particles. As illustrated by Rice et al. (2004), particles of this size are largely decoupled from the disc gas, are essentially unaffected by gas drag, and hence do not become significantly concentrated in the gaseous spiral structures. The spiral structures in the 'planetesimal' disc effectively match that of the gas disc and are produced primarily by the gravitational potential of the gaseous disc, and not by gas drag.

Fig. 5 shows the surface density structure of the simulation with 150-cm particles, with an initial surface density 1000 times smaller than that of the gas disc. In this case, the particles do become concentrated in the gaseous spiral structures, producing very narrow spiral structures in the 'planetesimal' disc. However, unlike the simulation with the higher initial surface density, the densities achieved do not, in this simulation with the lower initial surface density, lead to fragmentation and clump formation in the 'planetesimal' disc. It appears, therefore, that the gravitational collapse of solids requires particles that will become strongly concentrated in the spiral density and that have a sufficiently high initial surface density, relative to the surface density in the gas disc.

4 DISCUSSION AND CONCLUSIONS

The simulations presented here extend the work of Rice et al. (2004) which showed that particles of a particular size (in their case 100-cm particles) could become very strongly concentrated in the spiral structures present in a quasi-steady, self-gravitating gaseous protoplanetary disc.

In this work we show that if the initial 'planetesimal' surface density is sufficiently high, the 'planetesimal' disc may undergo fragmentation to produce bound objects. It is this process that may provide a mechanism for producing kilometre-sized and larger planetesimals. In the simulations here each bound fragment consists of ~100 pseudo-particles and hence they have masses of $\sim 0.5 {\rm M}_{\rm earth}$. These clumps are extremely strongly bound with the magnitude of the gravitational potential energy many times greater than the kinetic energy. It is therefore likely that each bound clump may represent a number of smaller fragments, rather than a single bound object.

We are therefore not suggesting that this process would lead, immediately, to Earth-mass-like objects, but simply that the densities and velocity dispersions of the particles concentrated in the self-gravitating spiral can lead to fragmentation of the 'planetesimal' disc.

We should stress, however, that the process here differs considerably from the standard Goldreich & Ward (1973) mechanism (see also Youdin 2005) which requires a low velocity dispersion for fragmentation of the 'planetesimal' disc. In our simulations, fragmentation occurs when the velocity dispersion of the particles is comparable to the sound speed of the disc gas. It is the enhanced density that leads to the fragmentation, rather than the reduced velocity dispersion.

It does appear, however, that for fragmentation to occur, the surface density of particles in the appropriate size range must be relatively high. This could occur if, while the disc is still massive, particle growth up to ∼metre sizes occurs through collisional growth. Growth beyond metre sizes may, however, be difficult because of the sticking efficiency becoming relatively small (Supulver et al. 1997), and there would therefore be a pile-up of these particles. Once the surface density of these particles reached a critical value, subsequent growth would then occur through gravitational collapse triggered by the density of these particles being enhanced in the spiral structures of the gaseous disc. Additionally, the dependence of this process on the solid to gas ratio is consistent with planet formation being more likely in higher metallicity environments (Santos, Israelian & Mayor 2004; Fischer & Valenti 2005).

The process described here potentially solves a major problem in the standard planet formation scenario. Rather than rapidly migrating into the central star, ∼metre-sized particles become concentrated in self-gravitating spiral structures. The densities achieved can then lead to planetesimal formation via direct gravitational collapse. In this scenario, kilometre-sized planetesimals form very early, removing a major bottleneck in the planet formation process.

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