# Simulation of continuous blood $\mathrm{O}_{2}$ equilibrium curve over physiological $\mathrm{pH}, \mathrm{DPG}$, and $\mathrm{PCo}_{2}$ range 

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#### Abstract

Winslow, Robert M., Michele Samaja, Nancy J. Winslow, Luigi Rossi-Bernardi, and Richard I. Shrager. Simulation of continuous blood $\mathrm{O}_{2}$ equilibrium curve over physiological $p H, D P G$, and $\mathrm{PCO}_{2}$ range. J. Appl. Physiol.: Respirat. Environ. Exercise Physiol. 54(2): 524-529, 1983.-We analyzed $56 \mathrm{O}_{2}$ equilibrium curves of fresh human blood, each from 0 to 150 Torr $\mathrm{Po}_{2}$. The data were collected over ranges of values for the 2,3-diphosphoglyceric acid-to-hemoglobin concentration ratio [DPG]/[Hb] of $0.2-2.7$, for pH of $7.0-7.8$, and for $\mathrm{PCO}_{2}$ of $7-$ 70 Torr. Each curve was characterized according to the Adair scheme for the stepwise oxygenation of Hb , and the resulting constants ( $a_{1}, a_{2}, a_{3}, a_{4}$ ) were analyzed to allow the simulation of the entire $\mathrm{O}_{2}$ equilibrium curve under any conditions of [DPG]/[Hb], pH , and $\mathrm{Pco}_{2}$ in the specified range. This analysis provides a powerful tool to study the affinity of Hb for $\mathrm{O}_{2}$ within the red blood cell and to predict the shape of the $\mathrm{O}_{2}$ equilibrium curve in various physiological and pathological states. Other attempts to predict blood $\mathrm{O}_{2}$ affinity have considered only $\mathrm{P}_{50}$ (the $\mathrm{Po}_{2}$ at one-half saturation with $\mathrm{O}_{2}$ ) or have provided too little data for continuous simulations.


hemoglobins; blood oxygen affinity; Adair equation; 2,3-diphosphoglycerate; carbon dioxide; pH ; Bohr effect
the position of the whole-blood $\mathrm{O}_{2}$ equilibrium curve (OEC) is usually represented by the partial pressure of $\mathrm{O}_{2}$ at which hemoglobin $(\mathrm{Hb})$ is half-saturated $\left(\mathrm{P}_{50}\right)$. Many physicochemical conditions within the red blood cell determine, in concert, its actual value. However, the principal effectors that vary under physiological conditions to regulate the $\mathrm{P}_{50}$ are 2,3-diphosphoglyceric acid (DPG), $\mathrm{H}^{+}$, and $\mathrm{CO}_{2}(2,4,11)$. Nomograms and equations have been developed to estimate $P_{50}$ from known values of $\mathrm{PCO}_{2}, \mathrm{pH}$, and the $[\mathrm{DPG}\rceil /[\mathrm{Hb}]$ molar ratio (13).
For two important reasons, we extended our work to a description of the entire OEC over a broad range of $\mathrm{Po}_{2}$. First, the allosteric effectors of $\mathrm{O}_{2}$ affinity, DPG, $\mathrm{H}^{+}$, and $\mathrm{CO}_{2}$ react differently with deoxy- and oxyhemoglobin and, therefore, influence the shape in addition to the position ( $\mathrm{P}_{50}$ ) of the OEC. Second, the $\mathrm{P}_{50}$ is a somewhat artificial index for physiological considerations. Under normal conditions ( $37^{\circ} \mathrm{C}, \mathrm{pH} 7.4, \mathrm{PcO}_{2} 40$ Torr) we have found the mean $\mathrm{P}_{50}$ to be $26.7 \pm 1.7$ Torr (23), whereas in resting humans at sea level arterial $\mathrm{Po}_{2}$ is 90 Torr and mixed venous $\mathrm{PO}_{2}$ is 40 Torr, placing $\mathrm{P}_{50}$ outside the operating range.

Many previous systematic studies of the effects of the allosteric regulators of $\mathrm{Hb}-\mathrm{O}_{2}$ affinity have been done at low Hb concentration ( $8,18-20$ ) and therefore are complicated by tetramer-dimer dissociation (9). Such dissociation further affects the shape and position of the OEC, and, therefore, the data do not apply directly to the conditions within the red blood cell. Furthermore, the data that are available from studies on whole blood or concentrated Hb solutions $(2,4,7)$ do not encompass ranges of $[\mathrm{DPG}] /[\mathrm{Hb}], \mathrm{pH}$, or $\mathrm{PCO}_{2}$ to allow a complete description of physiological functions.
In this report, we present a description of the entire OEC for $\mathrm{Po}_{2} 0-150$ Torr, $\mathrm{pH} 7.2-7.8, \mathrm{Pco}_{2} 7-70$ Torr, and $[\mathrm{DPG}] /[\mathrm{Hb}] 0.2-2.7 \mathrm{~mol} / \mathrm{mol}$. We have used a method for precisely measuring the OEC under controlled conditions (23) to obtain data for a wide range of $\mathrm{pH}, \mathrm{PCO}_{2}$, and $[\mathrm{DPG}] /[\mathrm{Hb}]$ molar ratio. The Adair equation (1) was used as a model to characterize the OEC, and the parameters $a_{1}, a_{2}, a_{3}$, and $a_{4}$ were obtained by methods we have previously described (24). The $a$ 's were then reduced to simple empirical functions of the three effectors so that an OEC can be reconstructed under any set of physiological conditions likely to be encountered.

## METHODS

Blood samples. Heparinized blood from two healthy male nonsmoking donors was obtained by venipuncture. Concentrations of Hb , carboxyhemoglobin, and methemoglobin in whole blood were measured with the use of a Microblood analyzer, kindly loaned by Carlo Erba Strumentazione, Milan, Italy. [DPG] was measured using kits obtained from Calbiochem. When no alterations of [DPG] were required blood was kept on ice at $4^{\circ} \mathrm{C}$ until use, no more than 4 h after collection. The two subjects were representative of a group of 38 normal individuals studied in a previously reported survey (23) using the same techniques.

Alteration of [DPG] and pH . To increase [DPG], red blood cells were washed in isotonic saline and incubated at $37^{\circ} \mathrm{C}$ in isotonic media containing inosine, pyruvate, and phosphate at various ratios for 90 min (3) as described previously (14). To decrease [DPG], cells were incubated in isotonic saline at $37^{\circ} \mathrm{C}$ for various times up to 12 h , depending on the degree of depletion required.

Incubations were followed by recombination of washed red blood cells with their native plasma. To alter the pH of the final suspension, small quantities (up to $10 \mu \mathrm{l} / \mathrm{ml}$ ) of either 4.45 M lactic acid or 10 M NaOH were added to the plasma. In these instances, plasma samples were equilibrated to the final $\mathrm{Pco}_{2}$ before red blood cells were added to avoid extremes of pH and consequent hemolysis.

OEC determinations. Samples of blood ( 2.5 ml ) were deoxygenated in an IL 237 tonometer (Instrumentation Laboratories, Lexington, MA) for 20 min using $\mathrm{N}_{2}$ containing 1, 5.6, or $10 \% \mathrm{CO}_{2}$. Gases were obtained from Lif-O-Gen, Cambridge, MD, and compositions were specified to $\pm 0.01 \%$. Blood OEC's were measured as previously described (23). During the oxygenation, the $\mathrm{Pco}_{2}$ and pH of the blood were maintained at the starting values. Reproducibility of the measurement was reported previously (23).
A total of 56 OEC's were measured at different pH , $\mathrm{Pco}_{2}$, and $[\mathrm{DPG}] /[\mathrm{Hb}]$ ratios. The conditions were chosen to encompass the range of variation expected under physiological conditions for the three effectors (Fig. 1). At least three OEC's were determined at different pH for each set of $\mathrm{Pco}_{2}$ and $[\mathrm{DPG}] /[\mathrm{Hb}]$ ratio.

Data analysis. As discussed previously $(23,24)$ the major source of error in this method is calibration of the $\mathrm{O}_{2}$ electrode. A separate method, which does not depend on $\mathrm{PO}_{2}$ measurement, for the precise determination of $\mathrm{P}_{50}$ only was devised (13). Briefly, the method consists of equilibrating small samples of blood with gas mixtures of known $\mathrm{O}_{2}-\mathrm{CO}_{2}-\mathrm{N}_{2}$ composition in sealed glass tonometer bottles. At the end of the equilibration period Hb saturation with $\mathrm{O}_{2}, \mathrm{pH}$, and DPG are measured. If the $\mathrm{Po}_{2}$ is chosen close to the expected $\mathrm{P}_{50}$ value, the Hill equation can be used to calculate $\mathrm{P}_{50}$ from the known $\mathrm{Po}_{2}$ and measured saturation. This technique was used to measure $\mathrm{P}_{50}$ over a wide range of $\mathrm{pH},[\mathrm{DPG}] /[\mathrm{Hb}]$, and $\mathrm{Pco}_{2}$, and a nomogram was constructed to relate the variables. On the assumption that any discrepancy in $\mathrm{P}_{50}$ observed using the two methods was due to the $\mathrm{O}_{2}$ electrode calibration in the continuous method, all $\mathrm{PO}_{2}$ values were adjusted by the ratio $\mathrm{P}_{50}\left(\mathrm{P}_{50}\right.$ nomogram) $/ \mathrm{P}_{50}$ (continuous OEC). In any case, the discrepancies were always small ( $<2$ Torr).
Experimental data. The Adair equation in its form for the present work is

$$
\begin{equation*}
Y=\frac{a_{1} \mathrm{P}+2 a_{2} \mathrm{P}^{2}+3 a_{3} \mathrm{P}^{3}+4 a_{4} \mathrm{P}^{4}}{4\left(1+a_{1} \mathrm{P}+a_{2} \mathrm{P}^{2}+a_{3} \mathrm{P}^{3}+a_{4} \mathrm{P}^{4}\right)} \tag{1}
\end{equation*}
$$

where $Y$ is the fractional saturation of Hb with $\mathrm{O}_{2}$ and P is the partial pressure of $\mathrm{O}_{2}$. This equation is a convenient representation of the entire OEC, since the four constants ( $a$ 's) can be used to determine $Y$ at any $P$.

The corrected data sets were transferred to a Decsys-tem-10 digital computer (Digital Equipment, Maynard, MA) for analysis of the Adair parameters as described previously (24). In this analysis, 100 experimental points were used for each OEC, and the $a$ 's were constrained to positive values. $\mathrm{P}_{50}$ was computed as a Newton-Rapheson solution of the Adair equation by finding P such that $Y(\mathrm{P})=0.5$. The value $n_{\text {max }}$ was found from the slope of the function $Y(\mathrm{P}) /[1-Y(\mathrm{P})]$ on a $\log -\log$ plot by finite differences.


FIG. 1. Conditions under which $\mathrm{O}_{2}$ equilibrium curves were measured. $\mathrm{PCO}_{2}=7,40$, and 70 Torr.

## RESULTS

Experimental data. Table 1 lists the $a$ 's determined for all the OEC's analyzed, along with the $\mathrm{P}_{50}, n_{\text {max }}$, and residual sum of squares for each determination. The correlation coefficient for the relationship between $\mathrm{P}_{50}$ using the continuous method and that predicted from the $\mathrm{P}_{50}$ nomogram was 0.99835 . Since the nomogram is considered to be more precise in $\mathrm{PO}_{2}$, we assume that the major source of error is the $\mathrm{Po}_{2}$ measurement in the continuous method. Although this assumption has not been fully substantiated by experimentation, it will suffice for the purpose of the present analysis.

The logarithms of the Adair constants are apparently a linear function of pH under all conditions investigated. Examples of these relationships are shown in Fig. 2. The behavior of the $a$ 's as functions of $[\mathrm{DPG}] /[\mathrm{Hb}]$ ratio at constant pH is not so simple as shown in Fig. 3. This result is not surprising, since the effect of DPG on $\mathrm{O}_{2}$ binding to Hb within the red blood cell is complex and includes a sizeable intracellar pH effect at [DPG]/[Hb] $>1$ (14). Moreover, the effects of DPG and $\mathrm{CO}_{2}$ are highly interactive (11).

Analysis of Adair parameters. To establish the relationship between the Adair parameters and the various allosteric effectors, the parameters were related to $[\mathrm{DPG}] /[\mathrm{Hb}]$ molar ratio and $\mathrm{Pco}_{2}$ at pH 7.0 and 7.8 according to the following procedures.
table 1. $O_{2}$ equilibrium curve parameters

| $\begin{aligned} & \text { OEC } \\ & \text { No. } \end{aligned}$ | [DPG]/ <br> [ Hb ] | $\mathrm{pH}_{\mathrm{e}}$ | $\mathrm{P}_{50}$, Torr | $\log a_{1}$ | $\log a_{2}$ | $\log a_{3}$ | $\log a_{4}$ | $n_{\text {max }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{PCO}_{2}=7$ Torr |  |  |  | $\begin{aligned} & -4.169 \\ & -3.660 \end{aligned}$ | -5.375 | 2.48 |
| L073 | 0.22 | 6.99 | 22.8 | -1.932 | -2.642 |  |  |  |
| L074 | 0.22 | 7.27 | 18.1 | -1.561 | $-2.536$ |  | -5.010 | 2.33 |
| L075 | 0.22 | 7.74 | 11.5 | -1.287 | $-2.213$ |  |  | $2.46$ |
| N072 | 0.73 | 7.26 | 27.6 | -1.614 | -2.796 |  | -4.190 | 2.46 |
| N071 | 0.73 | 7.68 | 19.9 | -1.730 | $-2.567$ |  | -5.119 | 2.55 |
| 2208 | 0.86 | 7.36 | 27.4 | -1.666 | -3.177 |  | $\left\lvert\, \begin{aligned} & -5.131 \\ & -5.131 \end{aligned}\right.$ | 2.76 |
| 2206 | 0.86 | 7.73 | 19.5 | -1.883 | -2.604 |  |  | 2.57 |
| 2207 | 0.86 | 7.51 | 24.0 | -1.604 | -2.845 |  | $\left\|\begin{array}{l} -5.405 \\ -6.876 \end{array}\right\|$ | 2.71 |
| H072 | 2.70 | 6.83 | 58.4 | -1.724 | -4.186 |  |  | 3.09 |
| H073 | 2.70 | 7.15 | 16.1 | -1.476 | -3.569 |  | -6.406 | 2.84 |
| H074 | 2.70 | $\begin{aligned} & 7.22 \\ & 7.48 \end{aligned}$ | 45.6 | -1.830 | -3.337 |  | -6.511 |  |
| H075 | 2.70 |  | 36.3 | -1.710 |  |  | -6.108 | 2.85 |
|  |  |  | $\mathrm{PCO}_{2}=40 \mathrm{Torr}$ |  |  |  |  |  |
| 0528 | 0.40 | 6.91 | $\left\|\begin{array}{l} 33.5 \\ 25.1 \end{array}\right\|$ | $\begin{aligned} & -2.210 \\ & -1.990 \end{aligned}$ | $\left\|\begin{array}{l} -2.971 \\ -2.721 \end{array}\right\|$ | $-5.052$ | -6.129 | 2.26 |
| 0523 | 0.40 | 7.26 |  |  |  | -4.355 | -5.622 |  |
| 0527 | 0.40 | 7.27 | 24.820.6 | -1.996 | $-2.886$ | $\left\lvert\, \begin{aligned} & -4.380 \\ & -4.151 \end{aligned}\right.$ | -5.672 | $2.36$ |
| 0524 | 0.40 | 7.52 |  | -2.295 | $-2.796$ |  |  | $2.53$ |
| 2004 | 0.80 | 6.95 | 20.6 39.9 | -1.842 | $-3.286$ | $-4.151$ | $\left\|\begin{array}{l} -.379 \\ -6.295 \end{array}\right\|$ | $2.71$ |
| 2023 | 0.80 | 7.12 | 39.9 34.8 | -1.812 | $-3.070$ |  |  | $\begin{aligned} & 2.11 \\ & 2.57 \end{aligned}$ |
| 2005 | 0.80 | 7.22 | $\begin{aligned} & 34.8 \\ & 31.9 \end{aligned}$ | $\begin{aligned} & -1.848 \\ & -1.740 \end{aligned}$ | -3.177 |  | $-5.979$ | $2.60$ |
| 2003 | 0.80 | 7.43 | 26.8 |  | $\left\lvert\, \begin{array}{l\|l\|} -2.879 \\ -2.833 \end{array}\right.$ |  | -5.680 | $2.66$ |
| 2021 | 0.80 | 7.56 | 24.1 | -1.750 |  |  | -5.442 | 2.70 |
| 2022 | 0.80 | $\begin{aligned} & 7.72 \\ & 6.97 \end{aligned}$ | 21.138.6 | -1.670 | $-2.706$ |  | -5.206 2.68 |  |
| N403 | 0.83 |  |  |  |  |  | -6.253 2.46 |  |
| N404 | 0.83 | 6.97 38.6 -1.807 -3.086 <br> 7.11 36.3 -1.625 -3.310 |  |  |  |  | -6.086 2.81 | . 68 |
| N401 | 0.83 | 7.35 | $\begin{aligned} & 29.8 \\ & 36.7 \end{aligned}$ | $\begin{array}{r} -1.824 \\ -1.866 \end{array}$ | $-2.967$ |  | -5.827 2.54 |  |
| 0518 | 1.00 |  |  |  | -3.101 |  | -6.161 | 2.55 |
| 0516 | 1.00 |  | $\begin{aligned} & 36.7 \\ & 31.6 \end{aligned}$ | $\begin{aligned} & -1.866 \\ & -1.785 \end{aligned}$ | $\begin{aligned} & -2.967 \\ & -2.907 \end{aligned}$ |  | -5.900 | 002.5 |
| 0515 | 1.00 | 7.42 | 29.0 | -1.660 |  |  | $\begin{aligned} & -5.728 \\ & -5.361 \end{aligned}$ | 2.56 |
| 0521 | 1.00 | 7.72 | $\begin{aligned} & 22.4 \\ & 45.9 \end{aligned}$ | -2.076 | -2.699 |  |  | 2.62 |
| 0826 | 1.80 | $\begin{aligned} & 7.08 \\ & 7.16 \end{aligned}$ |  | $\begin{aligned} & -1.833 \\ & -1.903 \end{aligned}$ | -3.644 |  | -6.520 2.96 |  |
| 0827 | 1.80 |  | $\begin{aligned} & 45.9 \\ & 43.0 \end{aligned}$ |  | -3.426 |  | -6.429 | 2.81 |
| 0823 | 1.80 | $7.28$ | 38.3 | -1.854 | -3.331 |  | -6.229 | 2.82 |
| 0828 | 1.80 | 7.46 | 33.5 | -1.708 | -3.688 |  | -6.049 | 2.81 |
| 0824 | 1.80 | 7.61 | 29.2 | -2.133 | -3.072 |  | -5.818 | 2.85 |
| 0602 | 2.50 | 6.97 | 56.2 | -1.818 | -4.076 |  | -6.870 | 2.94 |
| 0601 | 2.50 | 7.16 | 47.8 | -1.889 | $-3.866$ |  | -6.638 | 2.91 |
| 0604 | 2.50 | 7.17 | 35.8 | -1.660 | $-3.433$ |  | -6.072 | 2.95 |
| 0603 | 2.50 | 7.63 | 32.8 | -1.592 | -3.839 |  | -5.930 | 2.87 |
| 0605 | 2.50 | 7.70 | 30.3 | -1.527 | $-3.896$ |  | -5.807 | 2.94 |
| 0606 | 2.50 | 7.80 | 27.7 | -1.785 | $-3.136$ |  | -5.682 | 2.94 |
|  |  |  | Pco | 70 |  |  |  |  |
| L703 | 0.27 | 7.31 | 21.8 | -1.449 | -3.536 | -4.144 | $-5.345$ | 2.61 |
| L705 | 0.27 | 7.39 | 23.0 | -1.533 | -3.457 | -4.245 | -5.451 | 2.63 |
| L702 | 0.27 | 7.48 | 19.7 | -1.538 | -3.146 | -4.165 | -5.169 | 2.71 |
| L704 | 0.27 | 7.59 | 20.1 | -1.775 | -2.967 | -4.237 | -5.240 | 2.69 |
| 2117 | 0.80 | 6.93 | 41.8 | -1.979 | $-3.289$ |  | -6.406 | 2.63 |
| 2118 | 0.80 | 7.01 | 39.3 | -2.040 | $-3.195$ |  | -6.307 | 2.61 |
| 2116 | 0.80 | 7.24 | 32.3 | -1.955 | -3.059 |  | -5.967 | 2.66 |
| 2112 | 0.80 | 7.29 | 31.7 | -1.642 | -3.199 |  | -5.936 | 2.53 |
| 2113 | 0.80 | 7.37 | 29.9 | -1.839 | -3.032 |  | -5.815 | 2.72 |
| 2114 | 0.80 | 7.52 | 26.6 | -2.123 | -2.917 |  | -5.658 | 2.74 |
| 2115 | 0.80 | 7.66 | 23.9 | $-1.730$ | -3.042 |  | -5.450 | 2.84 |

$\mathrm{pH}_{\mathrm{e}}$, extracellar pH . See text for explanation of all other abbreviations.

1) The value of $\log a_{i}$ was fit, using the least-square method, to a second-order equation

$$
\begin{equation*}
\log a_{\mathrm{i}}=w\left(\mathrm{G}^{2}\right)+y(\mathrm{G})+z \tag{2}
\end{equation*}
$$

where $\mathrm{G}=[\mathrm{DPG}] /[\mathrm{Hb}]$. The values of $w, y$, and $z$ were estimated for each $a$ and $\mathrm{PCO}_{2}$ at pH 7.0 and 7.8. Thus each $a_{\mathrm{i}}$ can be empirically estimated at pH 7.0 and 7.8 as a function of $G$.
2) At each pH and $G$ value, $w, y$, and $z$ are functions of $\mathrm{PCO}_{2}$ only and can be related according to another second-order equation

$$
\begin{equation*}
w, y, \text { or } z=A\left(\mathrm{Pco}_{2}\right)^{2}+B\left(\mathrm{PcO}_{2}\right)+C \tag{3}
\end{equation*}
$$

From the relation of $w, y$, and $z$ to $\mathrm{PCO}_{2}$ for each $G$, values of $A, B$, and $C$ were further determined by leastsquares analysis.

We have avoided the use of subscripted variables, for clarity, but it is understood that there is a different set of $A, B$, and $C$ for each set of $w, y$, and $z$ and for each two pH 's and each of four $a$ 's, making $3 \times 2 \times 4=24$ sets of $A, B$, and $C$ in all. Saturation under all pertinent conditions is now specified by these 72 coefficients.

The values of all required $A$ 's, $B$ 's, and $C$ 's are given in Table 2.

Simulation of the OEC. The calculation of the four Adair parameters from known values of pH , [DPG]/ [Hb], and $\mathrm{PCO}_{2}$ can be obtained by using the following steps. 1) For pH 7.0 and 7.8, calculate $w, y$, and $z$ from $E q .3$ and the selected $\mathrm{PCO}_{2}$. 2) Using the values of $w, y$, and $z$ obtained in step 1 , calculate the four $\log a$ 's at pH 7.0 and 7.8 from Eq. 2 and the selected [DPG]/[Hb] ratio. 3) Since $\log a$ 's are linear functions of pH at each $[\mathrm{DPG}] /[\mathrm{Hb}]$ and $\mathrm{Pco}_{2}$ condition (Fig. 2), the $\log \alpha$ 's at the selected pH are calculated by linear interpolation. 4) The $a_{\mathrm{i}}$ 's are substituted in Eq. 1 to compute $Y$ at any P and, hence, to generate the entire OEC. 5) Evaluation of $P$ at any $Y$ can conveniently be accomplished using the Newton-Raphson method of successive approximation. To obtain a solution for P to an error $<0.1$ Torr, three to four iterations are usually required.

An example of a FORTRAN IV program for performing all of the above computations is available on request.

Error analysis. The use of the Adair model to fit the experimental OEC using our method of curve fitting has been discussed extensively (23, 24). Winslow et al. (23) demonstrated that the deviation of the directly measured saturation from that estimated from the Adair parameters was $0.5 \%$ on the range of $\mathrm{Po}_{2} 5-100$ Torr. This magnitude of discrepancy is routinely observed using the present methods, and we accept the Adair simulation as an accurate representation of the real OEC. The $\pm 0.5 \%$ deviation is certainly within the limits of our ability to measure saturation using the present experimental techniques.
To estimate the magnitude of the discrepancy between the experimental OEC and that calculated using the present algorithms, simulations were made for each set of conditions for the OEC's listed in Table 1. Then, at each $10 \%$ increment of saturation from 10 to $90 \%$, saturations were computed using both observed and simulated $a$ 's. For example: 1) Find $P_{\text {obs }}$ at $Y_{\text {obs }}=10 \%$ from an experimental OEC. 2) Using that $\mathrm{p}_{\text {obs }}$, find $Y_{\text {calc }}$ using the equations. 3) Find an error, $Y_{\text {obs }}-Y_{\text {calc }}$. These errors were computed for all the OEC's, and the standard deviations were calculated for them. The result is shown in Fig. 4, which illustrates that under all conditions encompassed by the experimental conditions, the saturation can be found from $\mathrm{Po}_{2}$ to a maximum standard deviation of -1.2 to $+2.0 \%$ saturation. The analysis also shows that the error in saturation estimation is greatest at around $50 \%$.


FIG. 2. Relationship of $\log a_{i}$ vs. pH for representative $\mathrm{O}_{2}$ equilibrium curves: $\mathrm{Pco}_{2}$ is 40 Torr, $[\mathrm{DPG}] /[\mathrm{Hb}]$ ratios are 0.4 (口), 0.8 (○), and


FIG. 3. Relationships of $\log \alpha_{i}$ vs. [DPG]/[Hb] at different $\mathrm{Pco}_{2}$ and for 2 extremes of pH . Curved lines were determined by least-squares fit to seconi-order equations as described in text. Under some conditions $a_{3}$ tended to negative values, shown by arrows.
2.7 ( $\Delta$ ). Straight lines are determined by least-squares regression. The $a_{i}$ 's were also linear with pH at $\mathrm{Pco}_{2} 7$ and 70 Torr and are not shown.
$P_{50}$ estimation. The procedures outlined above were used to calculate $P_{50}$ for each of the 56 sets of conditions from the experimental OEC's and from the computer model. Figure 5 demonstrates that the correlation is excellent for all conditions studied.

OEC shape estimation. The strength of the present approach is that the shape of the OEC can now be estimated under a variety of physiological conditions. In Fig. 6, we compare the behavior of $n$ (at $\mathrm{P}_{50}$ ) predicted from the present analysis with that actually found in a previous study (14). The agreement demonstrates that the present method does, indeed, account for these changes.

## discussion

In the past decade, the influences of the allosteric effectors of Hb function, $\mathrm{H}^{+}, \mathrm{CO}_{2}$, and DPG, have been well defined. Each of these molecules has different reactivities with the deoxy and oxy conformers of Hb , and they therefore have important effects on the shape as well as the position of the OEC. Before these effects were known, the shape of the OEC was considered to be invariant when its position was defined. The most frequently used equation for the OEC is that described by Hill (5)

$$
\begin{equation*}
\log Y=n \log \mathrm{P}+C \tag{5}
\end{equation*}
$$

A major drawback of Eq. 5 is that the slope, $n$, is not constant over the full range of oxygenation. Alchough the value at $P_{50}$ may be about 2.6 , it decreases to 1 at high
and low saturation (18). Thus, when calculation of saturation from $\mathrm{Po}_{2}$ (or vice versa) is required, significant errors can occur at extremes of saturation.
Some algorithms and nomograms for calculations based on the OEC use this simple representation, with the implicit assumption that the shape of the OEC does not vary with conditions. One of these is the blood gas calculator of Severinghaus (15, 16). In his original paper describing the use of the calculator (15), he stated these assumptions but gave recognition to the fact that the behavior of the shape under different conditions was unknown.

We previously reported that DPG and $\mathrm{H}^{+}$have impor-
table 2. Parameters for Eqs. 2 and 3

| $a_{i}$ | pH |  | A | $B$ | C |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $a_{1}$ | 7.0 | $w$ | $0.6675 \times 10^{-3}$ | $-0.3351 \times 10^{-1}$ | $-0.8114 \times 10^{-1}$ |
|  |  | $y$ | $-0.2077 \times 10^{-2}$ | 0.1057 | 0.2550 |
|  |  | $z$ | $0.1389 \times 10^{-2}$ | $-0.8233 \times 10^{-1}$ | -1.522 |
|  | 7.8 | $w$ | $-0.1323 \times 10^{-3}$ | $-0.1180 \times 10^{-2}$ | 0.5527 |
|  |  | $y$ | $0.3564 \times 10^{-3}$ | $0.8291 \times 10^{-2}$ | -1.815 |
|  |  | $z$ | $-0.257 \times 10^{-3}$ | $-0.1163 \times 10^{-9}$ | -0.7493 |
| $a_{2}$ | 7.0 | $w$ | $-0.4558 \times 10^{-3}$ | $0.1002 \times 10^{-1}$ | 0.3322 |
|  |  | $y$ | $0.1411 \times 10^{-2}$ | $-0.2945 \times 10^{-1}$ | $-1.643$ |
|  |  | $z$ | $-0.100 \times 10^{-2}$ | $0.3303 \times 10^{-1}$ | -2.432 |
|  | 7.8 | $w$ | $0.3562 \times 10^{-3}$ | $-0.243 \times 10^{-1}$ | 0.2996 |
|  |  | $y$ | $-0.9808 \times 10^{-3}$ | $0.6717 \times 10^{-1}$ | -1.016 |
|  |  | $z$ | $0.5823 \times 10^{-3}$ | $-0.4721 \times 10^{-1}$ | -1.768 |
| $a_{3}$ | 7.0 | $w$ | $0.1151 \times 10^{-2}$ | -0.1023 | -1.660 |
|  |  | $y$ | $0.2031 \times 10^{-2}$ | -0.1639 | 2.278 |
|  |  | $z$ | $-0.3756 \times 10^{1}$ | $0.3432 \times 10^{-1}$ | -4.632 |
|  | 7.8 | $w$ | $-0.7407 \times 10^{-3}$ | $0.5148 \times 10^{-1}$ | -3.654 |
|  |  | $y$ | $0.2855 \times 10^{-2}$ | $-0.2157$ | 2.78 |
|  |  | $z$ | $-0.1325 \times 10^{-2}$ | $0.8771 \times 10^{-1}$ | -4.129 |
| $a_{4}$ | 7.0 | $w$ | $0.3457 \times 10^{-3}$ | $-0.2257 \times 10^{-1}$ | 0.4241 |
|  |  | $y$ | $-0.1019 \times 10^{-2}$ | $0.6913 \times 10^{-1}$ | -1.774 |
|  |  | $z$ | $0.6414 \times 10^{-3}$ | $-0.4939 \times 10^{-1}$ | -4.796 |
|  | 7.8 | $w$ | $0.6947 \times 10^{4}$ | $-0.1197 \times 10^{-1}$ | 0.5114 |
|  |  | $y$ | $-0.3443 \times 10^{-3}$ | $0.5013 \times 10^{-1}$ | -2.214 |
|  |  | $z$ | $0.2638 \times 10^{-3}$ | $-0.3888 \times 10^{-1}$ | -3.541 |

See text for explanation of abbreviations.


FIG. 4. Deviation of computed from observed $\mathrm{O}_{2}$ equilibrium curves (OEC). Vertical bars represent ranges of error for all OEC in Table 1. See methods for discussion of calculations. Continuous curves represent errors for 3 standard curves ( $A, B$, and $C$ ).


FIG. 5. Correlation between experimental $\mathrm{P}_{50}$ and that calculated from Eqs. 2-4 and Table 2. Regression lines are shown. $\mathrm{P}_{50}, \mathrm{Po}_{2}$ at halfsaturation with $\mathrm{O}_{2}$.


FIG. 6. Effect of $[\mathrm{DPG}] /[\mathrm{Hb}]$ on shape of OEC. Ability of present simulation method to predict changes in shape of OEC is demonstrated. Hill's parameter ( $n$ ) has been estimated for present method (continuous curve) and compared with measured values (14) taken from Table 1. Blood conditions are $\mathrm{pH}=7.4$ and $\mathrm{PcO}_{2}=40$ Torr.
tant effects on Hill's parameter, $n$, at constant $\mathrm{Pco}_{2}$ (14). Furthermore, in whole blood the influence of these effectors is not uniform over the entire range of saturation (6, 21). Neville (12) has pointed out that such changes in the shape of the OEC can affect peripheral $\mathrm{O}_{2}$ delivery. Others, for example West (22), have attempted to calculate the influence of alterations of shape on physiological functions, but without adequate model data.

No comprehensive attempt to describe the effects of $\mathrm{H}^{+}, \mathrm{CO}_{2}$, and DPG has been reported heretofore, with the exception of the work of Arturson et al. (2). They reported experiments in which OEC's were measured using the Radiometer DCA-1 instrument at three pH values (7.20, 7.40, and 7.60), three values of [DPG] (3, 5,
and $7 \mathrm{~mol} / 1 \mathrm{blood}$ ), and three values of $\mathrm{PCO}_{2}$ (20, 40, and 80 Torr).

Several problems with the Arturson et al. data reduce their utility. First, they are cumbersome to use; i.e., data at only a few conditions are available, and linear interpolations must be made where it is known that linear relations do not exist. In contrast, our approach to use the Adair scheme means that continuous functions can be defined which allow calculations at any combination of conditions. Second, in the data of Arturson et al. [DPG] is given as moles of DPG per liter of blood. Without knowledge of [ Hb ], no regular pattern of effect of DPG can be defined; i.e., the DPG effect is a function of the $[\mathrm{DPG}] /[\mathrm{Hb}]$ molar ratio $(8,14)$, and the concentration unit mol/l blood will not account for variations in hematocrit or Hb values in different individuals. Finally, the tables of Arturson et al. (17) cannot be readily adapted to computer programs for continuous determinations

We have chosen the Adair equation to describe the OEC, because it is based on physicochemical considerations. It probably does not describe the OEC better than the Kelman equation (10) or other empirical relations, but it does allow the possibility of comparing complex cellular phenomena to data obtained from much simpler experiments with purified Hb solutions (4). We have previously described the methods used to obtain the values of $a$ 's and their uncertainties $(23,24)$.

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An obvious problem with the Adair scheme is the difficulty in estimation of $a_{3}$. This has been discussed extensively in our previous work on this subject $(23,24)$. Indeed, the OEC can be often described using only three of the Adair parameters (23). The high degree of interdependence among the parameters does not invalidate the Adair scheme, however. Rather, further work should be directed toward obtaining independent estimations of the parameters.
In this report we do not attempt a physical interpretation of the reactions of DPG, $\mathrm{H}^{+}$, and $\mathrm{CO}_{2}$ within the red blood cell. Instead, an initial estimation of the behavior of the Adair parameters is intended only as a starting point for the detailed investigation of the interactions of these effectors with Hb . Still more data are required to continue such an approach, particularly at different temperatures and over a wider range of conditions.

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