Mon. Not. R. Astron. Soc. **364,** L56–L60 (2005) doi:10.1111/j.1745-3933.2005.00105.x

Investigating fragmentation conditions in self-gravitating accretion discs

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Accepted 2005 September 14. Received 2005 August 23; in original form 2005 June 14

ABSTRACT

The issue of fragmentation in self-gravitating gaseous accretion discs has implications both for the formation of stars in discs in the nuclei of active galaxies, and for the formation of gaseous planets or brown dwarfs in circumstellar discs. It is now well established that fragmentation occurs if the disc is cooled on a time-scale smaller than the local dynamical time-scale, while for longer cooling times the disc reaches a quasi-steady state in thermal equilibrium, with the cooling rate balanced by the heating due to gravitational stresses. We investigate here how the fragmentation boundary depends on the assumed equation of state. We find that the cooling time required for fragmentation increases as the specific heat ratio γ decreases, exceeding the local dynamical time-scale for $\gamma = 7/5$. This result can be easily interpreted as a consequence of there being a maximum stress (in units of the local disc pressure) that can be sustained by a self-gravitating disc in quasi-equilibrium. Fragmentation occurs if the cooling time is such that the stress required to reach thermal equilibrium exceeds this value, independent of γ. This result suggests that a quasi-steady, self-gravitating disc can never produce a stress that results in the viscous α parameter exceeding $\sim$0.06.

Key words: accretion, accretion discs – gravitation – instabilities – stars: formation – galaxies: active – galaxies: spiral.

1 INTRODUCTION

It is becoming clearer that self-gravity may play an important role in the dynamical evolution of accretion discs. Active galactic nuclei (AGN) often show rotation curves that depart significantly from a Keplerian profile (Greenhill & Gwinn 1997; Lodato & Bertin 2003; Kondratko, Greenhill & Moran 2005), while protostellar disc masses may be a significant fraction of the central object mass during the early stages of star formation (Larson 1984; Lin & Pringle 1987). For example, a recent radio observation of a Class 0 object (a very young protostellar source, with age $<10^5$ yr and mass $M_\star \approx 0.8\,\mathrm{M}_\odot$, Rodriguez et al. 2005) shows a disc with a estimated mass $M_{\rm disc} =$ 0.3–0.4 $\mathrm{M}_\odot$ and a very suggestive two-armed spiral-like structure. In addition, there are also clues that massive accretion discs can be present around massive protostars (Beltran et al. 2004; Chini et al. 2004).

An important effect of disc self-gravity is that it provides an efficient mechanism for transporting angular momentum outwards, allowing mass to accrete on to the central object (Lin & Pringle 1987; Laughlin & Bodenheimer 1994). In protostellar discs, in which the ionization level is expected to be low, thus inhibiting magnetohydrodynamics-driven turbulence (Matsumoto & Tajima 1995; Gammie 1996), this may be the dominant mechanism, at least in certain regions of the disc. In general, however, the transport associated with self-gravitating disturbances may not be well described as a simple diffusion mechanism. Balbus & Papaloizou (1999) have shown that, for self-gravitating discs, the energy flux contains non-local terms that they associate with wave energy transport. On the other hand, Lodato & Rice (2004, 2005) have shown that, for a self-gravitating accretion disc in thermal equilibrium, the dissipation arising from gravitational stresses agrees reasonably well with the expectations based on the standard viscous theory (Shakura & Sunyaev 1973; Pringle 1981) (see, however, Mejia et al. 2005 for a different result).

In the simulations performed by Lodato & Rice (2004, 2005), thermal equilibrium is achieved by allowing the disc to heat up through $P\,\mathrm{d}V$ work and shock dissipation, and cooling it at a prescribed rate. They use a simple cooling term with a cooling time given by $t_{\rm cool} = \beta\Omega^{-1}$, where Ω is the angular frequency and $\beta =$ 7.5. In thermal equilibrium it can be shown (Pringle 1981; Gammie 2001) that, for a viscous disc, the parameter α (Shakura & Sunyaev 1973), which characterizes angular momentum transport, and the cooling time, $t_{\rm cool}$, are related through

$$\alpha = \frac{4}{9\gamma(\gamma-1)}\frac{1}{t_{\rm cool}\Omega}, \tag{1}$$

where γ is the ratio of the specific heats.

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The ultimate evolution of a self-gravitating accretion disc depends strongly on the rate at which the disc heats up, through the growth of the instability, and on the rate at which it cools. It has been shown, using a local, two-dimensional model (Gammie 2001), that if $t_{\rm cool} < 3\Omega^{-1}$, the disc will fragment into bound objects rather than evolve into a quasi-steady state, a result that was largely confirmed by Rice et al. (2003a) using global, three-dimensional models. This process has been suggested as a mechanism both for forming gaseous planets in protostellar discs (Kuiper 1951; Boss 1998; Mayer et al. 2004), and for forming stars in active galactic discs (Goodman & Tan 2004).

A major obstacle to the formation of objects via disc fragmentation is the requirement that the cooling time be smaller than the local dynamical time (Gammie 2001; Rice et al. 2003a). This may be possible in AGN discs (Johnson & Gammie 2003), but appears unlikely in protostellar discs (Rafikov 2005). The cooling time requirement was, however, determined using equations of state with specific heat ratios of $\gamma = 2$ (Gammie 2001) and $\gamma = 5/3$ (Rice et al. 2003a). It has been suggested (Lodato & Rice 2005) that since the stress required to balance the cooling rate (as measured by the viscous α) depends on the specific heat ratio, γ (see equation 1), the cooling time required for fragmentation may also depend on γ. In particular, if there is a maximum stress $\alpha_{\rm max}$ (in units of the local disc pressure) sustainable by a self-gravitating disc, then equation (1) states that fragmentation should occur if

$$t_{\rm cool}\Omega < \frac{4}{9\gamma(\gamma-1)}\frac{1}{\alpha_{\rm max}}. \qquad (2)$$

In this Letter we investigate, using global, three-dimensional simulations, how the fragmentation boundary (as measured by $t_{\rm cool}$) varies for different values of γ. We also consider various disc masses to study the suggestion by Rice et al. (2003a) that the fragmentation boundary may depend on the ratio of the disc mass to the mass of the central object. In Section 2, we describe the range of cooling times, disc masses and specific heat ratios, γ, that have been considered, and determine, for a given disc mass, the cooling time required for fragmentation. In Section 3 we discuss these results in light of the relationship between the stresses in the disc (as measured by the viscous α) and the imposed cooling time. We show that fragmentation is indeed easier in discs with smaller specific heat ratios. We therefore conclude that there is a maximum stress sustainable by a self-gravitating disc, and we quantify this maximum stress to be $\alpha_{\rm max} \approx 0.06$. In Section 4 we discuss our results and draw our conclusions.

2 SIMULATION RESULTS

The simulations performed here are very similar to those of Rice et al. (2003a) and Lodato & Rice (2004, 2005). The three-dimensional, gaseous discs are modelled using smoothed particle hydrodynamics (SPH) (Benz 1990; Monaghan 1992), a Lagrangian hydrodynamics code. The disc is represented by 250 000 SPH particles, while the central star is a point mass on to which gas particles may accrete if they approach to within the accretion radius (here taken to be at a radius of $R_{\rm acc} = 0.25$). In code units, the disc extends from $R_{\rm in} = 0.25$ to $R_{\rm out} = 25$, and the central object has a mass of $M_* = 1$. We consider disc masses of $M_{\rm disc} = 0.1$, 0.25 and 0.5, with initial surface density profiles of $\Sigma \propto R^{-1}$, and with a temperature that has a radial profile of $T \propto R^{-0.5}$. With these initial surface density and temperature profiles, the Toomre stability parameter, $Q = c_{\rm s}\Omega/\pi G\Sigma$, is not initially constant, but decreases with increasing radius. The temperature is therefore normalized such that at the beginning of the simulation the disc is stable, with a minimum $Q = 2$ at $R = 25$.

Since we are interested in how cooling influences the disc evolution, the disc gas is allowed to heat up owing to both $P\mathrm{d}V$ work and viscous dissipation, with the viscosity given by the standard SPH artificial viscosity (Monaghan 1992) with $\alpha_{\rm SPH} = 0.1$, and $\beta_{\rm SPH} = 0.2$. We use an adiabatic equation of state and consider specific heat ratios of $\gamma = 5/3$ and 7/5. A particle with internal energy per unit mass $u_{\rm i}$ is then cooled using

$$\frac{\mathrm{d}u_{\rm i}}{\mathrm{d}t} = -\frac{u_{\rm i}}{t_{\rm cool}}, \qquad (3)$$

where $t_{\rm cool} = \beta\Omega^{-1}$.

An important numerical issue to be considered in this context is the role of artificial SPH viscosity. The growth and the saturation of gravitational instabilities depend on the balance between external cooling and internal heating provided by the instability itself. Therefore we have to be sure that dissipation is dominated by gravitational instabilities rather than by artificial viscosity (which would provide an extra, undesired, stabilization term in the energy balance). It can be shown (Artymowicz & Lubow 1994; Murray 1996) that artificial SPH viscosity scales as $\nu_{\rm SPH} \propto \alpha_{\rm SPH}\langle h\rangle$, where $\langle h\rangle$ is the average SPH smoothing length. We therefore should achieve a high resolution (in order to keep the smoothing length small) and adopt a sufficiently low value of $\alpha_{\rm SPH}$, while preserving the ability of the code to resolve properly the shocks that arise in the simulation. We have already shown (Lodato & Rice 2004, appendix) that, with the setup described above ($N = 250\,000$, $\alpha_{\rm SPH} = 0.1$ and $\beta_{\rm SPH} = 0.2$), we are indeed able to resolve the shocks properly and to have an artificial dissipation smaller by more than one order of magnitude with respect to gravitationally induced dissipation. We are therefore confident that artificial viscosity is not going to affect our conclusions significantly.

For each disc mass, and for each γ, we have performed a large number of simulations with different values of β. We initially start with a β value that should result in fragmentation (Gammie 2001; Rice et al. 2003a). We stop the simulation once at least one clump/fragment has formed that has a density 2–3 orders of magnitude greater than that of the surrounding gas. The densest clump is then tested to check if it is bound. First, we determine the approximate size of the clump, by finding the distance from the centre of the clump at which the density has returned to a value comparable to that of the surrounding disc. All the particles within this spherical volume are then assumed to be part of this clump. In every case in which fragmentation occurred, the densest clump consisted of at least 100 particles, and in some cases as many as 500 particles. This more than satisfies the Jeans criterion (Bate & Burkert 1997), and we are therefore confident that the fragmentation in these simulations is not artificial. Once the clump size has been determined, we then calculate the total thermal energy and the gravitational potential energy. If the net energy is negative the clump is bound, the simulation is stopped, and a new simulation is started with the same initial conditions, but with $\beta_{\rm new} = \beta_{\rm old} + 1$. This new simulation is then run either until it also fragments or until the disc has evolved into a quasi-steady state (Gammie 2001; Rice et al. 2003a) without any signs of fragmentation. We also run the non-fragmenting simulations approximately an outer rotation period longer than the equivalent simulation that did undergo fragmentation.

Fig. 1 illustrates the procedure discussed above. The disc mass in all four figures is $M_{\rm disc} = 0.1$, the specific heat ratio is $\gamma = 5/3$, and the cooling times are $t_{\rm cool}\Omega = 3$ (top left), $t_{\rm cool}\Omega = 5$ (top right), $t_{\rm cool}\Omega = 6$ (bottom left) and $t_{\rm cool}\Omega = 7$ (bottom right). In

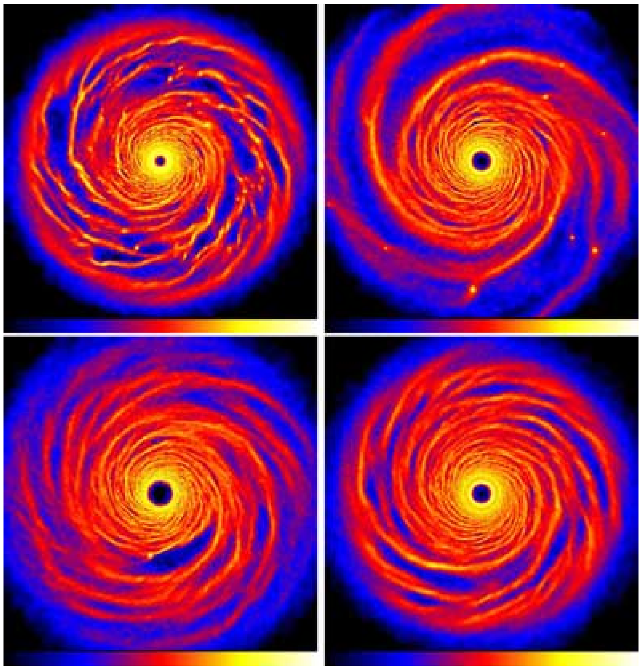

Figure 1. Surface density structure of discs with masses $M_{\rm disc} = 0.1$ and with cooling times of $t_{\rm cool}\Omega = 3$ (top left), $t_{\rm cool}\Omega = 5$ (top right), $t_{\rm cool}\Omega = 6$ (bottom left) and $t_{\rm cool}\Omega = 7$ (bottom right). The logarithmic colour scale in each figure is from 10 to 2×10^4 g cm $^{-2}$. The linear scale is from -25 to 25 for both axes.

this particular case we did not complete a $t_{\rm cool}\Omega = 4$ run since the $t_{\rm cool}\Omega = 5$ run had already shown signs of fragmentation prior to the completion of the $t_{\rm cool}\Omega = 4$ simulation. For $t_{\rm cool}\Omega = 3$ there are a large number of fragments, consistent with Rice et al. (2003b). For $t_{\rm cool}\Omega = 5$ there are a number of fragments, while for $t_{\rm cool}\Omega = 6$ there is only a single fragment that in the image can be seen just below the central object. For $t_{\rm cool}\Omega = 7$, which ran for almost 7 outer rotation periods, the disc is clearly unstable at all radii, but there are no signs of fragmentation. We repeated the above procedure for disc masses of $M_{\rm disc} = 0.25$ and 0.5, and for specific heat ratios of $\gamma = 5/3$ and $7/5$. The results are summarized in Table 1. The columns in Table 1 are the ratio of disc to central object mass, $M_{\rm disc}/M_*$, the specific heat ratio, γ, the cooling time, $t_{\rm cool}\Omega$, and, if fragmentation occurs, the total energy (in code units) of the densest clump, $E_{\rm tot}$, where $E_{\rm tot}$ is the sum of the thermal energy and gravitational potential energy (Bate, Bonnell & Price 1995). In the earlier work of Rice et al. (2003a) there was a suggestion that the cooling time required for fragmentation may depend on the total disc mass, relative to the mass of the central object. The results shown in Table 1 suggest that there is no disc mass dependence. Fragmentation occurs for $t_{\rm cool}\Omega$ between 6 and 7 when $\gamma = 5/3$, and between 12 and 13 when $\gamma = 7/5$, for all disc masses considered. The reason why there is a difference from the results of Rice et al. (2003a) is unclear. Their discs had slightly steeper surface density profiles ($\Sigma \propto R^{-7/4}$ rather than $\Sigma \propto R^{-1}$), and it is possible that their $t_{\rm cool}\Omega = 5$ simulation, which did not fragment, may have done so had it been run for longer. Table 1 also shows that in all the cases where clumps were detected, the total energy of the densest clump is negative and that at least the densest clump is bound.

Although Table 1 shows that the fragmentation boundary occurs for cooling times longer than that predicted by Gammie (2001), for $\gamma = 5/3$ the required cooling time is still smaller than the local dynamical time. It also shows that as the specific heat ratio decreases, the required cooling time increases and is almost twice the local dynamical time for $\gamma = 7/5$. The fragmentation boundary for a disc mass of $M_{\rm disc} = 0.25$ and for both of the specific heat ratios considered is shown in Figs 2 and 3. Fig. 2 shows the final surface density structures for $M_{\rm disc} = 0.25$, a specific heat ratio of $\gamma = 5/3$, and cooling times of $t_{\rm cool}\Omega = 6$ (left-hand panel) and $t_{\rm cool}\Omega = 7$ (right-hand panel). The $t_{\rm cool}\Omega = 7$ simulation was evolved for almost an outer rotation period longer than the $t_{\rm cool}\Omega = 6$ simulation, yet shows no signs of fragmentation. The discs shown in Fig. 3 have the same parameters as in Fig. 2 except that $\gamma = 7/5$, and the cooling times are $t_{\rm cool}\Omega = 12$ and 13. Again there is no sign of fragmentation in the right-hand panel which was also evolved for almost an outer rotation period longer than the simulation shown in the left-hand panel.

Table 1. Results of a series of simulations considering discs with masses between $M_{\rm disc} = 0.1$ and 0.5, specific heat ratios of $\gamma = 5/3$ and 7/5, and various cooling times. These results suggest that the fragmentation boundary does not depend on disc mass, and that for $\gamma = 7/5$ fragmentation may occur for cooling times almost twice the local dynamical time.

$M_{\rm disc}/M_*$	γ	$t_{\rm cool}\Omega$	$E_{\rm tot}$
0.1	5/3	3	-9.7×10^{-7}
0.1	5/3	5	-1.0×10^{-7}
0.1	5/3	6	-3.8×10^{-5}
0.1	5/3	7	no clumps
0.1	7/5	11	-8.8×10^{-7}
0.1	7/5	12	-6.6×10^{-8}
0.1	7/5	13	no clumps
0.25	5/3	5	-9.4×10^{-6}
0.25	5/3	6	-3.0×10^{-7}
0.25	5/3	7	no clumps
0.25	7/5	11	-8.2×10^{-7}
0.25	7/5	12	-7.2×10^{-7}
0.25	7/5	13	no clumps
0.5	5/3	6	-4.9×10^{-5}
0.5	5/3	7	no clumps
0.5	7/5	11	-1.0×10^{-5}
0.5	7/5	12	-7.5×10^{-6}
0.5	7/5	13	no clumps

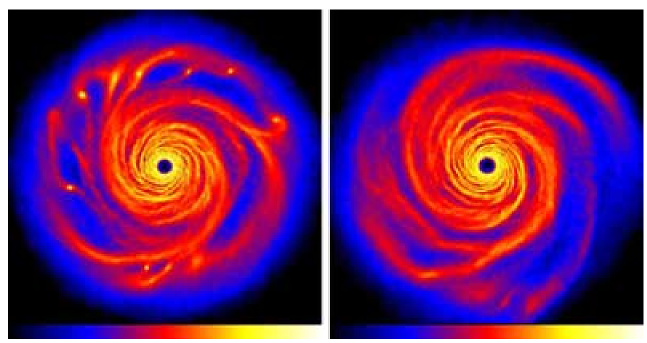

Figure 2. Surface density structure of discs with a mass of $M_{\rm disc} = 0.25$, a specific heat ratio of $\gamma = 5/3$, and cooling times of $t_{\rm cool}\Omega = 6$ (left-hand panel) and $t_{\rm cool}\Omega = 7$ (right-hand panel). The lack of fragmentation in the right-hand panel suggests that the fragmentation boundary is at a cooling time of between $t_{\rm cool}\Omega = 6$ and 7. The colour scale of the density and the linear scale of the image are the same as in Fig. 1.

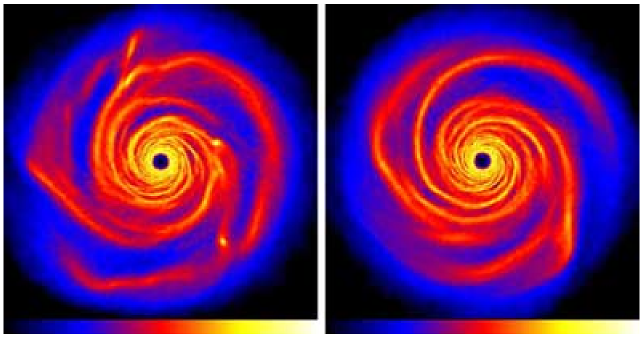

Figure 3. Surface density structure of discs with the same mass as in Fig. 2 but with a specific heat ratio of $\gamma = 7/5$, and cooling times of $t_{\rm cool}\Omega = 12$ (left-hand panel) and $t_{\rm cool}\Omega = 13$ (right-hand panel). The lack of fragmentation in the right-hand panel in this case suggests that for $\gamma = 7/5$ the fragmentation boundary is at a cooling time of between $t_{\rm cool}\Omega = 12$ and 13. The colour scale of the density and the linear scale of the image are the same as in Fig. 1.

As a further numerical check, we repeated one set of calculations using 125 000 particles rather than 250 000 particles. We considered only the case where $M_{\rm disc} = 0.25$ and $\gamma = 5/3$. The result with 125 000 particles was the same as the simulation with 250 000 particles. Fragmentation occurred for $t_{\rm cool} = 6\Omega^{-1}$ and did not occur for $t_{\rm cool} = 7\Omega^{-1}$. Therefore not only do the simulations that fragment satisfy the Jeans criterion for fragmentation (Bate & Burkert 1997), but it also appears that the results are resolution-independent.

3 A MAXIMUM VALUE FOR GRAVITATIONAL STRESSES

Based on the results summarized in Table 1, for every value of $M_{\rm disc}/M_*$ and γ, we can define a minimum cooling time for which no fragmentation occurs, $t_{\rm nf}$, and a maximum cooling time for which fragmentation does occur, $t_{\rm f}$. The boundary value of $t_{\rm cool}$ for fragmentation can therefore be defined as $t_{\rm frag} = 1/2(t_{\rm nf} + t_{\rm f})$, with a corresponding uncertainty given by $\Delta t_{\rm frag} = 1/2(t_{\rm nf} - t_{\rm f})$. The stress $\alpha_{\rm max}$, corresponding to $t_{\rm frag}$, can be computed from equation (1), and the corresponding uncertainty is given by $\Delta\alpha_{\rm max} = (\alpha_{\rm max}/t_{\rm frag})\Delta t_{\rm frag}$. The resulting values of $t_{\rm frag}$ and $\alpha_{\rm max}$ are shown as data points in Fig. 4, together with the curves defined by equation (1), for three values of $\gamma = 2$, 5/3 and 7/5. The filled green squares with error bars refer to the simulations presented here. The open blue triangle represent the value found by Gammie (2001) in his local, 2D simulations that assumed $\gamma = 2$. This is consistent with our result which suggests that, for $\gamma = 2$, fragmentation should occur between $t_{\rm cool}\Omega = 3$ and 4. In fact, it is worth noting that, since Gammie's simulations are 2D, we should not expect a perfect agreement between our 3D results and his ones. This can be partially seen already from Fig. 4. In particular, care should be taken in considering the role of the adiabatic index γ, which has a different physical interpretation in 2D and in 3D. However, as discussed in more detail in Gammie (2001), a mapping is possible between the 2D and the 3D adiabatic indices. In the case of self-gravitating discs, Gammie's choice of a 2D adiabatic index equal to 2 does correspond to $\gamma = 2$ also in 3D (Gammie 2001).

As can be seen, fragmentation occurs at an almost constant value of α ($\alpha_{\rm max} \sim 0.06$, indicated by the horizontal green line in Fig. 4), thus vindicating the idea that gravitational instabilities cannot provide (in a steady state) a stress larger than $\alpha_{\rm max}$. If the dissipation associated with $\alpha_{\rm max}$ is not sufficient to balance the cooling rate, then the reaction of the disc is to fragment into bound objects.

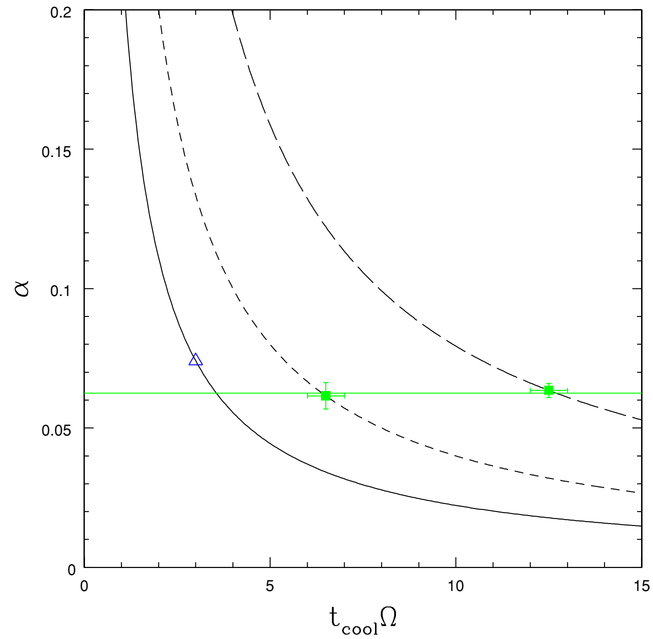

Figure 4. The relationship defined by equation (1) for $\gamma = 2$ (solid line), $\gamma = 5/3$ (short-dashed line) and $\gamma = 7/5$ (long-dashed line). The data points show the couples ($t_{\rm frag}\Omega$, $\alpha_{\rm max}$) as derived from the simulations: the green squares refer to our simulations, while the blue triangle refers to Gammie (2001). The horizontal green line illustrates the constant values $\alpha = 0.06251$.

4 DISCUSSION AND CONCLUSIONS

In this Letter we elucidate the processes that lead to the fragmentation of a massive disc. Our main result is the determination of a maximum value for the stress that can be provided by gravitational instabilities in a quasi-steady state. We then argue that fragmentation will occur whenever the external cooling requires, in order to be balanced by internal heating, a stress larger than this maximum value, which we estimate to be $\alpha_{\rm max} \sim 0.06$ (in units of the local disc pressure). As a consequence, discs with larger values of the ratio of the specific heats will be less susceptible to fragmentation. For $\gamma = 7/5$, for example, we estimate the fragmentation cooling time to be between $12\Omega^{-1}$ and $13\Omega^{-1}$, compared with between $3\Omega^{-1}$ and $4\Omega^{-1}$ for $\gamma = 2$ (Gammie 2001).

We wish to stress that the threshold value for α that we have found here refers to a quasi-steady state, in which the disc stays in thermal equilibrium and the relevant physical quantities do not change significantly on time-scales shorter than the thermal time-scale. We have already shown (Lodato & Rice 2005) how very massive discs (with masses comparable to that of the central object) can generate transient strong spiral episodes, with correspondingly large values of the stress α, which, however, do not last for longer than one dynamical time-scale (see details in Lodato & Rice 2005).

A further remark is in order, in reference to the possibility of non-local transport in self-gravitating discs. In all our simulations, we did not find any significant evidence for non-local transport of energy due to self-gravity (Lodato & Rice 2004, 2005). If the disc does not fragment, the dissipation provided by the gravitational stresses balances almost exactly the imposed cooling rate. However, this conclusion might depend on the simulation setup and, in particular, on the assumed radial dependence of the cooling time. Mejia et al. (2005) claim to find evidence for non-local energy transport in their simulations that employ a $t_{\rm cool}$ constant with radius (rather than $\propto \Omega^{-1}$, as we do). If this is the case, it might offer a possible escape route for the disc in order to avoid fragmentation. Consider the case where $t_{\rm cool}\Omega$ is such that the disc is stable against fragmentation in the inner regions, but would fragment in the outer regions, following our prediction in equation (2) (which is based on the implicit assumption that energy dissipation is viscous). In such a situation, the inner disc could 'help' the outer disc, by heating it up via non-local effects and preventing fragmentation. This process is not viable in the simulations presented here, since here the whole disc is uniformly stable or unstable with respect to fragmentation, and increasing the cooling rate of the inner disc via non-local energy transport would make it fragment. Clearly, further simulations of the fragmentation process in discs with a radius-dependent $t_{\rm cool}\Omega$ are needed to investigate this issue.

Finally, we note that the fragmentation requirements determined by Rafikov (2005) were calculated assuming the much shorter cooling times of Gammie (2001), and by assuming that the Toomre Q parameter must be unity or less for fragmentation to commence. Since the actual Q value required for fragmentation can be slightly greater than 1 (Pickett et al. 1998), and since the required cooling time in discs with $\gamma = 7/5$ [as used by Boss (1998, 2002)] can be larger than that predicted by Gammie (2001), the conditions for fragmentation may not be as stringent as those determined by Rafikov (2005).

ACKNOWLEDGMENTS

The computations reported here were performed using the UK Astrophysical Fluids Facility (UKAFF), and Datastar at the San Diego Supercomputing Center (SDSC). This work was supported by NASA under grants NAG5-13207 and NNG04GL01G from the Origins of Solar Systems and Astrophysics Theory Programs, and by the NSF under grant AST 0407040. WKMR acknowledges support from the Institute for Pure and Applied Mathematics (IPAM) at the University of California, Los Angeles, where some of this work was carried out. We thank Cathie Clarke and Matthew Bate for useful discussions, and Jim Pringle for a careful reading of the manuscript.

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