Some Issues on Optimal Non-Linear Taxation

Francesco Cohen

Supervisor: Prof. Paolo M. Panteghini
Supervisor: Prof. Enrico Minelli

Contents

Preface

In this thesis we apply an optimal taxation approach to analyze how citizens' private information may affect Social Planner' goal of redistribution across individuals. In first two Chapters, we allow individuals to emigrate to a laissez-faire country. Chapter 1 assumes the existence of two types of citizens, who are endowed with a different labor productivity. We let these two types be low- and high-skilled, respectively, and assume that emigration is costly. We prove that low-skilled agents' incentive compatibility constraint is never binding at the optimum. Moreover high-skilled citizens are net taxpayers and, hence, a lump-sum tax is levied by the Social Planner. Relative to the standard closed-economy model, we also find that, if the economy stands for two periods, in the absence of Government's commitment, redistribution is dramatically reduced and that an inducing first-period pooling tax scheme is much less likely to be implemented. This implies that a separating scheme is easier. In Chapter 2 we introduce a third type of citizens, who are characterized by an intermediate skill level. This allows us to better analyze the distributive effects of taxation. In particular, we will show that countervailing incentives may occur. Indeed, medium-type citizens may find it optimal to mimic high-skilled ones. In a such a context, a separating scheme can be implemented. However, a distortion at the top arises. As in Chapter 1, Chapter 3 studies an economy, that is populated by two continua of individuals, who differ in labor-productivity. We allow agents to save and let effort be private information. Therefore, both adverse selection and moral hazard problems may arise. In particular, we prove that, under adverse selection, it is optimal for the Social Planner to fully insure high-skilled agents and to over-insure low-skilled ones. Moreover, no high-type aggregate intertemporal wedge is implemented, while a positive L-aggregate intertemporal wedge is obtained. Finally, we study the joint effects of dynamic moral hazard and adverse selection on social welfare. In this case, both types are partially insured. Moreover, both the H-aggregate intertemporal wedge and the L-aggregate intertemporal wedge are positive.

Chapter 1

Dynamic Optimal Taxation under Migration

1.1 Introduction

Since Mirrlees (1971), who highlighted the trade-off between allocative efficiency and redistribution under adverse selection, the information asymmetry on individuals has been considered a significant constraint on tax policy design. In such a framework, when lump-sum taxation cannot be levied, a second-best solution arises, where the Social planner offers a menu of $\{$*gross income; consumption*$\}$, making sure that the optimal allocation designed for each $i-$agent is *incentive compatible.* Two relevant features of Mirrleesian model are that the economy stands just for one period and that citizens are forbidden to emigrate to other countries. Mirrlees (1971, p. 176) himself argues that, "since the threat of migration is a major influence on the degree of progression in actual tax systems, at any rate outside the United States", the assumption of impossibility of migration is very strong and, therefore, should be avoided. It is agreed that, given increasing globalization, high-skilled potential emigration to tax heaven or less redistributive countries, is a relevant constraint on government's policy. Wilson (1992) finds that high-type's elasticity of potential migration has a dramatic effect on the role of tax as a redistributional instrument. Simula and Trannoy (2006, 2010) analyze optimal taxation structure in an economy populated by a continuum of individuals, differing in labor-productivity, when option to migrate to a laissez-faire country is type-dependent. They find that high-skilled's migration opportunity may bring to a decrease in the optimal marginal tax rates, up to be strictly negative. Hence, the optimal average tax rate may decrease.

Within this literature, we want to find what happens if we move into a two-period setting, letting agents emigrate to a foreign country in both periods. With specific attention to the second issue, as underlined by Simula and Trannoy (2006, 2010), high-skilled's migration to tax heavens could be a heavy constraint on the government's ability at designing a redistributive tax schedule. If the Social Planner aims to avoid them to leave their country, it has to give them, at the optimum, an utility level at least identical to the one they would receive abroad, discounted by type-dependent migration costs. In line with Simula and Trannoy (2006, 2010), we will assume that there exist two countries, i.e. country "A", endowed with a benevolent Social Planner, and country "B", a pure laissez-faire economy, and that A's citizens can emigrate to B, bearing two different costs, a migration cost and a cost of living in a foreign country. The other issue we deal with, is what happens in a repeated game, instead of a one-shot one. When a government lacks credibility in committing to not use the information it learns about taxpayers' productivity to raise taxes, the Revelation Principle may no longer apply and a pooling equilibrium can be optimal from policy-maker's point of view. Brett and Weymark (2008) analyze a two-period closed economy in which individuals of two productivity types work and consume in both periods. Moreover, they are allowed to save. The government is assumed to lack of commitment; therefore, they provide an analysis of both a separating and a pooling tax regimes. Dillén and Lundholm (1996) study a dynamic closed economy, with the well known *ratchet effect*, where the Social Planner has as redistributional instruments, linear income tax. Apps and Rees (2008) show that in a two-period closed economy with Government's lack of commitment, a first-period partial pooling may be optimal.

In this Chapter we first introduce a basic model. In section 2, we analyze a simplified two-type version of the one-period Mirrlees Model, with a continuum of individuals of each type. In section 3, we analyze the effect of high skilled individuals' threat of migration on the ability of the Social Planner to design an optimal non-linear tax scheme in the one-period framework. In section 4, we consider two-period models, both in closed economy and in open economy, showing that, in the latter case, an inducing first-period pooling equilibrium tax schedule is less likely to be implemented. In section 5, we discuss the effects on the optimal tax scheme of time inconsistency of government's commitment and of possibility of migration.

1.2 Two-Type Models

1.2.1 Optimal Taxation in a closed one-period economy

In order to create a useful benchmark, we start from the very basic one-period model, a simplified version of the famous Mirrlees' model.

The economy is populated by two continuum of citizens, each one with a share $\beta_i > 0$, $i = L, H$, and $\sum_{i=H,L} \beta_i = 1$, differing only in their given labor productivity ω_i, $i = H, L$, $\omega_H > \omega_L$, constant over time.

Its benevolent utilitarian Social Planner aims at redistributing income between agents, in order to increase equality across population, at the minimum loss of efficiency.

All agents share an identical additive and separable (between consumption good and labor time) utility function. We assume that both the consumption good and the leisure time are normal goods.

The agents are both consumers and workers; in particular, labor market is perfectly competitive, having l_i as the only production input, with constant return to scale. So, agent i's gross income y_i (in terms of consumption good) is defined as:

$$y_i = \omega_i l_i, \ i = H, L. \tag{1.1}$$

Each agent knows his own ability, whereas such an information is unavailable to the Social Planner, which only knows the distribution of the ability across the whole population. Agent i's utility function is given by:

$$U_i = u(x_i) - v(l_i), \ \forall i = H, L, \tag{1.2}$$

where $x_i = y_i - T_i \geq 0$ represents i''s consumption as the difference between i's gross income and total tax paid in terms of consumption good (where T_i can be positive or negative), with $u(x)$ as an increasing and strictly concave function in x_i and $v(l)$ as an increasing and strictly convex function in l_i,both twice differentiable, $\forall i$; this means that U_i, $i = H, L$, is strictly concave.

1.2.2 First-Best in a closed one-period economy

In a symmetric-information setting, the government knows the ability of each individual. Assuming that it can use a lump-sum[1] tool, the policy maker finds optimal to redistribute resources according to agents' type. Therefore, its policy does not depend on consumption and labor supply and that it can tax and subsidy its citizens through a lump sum redistribution, function only of the consumer's type. By assumption, the government

$$\max_{(x_L,l_L,x_H,l_H)} W_{f.b.closed}\left(U_H; U_L\right) = \sum_{i=H,L} \beta_i \left[u(x_i) - v(l_i)\right] \tag{1.3}$$

s.t. the *budget balance constraint (BC)*

$$\sum_{i=H,L} \beta_i \left(\omega_i l_{i,A} - x_{i,A}\right) \geq 0, \quad (\lambda) \tag{1.4}$$

Proposition 1.2.1 *An allocation* $(x_L^*, l_L^*, x_H^*, l_H^*)$ *solving* (1.3) *s.t.* (1.4) *satisfies*

$$u'\left(x_{H,A}^*\right) = u'\left(x_{L,A}^*\right), \tag{1.5}$$

$$\frac{v'\left(l_{H,A}^*\right)}{\omega_H} = \frac{v'\left(l_{L,A}^*\right)}{\omega_L}, \tag{1.6}$$

where the Lagrangian multiplier of (1.4)*,* $\lambda > 0$.

from proposition (1.2.1), given (1.5) , we can immediately obtain

$$x_{H,A}^* = x_{L,A}^* \tag{1.7}$$

Moreover, by (1.6) and $\omega_h > \omega_l > 0$, we can see that:

$$v'\left(l_{H,A}^*\right) > v'\left(l_{L,A}^*\right). \tag{1.8}$$

Given the convexity of $v(\cdot)$, we therefore obtain

$$l_{H,A}^* > l_{L,A}^* \tag{1.9}$$

Substituting (1.7) and (1.9) into (1.2) we obtain

$$U_H^* = u(x_H^*) - v(l_H^*) < u(x_L^*) - v(l_L^*) = U_L^*. \tag{1.10}$$

[1] A lump sum tax on high productivity agents (and a subsidy on low type consumers) does not distort the optimal choice of consumers/workers, because it only generates an income effect. Instead, a marginal tax rate (or subsidy) also causes a substitution effect between leisure time and consumption, thereby affecting the optimal (x_i, v_i) allocation, $\forall i = H, L$.

Given these results, we see that, at the social optimum, the two types of agents consume identically.

However, high-skilled agents work more. Due to this fact, the utility of a low skilled agent is higher than the utility of a high skilled one. We can, therefore, say that the government taxes high-skilled workers to subsidize the low-skilled ones, under (1.4).

Of course, this socially optimal allocation is not appreciated by high types, who have an incentive to mimic the behavior of low type individuals.

1.2.3 Second-Best in a closed one-period economy

Let us now assume that personal ability is private information, i.e., each agent knows her productivity type, whereas the tax authority does not. It just knows the overall distribution of the ability across his citizens.

In particular, the government is unable to observe each individual's skill level ω_i and units of labor supply l_i, but it can observe both her levels of consumption x_i and gross labor income y_i. By assumption, it also knows the functional form of U_i, $i = L, H$. In order to achieve its target, the Social Planner can levy both a lump sum tax and a marginal tax rate on visible gross income y_i.

From the point of view of the Social Planner, i-agent's utility function becomes

$$u(x_i) - v\left(\frac{y_i}{\omega_i}\right) \equiv u(x_i) - v(l_i), \tag{1.11}$$

where $\frac{y_i}{\omega_i} = l_i$, and $\frac{v'\left(\frac{y_i}{\omega_i}\right)}{\omega_i} > 0$, $\frac{v''\left(\frac{y_i}{\omega_i}\right)}{\omega_i} > 0$. Therefore, we obtain

$$MRS_{y_i,x_i} = \frac{v'(l_i)}{u'(x_i)} = \frac{v'(\frac{y_i}{\omega_i})}{\omega_i u'(x_i)}. \tag{1.12}$$

From (1.12), keeping constant y_i and x_i levels, if we increase the productivity level ω_i, we see that the MRS_{y_i,x_i} decreases. Producing an additional unit of output is less labor demanding. This means that, in laissez faire, the *Spence-Mirrlees single crossing condition* holds: high-skilled individuals produce and consume more, than lower type ones.

Since individual productivity is not known to the Social Planner, it must offer an incentive compatible tax scheme[2], where each $i-type$ citizen will weakly prefer, at the optimum, the allocation (x_i^*, y_i^*) designed for her to the one designed for a $j - type$ individual. In particular, if (x_H^*, y_H^*) is chosen by high skilled citizens, they will reveal their characteristics and the Social Planner will be able to redistribute from them to the low-skilled individuals.

Given asymmetric information, the social Planner

$$\max_{(x_L,x_H,y_L,y_H)} W_{s.b.closed}(U_H; U_L) = \sum_{i=H,L} \beta_i \left[u(x_i) - v\left(\frac{y_i}{\omega_i}\right)\right] \tag{1.13}$$

s.t.

$$\sum_{i=H,L} \beta_i (y_i - x_i) \geq 0, \quad (\lambda) \tag{1.14}$$

[2] Actually, since there are only two types of agents, by the *revelation principle* it will be optimal for the tax authority to set just two optimal allocations $(x_L^*; y_L^*)$ and $(x_H^*; y_H^*)$, since each allocation has to be incentive compatible for the agent for whom it has been designed.

In addition, the $\left(x_H^{*sb}, y_H^{*sb}\right)$ has to be *incentive compatible* for high skilled citizens (ICH)

$$u(x_H) - v\left(\frac{y_H}{\omega_H}\right) \geq u(x_L) - v\left(\frac{y_L}{\omega_H}\right), \quad (\gamma) \tag{1.15}$$

while $\left(x_L^{*sb}, y_L^{*sb}\right)$ has to be incentive compatible for low-skilled agents (ICL)

$$u(x_L) - v\left(\frac{y_L}{\omega_L}\right) \geq u(x_H) - v\left(\frac{y_H}{\omega_L}\right). \tag{1.16}$$

We guess that (1.16) is strictly satisfied at the constrained optimum, since Social Planner's problem comes from ω_H ability of claiming to be low-skilled, in order to limit redistribution at her expenses. Therefore, we can neglect this latter constraint into the maximization process and check ex-post if $\left(x_L^{*sb}, y_L^{*sb}\right)$ satisfies it.

Proposition 1.2.2 *At the optimal allocation* $\left(x_H^{*sb}, y_H^{*sb}, x_L^{*sb}, y_L^{*sb}\right)$, $\lambda > 0$, $\gamma > 0$ *and* (1.16) *is strictly satisfied. Moreover, the MRS are:*

$$MRS_{y_{H,A}, x_{H,A}} = \frac{v'\left(\frac{y_{H,A}^{*sb}}{\omega_i}\right)}{\omega_H u'\left(x_{H,A}^{*sb}\right)} = 1 \tag{1.17}$$

and

$$MRS_{y_L, x_L} = \frac{v'\left(\frac{y_L^{*sb}}{\omega_L}\right)}{\omega_L u'\left(x_L^{*sb}\right)} = 1 - \frac{\gamma}{\beta_L - \gamma}\left[\frac{v'\left(\frac{y_L^{*sb}}{\omega_L}\right)}{\omega_L u'\left(x_L^{*sb}\right)} - \frac{v'\left(\frac{y_L^{*sb}}{\omega_H}\right)}{\omega_H u'\left(x_L^{*sb}\right)}\right] < 1 \tag{1.18}$$

where

$$\left[\frac{v'\left(\frac{y_L^{*sb}}{\omega_L}\right)}{\omega_L u'\left(x_L^{*sb}\right)} - \frac{v'\left(\frac{y_L^{*sb}}{\omega_H}\right)}{\omega_H u'\left(x_L^{*sb}\right)}\right] > 0$$

measures the difference between the slopes of L-type and H-type agents' at the optimal low skilled individuals' allocation $\left(x_L^{*sb}, y_L^{*sb}\right)$.

It shows the trade off between the cost derived from the underproduction of low-type agents and the gain of increasing the redistribution from high skilled people to low skilled ones: the higher is the difference $(\omega_H - \omega_L)$, the higher is the advantage for H-type citizens to mimic L-type ones (i.e. (1.15) is relatively tighter), the higher is the downward distortion in the constrained optimal L-type allocation $\left(x_L^{*sb}, y_L^{*sb}\right)$.

Condition (1.17) shows that there is no distortion at the top*:* the decision of H-individuals is not distorted at the margin, just as in the first best case. It means that the tax authority will levy only a lump sum tax on them and, so, that the marginal tax rate on labor income is zero[3].

Condition (1.18) reveals us that there is a distortion at the bottom: L-individuals underproduce, comparing to the first best case. From (1.18) we can see that

$$\frac{\partial\left\{1 - \frac{\gamma}{\beta_L - \gamma}\left[\frac{v'\left(\frac{y_L^{*sb}}{\omega_L}\right)}{\omega_L u'\left(x_L^{*sb}\right)} - \frac{v'\left(\frac{y_L^{*sb}}{\omega_H}\right)}{\omega_H u'\left(x_L^{*sb}\right)}\right]\right\}}{\partial \beta_L} < 0.$$

[3] The implicit marginal tax rate on i's gross labor income is equal to $\left(1 - MRS_{y_i, x_i}\right)$.

The negative sign of this derivative means that the higher the fraction of low-skilled population, the lower will be the optimal distortion in L-type's optimal choice. This is due to the fact that the tax authority has a weaker incentive to create separation between the two types of individual. Low-skilled workers face a positive marginal tax rate

$$\tau_L^* = \frac{\gamma}{\beta_L - \gamma}\left[\frac{v'\left(\frac{y_L^{*sb}}{\omega_L}\right)}{\omega_L u'\left(x_L^{*sb}\right)} - \frac{v'\left(\frac{y_L^{*sb}}{\omega_H}\right)}{\omega_H u'\left(x_L^{*sb}\right)}\right]$$

on their gross income, receiving a lump-sum subsidy, that overcompensates them for τ_L^*. Moreover, we have that

$$\frac{u'\left(x_H^{*sb}\right)}{u'\left(x_L^{*sb}\right)} = \frac{\left(1 - \frac{\gamma}{1-\beta_H}\right)}{\left(1 + \frac{\gamma}{\beta_H}\right)} < 1.$$

By concavity of $u\left(x\right)$ we can derive that

$$x_H^{*sb} > x_L^{*sb} \tag{1.19}$$

Since constraint (1.15) is binding at the optimum, inequality (1.19) implies that

$$0 < u\left(x_H^{*sb}\right) - u\left(x_L^{*sb}\right) = v\left(\frac{y_H^{*sb}}{\omega_H}\right) - v\left(\frac{y_L^{*sb}}{\omega_H}\right).$$

According to this result, in a second-best scenario, high-skilled individuals still work more than low skilled ones. Since consumption and leisure time are normal goods, however, at the optimum high-type workers both consume more and work less, than in the first-best case.

1.3 Optimal taxation problem in an open one-period economy

So far, we have assumed that the economy (from now on, country A) was closed to citizens' migration. This meant that the government could levy its optimal tax schedule to its citizens, taking into account only the social budget constraint, BC, and H-type's incentive compatibility constraint, ICH. Let us next assume that there is another country, denoted as B. By assumption, country B is a simple laissez-faire economy, without taxation on gross labor income.

At the very beginning, the whole population bears in A. Then, after Government presents country A's tax scheme, each agent can move from one country to the other.

Therefore, we need to slightly modify the previous notation; i's utilities, when living in country A and in country B, become

$$U_{i,A} = u(x_{i,A}) - v(l_{i,A}),$$

and

$$U_{i,B} = u(x_{i,B}) - v(l_{i,B}),$$

respectively.

Therefore, country A's citizens are free to emigrate to B. There are assumed to exist two different costs related to the migration:

Assumption 1.3.1 $c_{i,B} > 0$, $\forall i = H, L$, *the cost of moving from one country to the other one. This cost is identical going from A to B and vice versa.*

Assumption 1.3.2 $k_{i,B} > 0$, $\forall i = H, L$, *the cost of living "abroad". This cost is borne by i-individual in each period spent in country B.*

We let both $c_{i,B}$ and $k_{i,B}$ be common knowledge. It is worth noting that, since country B is a laissez-faire economy, the utility of i when living in country B, i.e. $U_{i,B}$ (including the costs of moving and living abroad, respectively $c_{i,B}$ and $k_{i,B}$), is increasing in ω_i. In particular, assumed that there only two types of A's citizens, i.e. ω_H and ω_L, low-skilled individuals would not have any incentive to emigrate from country A, in which they would gain from the redistributive tax system, to a laissez-faire country B, facing also two different costs, $c_{i,B}$ and $k_{i,B}$. Differently, we need to assume that high skilled agents' utility level in (if country A was a closed economy) is lower than the one they would receive in the laissez-faire country B, discounted by the costs, otherwise there is no reason to consider the issue of migration.

Given this framework, we get that only H-workers can credibly threat to emigrate from A to B (setting up a ratio to insert a high-skilled agents' *participation constraint, PCH,* into the social planner's maximization problem).

1.3.1 The First-Best

Let us start a first-best setting with symmetric information. Since high-skilled workers can credibly move to B, if taxes are too high, country A's government has to maximize the modified social welfare function

$$\underset{(x_{L,A}, l_{L,A}, x_{H,A}, l_{H,A})}{W_{f.b.open}} (U_{H,A}, U_{L,A}) = \sum_{i=H,L} \beta_i \left[u(x_{i,A}) - v(l_{i,A}) \right] \tag{1.20}$$

s.t. both the BC

$$\sum_{i=H,L} \beta_i \left(\omega_i l_{i,A} - x_{i,A} \right) \geq 0 \;\; (\lambda) , \tag{1.21}$$

and the following PCH

$$u\left(x_{H,A}\right) - v\left(l_{H,A}\right) \geq \widehat{U}_{H,B} \;\; (\mu) , \tag{1.22}$$

where $\widehat{U}_{H,B}$ is the maximum utility level that a high-skilled worker would obtain by moving to country B and living abroad for one period[4]. We guess that low-type agents' participation constraint will not bind at the optimum; therefore we can neglect it and check ex-post that it is slack at the optimal allocation.

[4] $\widehat{U}_{H,B} = U\left(\widehat{x}_{H,B}; \widehat{l}_{H,B}\right)$ is the result of the following high-skilled worker's maximization problem:

$$\max \; U_{H,B} = u\left(x_{H,B}\right) - v\left(l_{H,B}\right)$$

s.t.

$$x_{H,B} \leq \omega_H l_{H,B} - \left(c_{H,B} + k_{H,B}\right)$$

Proposition 1.3.1 *At the optimal allocation* $\left(x^*_{L,A}, l^*_{L,A}, x^*_{H,A}, l^*_{H,A}\right)$, $\lambda > 0$ *and* $\mu > 0$. *Moreover, the neglected L-type participation constraint is slack and the equalities*

$$\left(1 + \frac{\mu}{\beta_H}\right) u'\left(x^*_{H,A}\right) = u'\left(x^*_{L,A}\right) \tag{1.23}$$

and

$$\frac{\left(1 + \frac{\mu}{\beta_H}\right) v'\left(l^*_{H,A}\right)}{\omega_H} = \frac{v'\left(l^*_{L,A}\right)}{\omega_L} \tag{1.24}$$

hold.

Given the concavity of $u\left(x_{i,A}\right)$ and the convexity of $v\left(l_{i,A}\right)$,the normality of x and leisure time and $\omega_H > \omega_L > 0$, from (1.23), (1.24) and (1.6) we can show that

$$x^*_{H,A} > x^*_{L,A}$$

and

$$\begin{cases} x^*_{H,A} > x^*_{H,cl.ec.} \\ l^*_{H,A} < l^*_{H,cl.ec.} \end{cases}$$

$$\begin{cases} x^*_{L,A} < x^*_{L,cl.ec.} \\ l^*_{L,A} > l^*_{L,cl.ec.} \end{cases}$$

Even if PCH reduce the extent of redistribution from the high-skilled fraction to the low one, because of the existence of $c_{H,B}$ and $k_{H,B}$, it is still feasible to levy positive lump-sum taxes on H-individuals, until the PCH becomes binding, subsidizing with this tax revenue L-consumers[5]. As in first-best closed economy, no distorting marginal tax rate τ on labor income is levied on A's citizens.

Moreover, since PCH is binding at the optimum, high-productivity types are indifferent between staying in A or moving to B; so, they will remain in their own country, A. In particular, since at the solution of the problem $U^*_{H,A} = \widehat{U}_{H,B}$ and $MRS_{l_{H,A},x_{H,A}} = MRS_{l_{H,B},x_{H,B}}$, we obtain

$$T^*_{H,A} = c_{H,B} + k_{H,B}$$

and

$$l^*_{H,A} = l^*_{H,B}$$
$$x^*_{H,A} = \omega_H l^*_{H,A} - T^*_{H,A} = \omega_H l^*_{H,B} - \left(c_{H,B} + k_{H,B}\right)$$

It is worth noting that the omitted PCL is strictly satisfied at the optimum, because in country A, low-skilled people receive a lump sum subsidy. Therefore, they consume more than they produce. If they emigrated to B, they would consume less than they produce, due to the costs of emigrating and living abroad.

[5] These tax schedules, together with the fact that $U^*_{L,A}$ indifference curve is always steeper than $U^*_{H,A}$. This implies that $l^*_{H,A} > l^*_{L,A}$.

1.3.2 The Second-Best

Let us next focus on a second-best case, where productivity is private information.

Accordingly, the Social Welfare Function in (1.13), the budget constraint in (1.14), the ICH in(1.15), and the PCH[6] in (1.22), must be modified in order to account for such an information asymmetry. Therefore, the Social Planner

$$\max_{(x_{L,A},y_{L,A},x_{H,A},y_{H,A})} W_{s.b.open}\left(U_{H,A};U_{L,A}\right)=\sum_{i=H,L}\beta_i\left[u(x_{i,A})-v\left(\frac{y_{i,A}}{\omega_i}\right)\right] \tag{1.25}$$

s.t.

$$\sum_{i=H,L}\beta_i\left(y_{i,A}-x_{i,A}\right)\geq 0,\ \ (\lambda) \tag{1.26}$$

$$u(x_{H,A})-v\left(\frac{y_{H,A}}{\omega_H}\right)\geq u(x_{L,A})-v\left(\frac{y_{L,A}}{\omega_H}\right),\ \ (\gamma) \tag{1.27}$$

$$u\left(x_{H,A}\right)-v\left(\frac{y_{H,A}}{\omega_H}\right)\geq \widehat{U}_{H,B},\ \ (\mu) \tag{1.28}$$

It is worth noting that the value of $\widehat{U}_{H,B}$ is crucial to understand how constraints (1.27) and (1.28) interact. There are three possible cases.

Case 1.3.1 *If $\widehat{U}_{H,B}$ is lower than U_H^{*sb}, the utility high-type citizens can get by choosing the former contract $\left(x_H^{*sb},y_H^{*sb}\right)$, the problem the Social Planner faces corresponds to the Mirrlees one, i.e. the "Second-Best Optimal Taxation Problem in a closed one-period economy". Therefore, the only binding at the optimum constraints in the social welfare maximization problem are* (1.26) *and* (1.27).

Case 1.3.2 *If $\widehat{U}_{H,B}$ coincides with U_H^{*sb}, i.e., the two "outside options" are identical from the point of view of country A's Government, it is optimal to implement an optimal tax scheme coincident with the one in the standard "Second-Best Optimal Taxation Problem in a closed one-period economy", by levying a (distortionary) positive marginal tax rate on low-skilled agents, and a zero marginal tax rate on high-skilled ones.*

Case 1.3.3 *If, finally, $\widehat{U}_{H,B}$ is strictly higher than U_H^{*sb}, there are two possible situations the Social Planner can face:*

1. *if $\widehat{U}_{H,B}$ is sufficiently high (i.e., $c_{H,B}+k_{H,B}$ low enough), only* (1.28) *binds at the optimum and, therefore, H-agents have non incentive on mimicking L-agents (i.e.,* (1.27) *is slack at the optimal allocation). Therefore, that no reason to cause a distortion at the bottom, i.e., $\tau_L^{*sb\ open}=0$. This means that, from Social Planner's point of view, the "Second-Best Optimal Taxation Problem in an open one-period economy" coincides with a "First-Best Optimal Taxation Problem in an open one-period economy". This case is likely to be faced whenever the difference in labor productivity $(\omega_H-\omega_L)$ is considerable, or when H-types' migration opportunity is a relevant option.*

[6] The RHS of new PCH (i.e. $\widehat{U}_{H,B}$), coincides with the one in (1.22), because A's tax authority knows that only L-individuals could have the incentive to move to country B; therefore he can anticipate the optimal utility level $\widehat{U}_{H,B}$ they can achieve by moving abroad.

2. *Otherwise, both* (1.28) *and* (1.27) *may be binding at the optimum. In this case, the Closed Second-Best solution is no longer valid. Since H-skilled agents must receive at least* $\widehat{U}_{H,B} > U_H^{*sb}$*, they will pay a lower lump-sum tax. Keeping* τ_L^* *constant and reducing subsidy on low-skilled agents (since H-type ones pay less lump-sum taxes), given assumptions on utility function, L-citizens would produce more and consume less, than in second-best closed setting. Therefore,* (1.27) *becomes slack. Then, the Social Planner can reduce L-type citizens' optimal marginal tax rate* τ_L^**; since both leisure time and consumption good are normal and the former is now relatively more expensive, L-agents choose to optimally produce and consume more. If this new L-type's optimal allocation, i.e.,* $\left(x_L^{*sb\ open}, y_L^{*sb\ open}\right)$ *is such that* $x_L^{*sb\ open} > x_L^{*sb}$ *(certainly,* $y_L^{*sb\ open} > y_L^{*sb}$*), it may be the case that both* (1.27) *and* (1.28) *are simultaneously binding. Therefore,* τ_L^* *is reduced, but not set to zero. In particular, we have*

$$\begin{aligned} 0 \ < \ \tau_L^{*sb\ open} &= \frac{\gamma}{\beta_L - \gamma}\left[\frac{v'\left(\frac{y_L^{*sb\ open}}{\omega_L}\right)}{\omega_L u'\left(x_L^{*sb\ open}\right)} - \frac{v'\left(\frac{y_L^{*sb\ open}}{\omega_H}\right)}{\omega_H u'\left(x_L^{*sb\ open}\right)}\right] \\ < \ \tau_L^* &= \frac{\gamma}{\beta_L - \gamma}\left[\frac{v'\left(\frac{y_L^{*sb}}{\omega_L}\right)}{\omega_L u'\left(x_L^{*sb}\right)} - \frac{v'\left(\frac{y_L^{*sb}}{\omega_H}\right)}{\omega_H u'\left(x_L^{*sb}\right)}\right]. \end{aligned}$$

Clearly, in both cases (1) *and* (2)*, high-types' optimal utility level increases (from* U_H^{*sb} *to* $\widehat{U}_{H,B}$*) and, therefore, low-types' one decreases (i.e. less than* U_L^{*sb}*), otherwise the allocation obtained in the "Second-Best Optimal Taxation Problem in a closed one-period economy" would not be optimal (i.e., a Pareto improvement would be feasible). However, case* (1) *is more plausible, and therefore, of much interest. Due to market openess, the higher high-skilled agents' migration opportunity, the higher the likelihood of the case to be faced and, therefore, the lower the redistribution of resources to low-skilled agents.*

The foregoing reasonings follow also from the omission of both *PCL* and *ICL*. Is it reasonable?

Let us begin with *PCL*. If (1.27) is binding at the optimum, we are in a standard Mirrlees framework. As we said, the optimal allocation to low-skilled agents is such that they face a $\tau_L^* > 0$ on their gross labor income and receive a lump-sum transfer that more than repays for the implicit positive marginal tax rate on y_L^{*sb}.

On the other hand, if $\widehat{U}_{H,B} \geq U_H^{*sb}$, PCH is binding; at the optimum, high-type fraction of A's population is indifferent to emigrate or to stay in its country and choosing the allocation designed for it. In the latter case, it will transfer a positive lump-sum amount to the low-type one ($\widehat{T}_{H,A} = c_{H,B} + k_{H,B}$). Therefore,L-individuals prefer to stay in A.

In both cases, low-type individuals enjoy a strictly higher utility, compared to what they could obtain moving abroad.

ICL is not binding at the optimum. Also, notice that, if case 1 (or 3) is the relevant one, at the optimum

$$u(x_H^{*sb}) - v\left(\frac{y_H^{*sb}}{\omega_H}\right) = u(x_L^{*sb}) - v\left(\frac{y_L^{*sb}}{\omega_H}\right)$$

or

$$u(x_H^{*sb}) - u(x_L^{*sb}) = v\left(\frac{y_H^{*sb}}{\omega_H}\right) - v\left(\frac{y_L^{*sb}}{\omega_H}\right). \tag{1.29}$$

Notice that the *ICL* is not binding at the optimum if

$$u(x_H^{*sb}) - u(x_L^{*sb}) < v\left(\frac{y_H^{*sb}}{\omega_L}\right) - v\left(\frac{y_L^{*sb}}{\omega_L}\right). \tag{1.30}$$

Using (1.29) and rearranging, gives

$$v\left(\frac{y_H^{*sb}}{\omega_L}\right) - v\left(\frac{y_H^{*sb}}{\omega_H}\right) > v\left(\frac{y_L^{*sb}}{\omega_L}\right) - v\left(\frac{y_L^{*sb}}{\omega_H}\right). \tag{1.31}$$

Since $v(\cdot)$ is increasing and convex in y_i, the inequality (1.31) holds and the *ICL* is never binding.

Moreover, in case 2, we must have that *ICH* is slack at the optimum. Indeed, the existence of $c_{H,B}$ and $k_{H,B}$, assures that $\widehat{T}_{H,A} > 0$. Therefore, the government is not induced to extract resources from the low-type agents to redistribute it to high-skilled ones. Hence, the *ICL* is unbinding at the optimum.

Therefore, we can conclude that the omitted *ICL* and *PCL* are never binding at the optimum. Given these results, the Social Planner's problem is as follows:

Lemma 1.3.1 *The social planner chooses the optimal allocation* $\left(x_{L,A}^*, y_{L,A}^*, x_{H,A}^*, y_{H,A}^*\right)$ *in order to maximize* (1.25) *subject to* (1.26) *and* (1.27)*, or* (1.28)*, or both.*

1.4 Dynamic Optimal Taxation

The Mirrlees model shows that, in a static contest, the best result achievable by a benevolent government (who cares about utility redistribution to worse-off individuals, minimizing the loss of efficiency) is to offer each $i-$agent an incentive compatible allocation (x_i, y_i). The Revelation Principle ensures that she picks only the allocation designed for her, thereby revealing her own type to the government. The result of this social welfare constrained maximization problem is an optimal non linear taxation, i.e. a combination of lump-sum taxes and subsidies and distorting marginal tax rates on gross labor income.

In a dynamic setting, the Revelation Principle may no longer apply, since it could be too expensive for the tax authority to design an inducing separation tax schedule. Actually, high-skilled individuals may fear that the Social Planner could use in future the information obtained about their type, in order to achieve a first-best solution, extracting more taxes from them. Therefore, an incentive compatible allocation on high-types could result to be so costly, to induce the government to find optimal a non-separating tax schedule.

The ratchet effect (i.e., the possibility that the government uses the previously gained information in order to increase future redistribution) would not be a dramatic issue, if the Social Planner can commit not to use these additional information on citizens' type in subsequent periods. If commitment is possible, the dynamic adverse selection game is just a repeated one-shot game and its optimal solution is to repeat each period the Mirrlees' optimal tax schedule in a one-period economy.

Unfortunately, this kind of commitment is hard to be achieved and a time inconsistency problem is likely to arise.

In order to analyze a two-period model, we need to further modify i's utility function:

$$U_i = u(x_i^1) - v(\frac{y_i^1}{\omega_i}) + \delta[u(x_i^2) - v(\frac{y_i^2}{\omega_i})], \forall i = H, L, \; t = \{1, 2\}, \; 0 < \delta \leq 1. \qquad (1.32)$$

In both periods all the individuals work and consume, but neither they, nor the government aren't allowed to save, lend and borrow money.

1.4.1 The Optimal tax problem in a closed economy without commitment

If the Social Planner, while designing the optimal second-period tax schedule, is unable to commit to not use in period 2 the information obtained in period 1, a first-period inducing types separation tax policy may no longer be optimal. The Social Planner's has two alternatives:

1. *to design a first-period tax schedule inducing pooling.*

   The government's gain while adopting this strategy is that it is likely to be less costly than a tax policy encouraging high-skilled individuals to reveal their type. On the other hand, its loss is related to the fact that he cannot obtain any information about citizens' type and, therefore, in period two it will face the classic "Second-best optimal taxation problem in a one-period closed economy".

2. *To design a first-period tax schedule inducing separation.*

   The trade off the Social Planner has to consider, is between the gain in second period, since it will face a "First-best optimal taxation problem in a one-period closed economy", and the first-period cost to be borne to induce high-skilled workers to reveal their own type[7].

Optimal Taxation Problem in a closed two-period economy when pooling arises in the first period

1. We start solving the second-period social welfare maximization problem, assuming that in period 1 the Social Planner designs a tax schedule inducing the whole population to choose the same allocation $\left(x^{1*}, y^{1*}\right)$, i.e. a pooling equilibrium. Therefore, it does not arise any information about its citizens' types and cannot update its belief about any individual. In this case, the policy-maker's problem is the one solved in (1.2.3), namely, a "*Second-best optimal taxation problem in a one-period closed economy*".

   At the beginning of time 1, while designing the optimal first-period pooling allocation, i.e. $\left(x^{1*}, y^{1*}\right)$, the Social Planner already anticipates the level of the maximized second period social welfare function. Nevertheless, the latter choice (period 2) only depends on second-period variables x_i^2, y_i^2, $i = H, L$. Moreover, since the tax authority does not aim to separate types, BC is the only binding at the optimum constraint.

Therefore, the Social Planner

[7] Actually, it is also possible to adopt a mixed strategy, inducing only a portion of ω_H A's citizens to reveal their type and letting the remaining high-skilled agents to pool with all the low types, i.e a partial pooling.

$$\max_{(x^1,y^1)} W^1_{closed\ pool} = \sum_{i=H,L} \beta_i \left[u(x^1) - v\left(\frac{y^1}{\omega_i}\right)\right] + W^{2*}_{closed\ pool} \tag{1.33}$$

s.t.

$$\left(y^1 - x^1\right) \geq 0, (\lambda_1) \tag{1.34}$$

Proposition 1.4.1 *At the optimal solution, i.e., $\left(x^{1*}, y^{1*}\right)$, $\lambda_1 > 0$. Moreover,*

$$\beta_H \frac{v'\left(\frac{y^{1*}}{\omega_H}\right)}{\omega_H u'\left(x^{1*}\right)} + (1-\beta_H) \frac{v'\left(\frac{y^{1*}}{\omega_L}\right)}{\omega_L u'\left(x^{1*}\right)} = 1. \tag{1.35}$$

By single-crossing condition, at the $\left(x^{1*}, y^{1*}\right)$ allocation, low-type individuals' indifference curve is steeper than higher skilled agents' one. Therefore, we can say that

$$MRS_{y_L,x_L} > 1 > MRS_{y_H,x_H} \tag{1.36}$$

Since the pooling equilibrium is due to the behavior of the H-citizens who mimic L-ones, the Government's optimal tax schedule is based on these latter.

Hence, we obtain $\tau^{1*} = \tau^{1*}_L = (1 - MRS_{y_L,x_L}) < 0$ (i.e., low-skilled agents are subsidized at the margin). Because of (1.34), the tax authority must also levy a lump-sum tax on tax payers. Conversely, since $MRS_{y_H,x_H} < 1$, high-types face (an implicit) positive marginal tax rate on their gross labor income.

Finally, in order to prevent high-skilled people from working and thus earning more (they could be better off by doing so), the Social Planner must set $\tau^{1'} = 100\%$, $\forall y' > y^{1*}$.

Optimal Taxation Problem in a closed two-period economy when separation of types arises in the first period

Let us now focus on the second pure strategy. In this case, the Social Planner can design an optimal two-period tax schedule inducing separation of types in period 1.

In period 2, assuming that in period 1 high-skilled individuals revealed themselves, by choosing the optimal incentive compatible first-period allocation $\left(x^{1*}_H, y^{1*}_H\right)$ designed for them, the government faces a "*First-best optimal taxation problem in a closed economy*" shown in (**??**) .

This means that there are only non-distortionary lump-sum taxes on high-skilled individuals and lump-sum subsidies to the low-skilled ones, aimed at equalizing all citizens' consumption, i.e. $x^{2*}_H = x^{2*}_L$.

Moreover, at the optimum, H-agents will receive a lower net utility, than in the pooling case of period 1. This loss is equal to

$$\Omega = u(x^{2*}_L) - v\left(\frac{y^{2*}_L}{\omega_H}\right) - \left[u(x^{2*}_H) - v\left(\frac{y^{2*}_H}{\omega_H}\right)\right], \tag{1.37}$$

where $\left[u(x_L^{2*}) - v\left(\frac{y_L^{2*}}{\omega_H}\right)\right]$ corresponds to the utility high-skilled agents receive at the constrained optimum, in "*Second-best optimal taxation problem in a one-period closed economy*" setting and $\left[u(x_H^{2*}) - v\left(\frac{y_H^{2*}}{\omega_H}\right)\right]$ is what they gain at the optimal allocation in "*First-best optimal taxation problem in a closed economy*" one. In period 1 high-skilled individuals expect that, by revealing their type, they lose Ω in period 2. Therefore, when the Social Planner designs a first-period inducing separation optimal tax scheme, it must account for high-skilled workers expectations. This implies that the ICH must be such that

$$\begin{aligned} & u(x_H^1) - v\left(\frac{y_H^1}{\omega_H}\right) + \delta\left[u(x_H^{2*}) - v\left(\frac{y_H^{2*}}{\omega_H}\right)\right] \\ \geq \quad & u(x_L^1) - v\left(\frac{y_L^1}{\omega_H}\right) + \delta\left[u(x_L^{2*}) - v\left(\frac{y_L^{2*}}{\omega_H}\right)\right] \end{aligned} \tag{1.38}$$

or, rearranging (1.38),

$$u(x_H^1) - v\left(\frac{y_H^1}{\omega_H}\right) \geq u(x_L^1) - v\left(\frac{y_L^1}{\omega_H}\right) + \delta\Omega, (\gamma_1) \tag{1.39}$$

In period 1, the government

$$\max_{\left(x_L^1, y_L^1, x_H^1, y_H^1\right)} W_{closed\ sep}^1 = \sum_{i=H,L} \beta_i \left[u(x_i^1) - v\left(\frac{y_i^1}{\omega_i}\right)\right] + W_{closed\ sep}^{2*} \tag{1.40}$$

s.t. the *BC*

$$\sum_{i=H,L} \beta_i \left(y_i^1 - x_i^1\right) \geq 0, \ \ (\lambda_1) \tag{1.41}$$

and (1.39).

Proposition 1.4.2 *At the optimal allocation* $\left(x_L^{1*}, y_L^{1*}, x_H^{1*}, y_H^{1*}\right)$*, both* γ_1 *and* λ_1 *are strictly positive. Since* W_{sep}^{2*} *and* $\delta\left[u(x_L^{2*}) - h_H\left(y_L^{2*}\right) - u(x_H^{2*}) - h_H\left(y_H^{2*}\right)\right]$ *are not affected by variables* $\left(x_L^1, y_L^1, x_H^1, y_H^1\right)$*, the optimal tax scheme highlights qualitatively identical properties of the standard "Second-Best Optimal Taxation Problem in a closed one-period economy": no distortion at the top and distortion at the bottom. However, at the optimum, the allocation chosen by high skilled individuals, i.e.,* $\left(x_H^{1*}, y_H^{1*}\right)$*, will be strictly better than the one they would obtain in the standard Mirrleesian one-shot game, i.e.* $\left(x_H^{*sb}, y_H^{*sb}\right)$. *The gain is just equal to* $\delta\Omega$.

1.4.2 Optimal taxation problem in an open economy without commitment

Let us next assume that country A's citizens can move across countries. Relative to a closed two-period economy, H-agents have three additional options:

1. to emigrate in period 1 to country B and to stay there in both periods. Thus, they reach the following utility level:

$$\widehat{U}_{H,B}^1 + \delta\left(\widetilde{U}_{H,B}^2\right) \tag{1.42}$$

where $\widehat{U}^1_{H,B}$ is time invariant. This is due to the fact that $\widehat{U}^1_{H,B} \equiv \widehat{U}^2_{H,B}$, since parameters $c_{H,B}$ and $k_{H,B}$ are reasonably constant across periods and ω_H is exogenously given, by definition) and $\widetilde{U}^2_{H,B}$ is the maximum utility level that high-skilled workers would obtain by living in country B in period 2, given that they emigrated to the laissez-faire country in period 1[8] (i.e., in period 2 they only pay the lump-sum cost $k_{H,B}$).

2. To emigrate in period 2 to country B and to come back in the second one. Note that by moving to country B, they implicitly reveal their own high-type to the Social Planner, because the low-skilled agents have no incentive to emigrate. Therefore, in period 2 they would have to face a *"First Best closed economy allocation"*, with the additional charge of $c_{H,B}$.So, it is straightforward that this option is never optimal.

3. To stay in country A in period 1 and to move abroad in the second one.

Optimal taxation problem in an open two-period economy, when the Social Planner designs an inducing first-period separation tax schedule.

If the Government chooses a first-period tax schedule aimed at inducing separation, it is aware that the more stringent between the following two high-type constraints

$$\begin{aligned} & (x^1_{H,A}) - v\left(\frac{y^1_{H,A}}{\omega_H}\right) + \delta \widehat{U}^2_{H,B} \\ \geq \quad & u(x^1_{L,A}) - v\left(\frac{y^1_{L,A}}{\omega_H}\right) + \delta \max\left\{\left[u(x^{2*}_{L,A}) - v\left(\frac{y^{2*}_{L,A}}{\omega_H}\right)\right]; \widehat{U}^2_{H,B}\right\} \end{aligned} \tag{1.43}$$

and

$$(x^1_{H,A}) - v\left(\frac{y^1_{H,A}}{\omega_H}\right) + \delta \widehat{U}^2_{H,B} \geq \widehat{U}^1_{H,B} + \delta\left(\widetilde{U}^2_{H,B}\right) \tag{1.44}$$

will hold. Moreover, $\left[u(x^{2*}_{L,A}) - v\left(\frac{y^{2*}_{L,A}}{\omega_H}\right)\right]$ (corresponding to the utility high-skilled workers reach at

[8] Notice that $\widetilde{U}^2_{H,B} = U\left(\widetilde{x}^2_{H,B}; \widetilde{l}^2_{H,B}\right)$ is the optimal solution of the following problem:

$$\max_{l^2_{H,B}} U^2_{H,B} = u\left(x^2_{H,B}\right) - v\left(l^2_{H,B}\right)$$

s.t.

$$x^2_{H,B} \leq \omega_H l^2_{H,B} - k_{H,B}$$

Again, since both x^t_i and leisure time are normal goods, and given the absence of $c_{H,B}$, we can derive that

$$\begin{cases} \widetilde{x}^2_{H,B} > \widehat{x}^2_{H,B} \\ \widetilde{l}^2_{H,B} < \widehat{l}^2_{H,B} \end{cases}$$

and, consequently,

$$\widetilde{U}^2_{H,B} > \widehat{U}^2_{H,B}$$

the optimum in "*Second-best one period closed economy*" case) cannot be less than $\left[u(x^1_{L,A}) - v\left(\frac{y^1_{L,A}}{\omega_H}\right)\right]$ at the constrained optimal $\left(x^{1*sb}_{L,A}; y^{1*sb}_{L,A}\right)$, and $\widetilde{U}^2_{H,B} > \widehat{U}^2_{H,B}$.

If

$$\max\left\{\left[u(x^{2*}_{L,A}) - v\left(\frac{y^{2*}_{L,A}}{\omega_H}\right)\right]; \widehat{U}^2_{H,B}\right\} = \widehat{U}^2_{H,B} = \widehat{U}^1_{H,B},$$

then

$$\widehat{U}^1_{H,B} > \left[u(x^1_{L,A}) - v\left(\frac{y^1_{L,A}}{\omega_H}\right)\right]$$

and, therefore, (1.44) is the binding constraint[9]. In this case, high-skilled agents' migration is the only credible threat. This means that, in period 2, the government faces a *"First-Best Optimal Taxation Problem in an open one-period economy"*.

In period 1, Social Planner's problem is as follows:

$$\max_{x_{H,A}, y_{H,A}, x_{L,A}, y_{L,A}} W^1_{open\ sep} = \sum_{i=H,L} \beta_i \left[u(x^1_{i,A}) - v\left(\frac{y^1_{i,A}}{\omega_i}\right)\right] + W^{2*}_{f.b.open} \tag{1.45}$$

s.t.

$$(1.41)$$

and

$$(1.44)\,.$$

Notice that (1.45) is similar to the problem faced by the tax authority in period 2, a part from the fact that redistribution is further lower (but still positive), given the more stringent *PCH* it has to take into account.

If

$$\max\left\{\left[u(x^{2*}_{L,A}) - v\left(\frac{y^{2*}_{L,A}}{\omega_H}\right)\right]; \widehat{U}^2_{H,B}\right\} = \widehat{U}^2_{H,B} = \widehat{U}^1_{H,B},$$

a first-period pooling equilibrium cannot be achieved, since to be chosen by high-skilled individuals, it would require in period 1 (the second one is identical in both cases) an optimal allocation $\left(x^{1*}_A, y^{1*}_A\right)$, that gives them an even higher utility, compared to the one they obtain by threatening emigration to country B. So, there would be no trade off: a more costly (in terms of redistribution) first-period tax scheme (when compared to the separating solution), would be implemented. However, in period 2 there would be no gain in social welfare.

If, conversely, (1.43) is binding at the optimum,

$$\left[u(x^{1*}_{H,A}) - v\left(\frac{y^{1*}_{H,A}}{\omega_H}\right)\right] = \left[u(x^{1*}_{L,A}) - v\left(\frac{y^{1*}_{L,A}}{\omega_H}\right)\right] + \delta\left\{\left[u(x^{2*}_{L,A}) - v\left(\frac{y^{2*}_{L,A}}{\omega_H}\right)\right] - \widehat{U}^2_{H,B}\right\}.$$

Since in $t = 1$, the term $\delta\left\{\left[u(x^{2*}_{L,A}) - v\left(\frac{y^{2*}_{L,A}}{\omega_H}\right)\right] - \widehat{U}^2_{H,B}\right\}$ is fixed and is strictly lower than Ω, differently from the closed-economy case, the Social Planner can levy a higher lump-sum tax on H-individuals.

[9] This argument also holds if $\left[u(x^{2*}_{L,A}) - v\left(\frac{y^{2*}_{L,A}}{\omega_H}\right)\right] = \widehat{U}^2_{H,B}$.

A necessary, but not sufficient, condition[10] for (1.43) to be the relevant H-type constraint in the Social Welfare maximization problem is that

$$\left[u(x_{L,A}^{1*}) - v\left(\frac{y_{L,A}^{1*}}{\omega_H}\right)\right] > \widehat{U}_{H,B}^{1}.$$

and, therefore,

$$\max\left\{\left[u(x_{L,A}^{2*}) - v\left(\frac{y_{L,A}^{2*}}{\omega_H}\right)\right]; \widehat{U}_{H,B}^{2}\right\} = \left[u(x_{L,A}^{2*}) - v\left(\frac{y_{L,A}^{2*}}{\omega_H}\right)\right].$$

In this case, emigration in period 1 is not a real threat: migration costs are so high, that the Social Planner is not worried about the possibility that H-citizens immediately move abroad. The first-period maximization problem is similar to the "*First-period Optimal Taxation Problem inducing separation of types in a closed economy*". The only difference is that now separation is less costly: in fact, high-skilled agents are aware that, in period 2, they will receive exactly $\widehat{U}_{H,B}^{2}$ (it corresponds to the "*First-Best Optimal Taxation Problem in an open one-period economy*"), that is assumed to be strictly higher than the utility they would achieve in the "*First-best closed economy*" environment. Therefore, they are no longer so worried about revealing their own productivity level. Since (1.43) ensures a first-period separating equilibrium, in period 2, high-skilled migration is a credible threat. Consequently, in period 2 the Social Planner will face a "*First-Best Optimal Taxation Problem in an open one period economy*".

Optimal tax problem in an open two-period economy, when the Social Planner designs an inducing first-period pooling tax schedule.

It is worth pointing out that migration reduces the social value of productivity information, since the first-best allocation is constrained by high-type outside opportunity. In period 1, a separation tax scheme is less costly, since the second-period social gain is lower. Therefore, a "*First period Optimal Taxation Problem with pooling*" is unlikely to be faced. A necessary condition for such a tax schedule to be implemented is that

$$\max\left\{\left[u(x_{L,A}^{2*}) - v\left(\frac{y_{L,A}^{2*}}{\omega_H}\right)\right]; \widehat{U}_{H,B}^{2}\right\} = \left[u(x_{L,A}^{2*}) - v\left(\frac{y_{L,A}^{2*}}{\omega_H}\right)\right].$$

The Social planner is aware that, before its decision (policy intervention), H-type agents can make two alternative decisions:

$$U_H^{1,2} = (x_A^1) - v\left(\frac{y_A^1}{\omega_H}\right) + \delta \max\left\{\left[u(x_{L,A}^{2*}) - v\left(\frac{y_{L,A}^{2*}}{\omega_H}\right)\right]; \widehat{U}_{H,B}^{2}\right\} \tag{1.46}$$

or

$$U_H^{1,2} = \widehat{U}_{H,B}^{1} + \delta\left(\widetilde{U}_{H,B}^{2}\right) \tag{1.47}$$

[10] Conversely, a sufficient condition for it is that

$$\left[u(x_{L,A}^{1*}) - v\left(\frac{y_{L,A}^{1*}}{\omega_H}\right)\right] > \widetilde{U}_{H,B}^{2}.$$

If $\left[u(x_{L,A}^{2*}) - v\left(\frac{y_{L,A}^{2*}}{\omega_H}\right)\right] \leq \widehat{U}_{H,B}^2 \equiv \widehat{U}_{H,B}^1 < \widetilde{U}_{H,B}^2$, in order to have pooling, we should have that

$$(x_A^1) - v\left(\frac{y_A^1}{\omega_H}\right) \geq \widehat{U}_{H,B}^1 + \delta\left(\widetilde{U}_{H,B}^2 - \widehat{U}_{H,B}^2\right). \tag{1.48}$$

As pointed out, in period 1, a pooling equilibrium strategy is suboptimal, compared to the inducing-separation one. In both strategies, the Social Planner knows that in the second period has to give high-skilled individuals at least $\widehat{U}_{H,B}^2$, in order to prevent their migration. Moreover, $\left(x_A^{1*}, y_A^{1*}\right)$ (first-period pooling optimal allocation) must give them the same utility of the inducing separation optimal H-bundle (i.e. $\left(x_{H,A}^{1*sep.}, y_{H,A}^{1*sep.}\right)$), since the R.H.S. of (1.44) corresponds to the one in (1.48). Since $\left(x_A^{1*}, y_A^{1*}\right) \neq \left(x_{H,A}^{1*sep.}, y_{H,A}^{1*sep.}\right)$, and given that latter bundle ensures revelation of types, $\left(x_{H,A}^{1*sep.}, y_{H,A}^{1*sep.}\right)$ is preferable from the social point of view.

Assume now $\left[u(x_{L,A}^{2*}) - v\left(\frac{y_{L,A}^{2*}}{\omega_H}\right)\right] > \widehat{U}_{H,B}^2 \equiv \widehat{U}_{H,B}^1 < \widetilde{U}_{H,B}^2$. Therefore, in period 2, if high-productive citizens are still in country A, the Social Planner will face a "*Second-Best Optimal Taxation Problem in a open (or, in this case equivalently, closed) one period economy*", where the only relevant constraints are (1.26) and (1.27).

In period 1, in order to implement an inducing pooling tax schedule that also prevents H-type agents' migration, the government must satisfies the following constraint:

$$(x_A^1) - v\left(\frac{y_A^1}{\omega_H}\right) \geq \widehat{U}_{H,B}^1 + \delta\left\{\widetilde{U}_{H,B}^2 - \left[u(x_{L,A}^{2*}) - v\left(\frac{y_{L,A}^{2*}}{\omega_H}\right)\right]\right\}. \tag{1.49}$$

Whenever $\left[u(x_{L,A}^{2*}) - v\left(\frac{y_{L,A}^{2*}}{\omega_H}\right)\right] > \widehat{U}_{H,B}^2$, there is a rationale for a pooling strategy. Indeed, a pooling strategy gives high-type workers a higher utility in period 2 (compared to the one, $\widehat{U}_{H,B}^2$, given through a first-period separating strategy), while preventing emigration in period 1 is less costly from the social welfare point of view. The lower $\left\{\widetilde{U}_{H,B}^2 - \left[u(x_{L,A}^{2*}) - v\left(\frac{y_{L,A}^{2*}}{\omega_H}\right)\right]\right\}$ (i.e. high-type agents have higher migration costs), the easier will be such a tax scheme.

1.5 Conclusions

In this Chapter we have shown that the threat of migration affects negatively redistribution across individuals in Mirrleesian framework. When there exist only two types of agents (i.e., high-type and low-type ones), the presence of strictly positive migration costs for high-skilled workers ensures a positive redistribution from them to the less productive ones. In such a model, countervailing incentives are not an issue, neither in one-period models, nor in two-period ones. In particular, when the economy stands for only one period, at least one, between high-type's incentive constraint and high-type's participation constraint, is binding at the optimum (besides social budget constraint, being always binding at the optimum). Whenever high-type's incentive constraint is binding at the optimum (both in the case that PCH is slack at the optimum and in the case that PCH is binding, too) high-skilled workers' labor supply is not distorted at the margin, while there is a downward marginal distortion for low-skilled ones. When only high-type's participation constraint is binding at the optimum, the most efficient tax schedule is obtained through lump-sum tax to high-skilled agents and lump-sum subsidy to low-skilled ones: no marginal tax rate is levied on citizens. On the other hand, when the economy lives for two periods and the Social Planner is unable to commit itself to not use in period 2 the information gained during period 1, in order to reach the best social outcome, it is common knowledge that the revelation principle may not apply. In the two-period setting, highly productive workers can choose some possible strategies due to their ability to move abroad. For instance, by moving abroad in the first period and staying in country B in the second one, they get more than twice the utility they obtain by migrating (since during the second period abroad they don't have to pay $c_{H,B}$, the cost of moving from country A to laissez-faire country B). However the existence of migration costs $c_{H,B}$ and $k_{H,B}$ ensures that high-type citizens are, at the optimal solution, net tax payers. Hence, freedom to migrate in period 2 makes first-period pooling strategy less likely to be implemented, since redistribution in period 2 is limited by second-period high-type participation constraint: the higher is high-type's outside opportunity option, the lower is the likelihood of implementing an optimal first-period pooling strategy. An interesting extension to this model is to allow for the presence of a third type of citizens, i.e., a *medium-type* one. If country 2 is endowed with three types of agents, countervailing incentives may be a relevant issue to be taken into account.

Chapter 2

Three-Type Models

2.1 Introduction

In this Chapter we generalize our previous findings by introducing a third type of citizens, who are characterized by an intermediate skill level. It is worth noting that this hypothesis allows us to better analyze the distributive effects of taxation. In particular, we will show that *countervailing incentives* may occur. Medium-type citizens may find it optimal to mimic high-skilled ones as a consequence of the interaction between Social Planner's desire of redistribution to low-skilled individuals and high-type's migration opportunity to a more favorable tax jurisdiction. In our framework, a separating scheme can be implemented. However, when countervailing incentives are a credible issue for the Social Planner, a distortion at the top arises. The term "countervailing incentives" is due to Lewis and Sappington (1989); theoretical analyses of optimal contracting under countervailing incentives are provided by Maggi and Rodriguez (1995) and Jullien (2000).

This Chapter is organized as follows. In the next section, we first introduce a third type of agents, i.e., a medium-skilled citizen. Then, we characterize the optimal non-linear tax scheme in a second-best closed economy. In section 3, we allow the economy to be open to emigration of its citizens. We first analyze the first-best open economy setting, finding that if only high-skilled agents can credibly threat to emigrate, there is the well-known "curse" of middle-skilled workers. Otherwise, redistribution to low-productive agents is heavily bounded. Next, we analyze different possible scenarios, assuming labor productivity as private information and that migration is possible. Depending on the utility level that both medium-skilled and high-skilled can receive by moving abroad, countervailing incentives may arise. Finally, in section 4, the results are briefly discussed.

2.2 Three-Type Models

We have seen previously that with only two types of citizens, it is never the case that $L-$type agents' incentives are binding-at-the-optimum, neither to emigrate to country B, nor to mimic the high skilled ones. This implies that countervailing incentive is not a credible issue for the Social Planner.

This is no longer the case, when we allow for the existence into A's economy of a third continuum

β_M (such that $\sum_{i=H,M,L} \beta_i = 1$) of agents, endowed with a labor productivity ω_M, in between ω_L and ω_H ones. Moreover, in order to avoid bunching in asymmetric information settings, we impose the sufficient condition *monotonicity of hazard rate* on the distribution of labor productivity, such that

$$\frac{\beta_H}{\beta_M} < \frac{\beta_H + \beta_M}{\beta_L}. \tag{2.1}$$

(2.1) ensures that the downward output distortion decreases when labor productivity increases. Finally, to further simplify the analysis, we will assume that

$$\begin{cases} \omega_H = (1+\alpha)\,\omega_M \\ \omega_M = (1+\rho)\,\omega_L \end{cases}, \ \alpha > 0, \ \rho > 0. \tag{2.2}$$

In *First-Best Closed One-Period Economy* setting, the solution to the optimal taxation problem is such that $\frac{v'\left(\frac{y_i}{\omega_i}\right)}{\omega_i u'(x_i)} = 1$, $\forall i$. However, this tax schedule obtained through personalized lump-sum taxes and subsidizes is not feasible, owing to the existence both of information asymmetry and opportunity of emigration constraints into the Social Welfare maximization.

2.2.1 Second-Best Optimal Taxation Problem in a closed three-type economy

By assumption, the tax authority cannot measure each agent's labor productivity. If migration to country B is forbidden, only *i-type incentive constraints* (i.e. $n*(n-1)$; in this problem there are 6 incentive constraints) and *budget balance constraint* are real issues. The latter is always binding at the optimum, but only some of these former are binding as well.

In particular, following the standard literature, we will replace the set of incentive constraints with two *local downward incentive constraints*[1]*:*

$$u(x_H) - v\left(\frac{y_H}{\omega_H}\right) \geq u(x_M) - v\left(\frac{y_M}{\omega_H}\right), \ \ (\gamma_{HM}) \tag{2.3}$$

$$u(x_M) - v\left(\frac{y_M}{\omega_M}\right) \geq u(x_L) - v\left(\frac{y_L}{\omega_M}\right), (\gamma_{ML}) \tag{2.4}$$

Therefore, Social Planner

$$\max_{x_H, y_H, x_M, y_M, x_L, y_L} W_{s.b.closed}\,(U_H; U_M; U_L) = \sum_{i=H,M,L} \beta_i \left[u(x_i) - v\left(\frac{y_i}{\omega_i}\right)\right] \tag{2.5}$$

s.t. (2.3), (2.4) and the budget constraint

$$\sum_{i=H,M,L} \beta_i\,(y_i - x_i) \geq 0, \ \ (\lambda) \tag{2.6}$$

[1]The monotonicity of hazard rate condition ensures that monotonicity constraint (i.e. $\frac{y_H^{*sb}}{\omega_H} > \frac{y_M^{*sb}}{\omega_M} > \frac{y_L^{*sb}}{\omega_L}$) is strictly satisfied at the optimal allocation. Therefore, the two local downward incentive constraints imply

$$u\left(x_H^{*sb}\right) - v\left(\frac{y_H^{*sb}}{\omega_H}\right) > u\left(x_L^{*sb}\right) - v\left(\frac{y_L^{*sb}}{\omega_H}\right),$$

that is, H-type agents are strictly better by choosing $\left(x_H^{*sb}; y_H^{*sb}\right)$, instead of $\left(x_L^{*sb}; y_L^{*sb}\right)$.

Solving this problem, leads to the following:

Proposition 2.2.1 *At the optimal tax schedule* (2.6), (2.3) *and* (2.4) *are binding and, given assumption* (2.1), *the monotonicity requirement is strictly satisfied* ($y_H^* > y_M^* > y_L^*$, *since* $\frac{y_H^*}{\omega_H} > \frac{y_M^*}{\omega_M} > \frac{y_L^*}{\omega_L}$). *Therefore, low-type incentive constraints are slack at the optimal allocation. Moreover,*

$$MRS_{y_H,x_H} = \frac{v'\left(\frac{y_H^{*sb}}{\omega_H}\right)}{(1+\alpha)(1+\rho)\,\omega_L u'\left(x_H^{*sb}\right)} = 1 \tag{2.7}$$

$$MRS_{y_M,x_M} = 1 - \frac{\gamma_{HM}}{\beta_M + \gamma_{ML} - \gamma_{HM}}\left[\frac{v'\left(\frac{y_M^{*sb}}{\omega_M}\right)}{(1+\alpha)\,\omega_L u'\left(x_M^{*sb}\right)} - \frac{v'\left(\frac{y_M^{*sb}}{\omega_H}\right)}{(1+\alpha)(1+\rho)\,\omega_L u'\left(x_M^{*sb}\right)}\right] < 1 \tag{2.8}$$

where

$$0 < \tau_M^* = \frac{\gamma_{HM}}{\beta_M + \gamma_{ML} - \gamma_{HM}}\left[\frac{v'\left(\frac{y_M^{*sb}}{\omega_M}\right)}{(1+\alpha)\,\omega_L u'\left(x_M^{*sb}\right)} - \frac{v'\left(\frac{y_M^{*sb}}{\omega_H}\right)}{(1+\alpha)(1+\rho)\,\omega_L u'\left(x_M^{*sb}\right)}\right] < 1$$

$$MRS_{y_L,x_L} = 1 - \frac{\gamma_{ML}}{\beta_L - \gamma_{ML}}\left[\frac{v'\left(\frac{y_L^{*sb}}{\omega_L}\right)}{\omega_L u'\left(x_L^{*sb}\right)} - \frac{v'\left(\frac{y_L^{*sb}}{\omega_M}\right)}{(1+\alpha)\,\omega_L u'\left(x_L^{*sb}\right)}\right] < 1 \tag{2.9}$$

where

$$0 < \tau_M^* < \tau_L^* = \frac{\gamma_{ML}}{\beta_L - \gamma_{ML}}\left[\frac{v'\left(\frac{y_L^{*sb}}{\omega_L}\right)}{\omega_L u'\left(x_L^{*sb}\right)} - \frac{v'\left(\frac{y_L^{*sb}}{\omega_M}\right)}{(1+\alpha)\,\omega_L u'\left(x_L^{*sb}\right)}\right] < 1$$

High-skilled agents' behavior is not distorted at the margin , while M-type and L-type citizens' are. More precisely:

- $\tau_H^* = 0$. Most efficient agents face a zero marginal tax rate on income revenue; they only pay a positive lump-sum tax on y_H^*.
- Both τ_L^* and τ_M^* are strictly positive. In particular, by monotone hazard rate,

$$\begin{aligned}\tau_L^* &= \frac{\gamma_{ML}}{\beta_L - \gamma_{ML}}\left[\frac{v'\left(\frac{y_L^{*sb}}{\omega_L}\right)}{\omega_L u'\left(x_L^{*sb}\right)} - \frac{v'\left(\frac{y_L^{*sb}}{\omega_M}\right)}{(1+\alpha)\,\omega_L u'\left(x_L^{*sb}\right)}\right] > \\ \tau_M^* &= \frac{\gamma_{HM}}{\beta_M + \gamma_{ML} - \gamma_{HM}}\left[\frac{v'\left(\frac{y_M^{*sb}}{\omega_M}\right)}{(1+\alpha)\,\omega_L u'\left(x_M^{*sb}\right)} - \frac{v'\left(\frac{y_M^{*sb}}{\omega_H}\right)}{(1+\alpha)(1+\rho)\,\omega_L u'\left(x_M^{*sb}\right)}\right] > 0.\end{aligned}$$

- Since (2.6) is binding at the optimum, low-skilled individuals receive a positive lump-sum transfer, while, depending on the value of parameters α, ρ, β_L, β_M and β_H, medium-type ones could be subsidized or taxed by a lump-sum amount.

Corollary 2.2.1 *Being monotonicity condition strictly satisfied at the optimal solution and since both the Local Downward Incentive constraints are binding, too, we can derive that:*

$$\begin{cases} \frac{y_H^{*sb}}{\omega_H} > \frac{y_M^{*sb}}{\omega_M} > \frac{y_L^{*sb}}{\omega_L} \\ x_H^{*sb} > x_M^{*sb} > x_L^{*sb} \end{cases}$$

and that

$$U_H^{*sb} > U_M^{*sb} > U_L^{*sb}$$

Moreover, just like in the two-type case,

$$\begin{cases} U_H^{*sb} > U_H^{*} \\ U_L^{*sb} < U_L^{*} \end{cases},$$

since

$$\begin{cases} \frac{y_H^{*sb}}{\omega_H} = l_H^{*sb} < l_H^{*} \\ x_H^{*sb} > x_H^{*} \\ \frac{y_L^{*sb}}{\omega_L} = l_L^{*sb} > l_L^{*} \\ x_L^{*sb} < x_L^{*} \end{cases}$$

The global effect on M-agents is still ambiguous, and it depends from the parameters.

2.3 Optimal taxation problem in a static open model

As previously, we let now country A's citizen migrate to the laissez-faire country B. The only difference with section (1.3), we have a median type of agents. Still assuming that high skilled citizens could gain from moving, while it is never the case for low ones, depending on medium type migration costs $c_{i,B}$ and $k_{i,B}$, we have to deal with two possible scenarios:

- both high and medium skilled agents can use the threat of migration; it will reduce substantially the redistribution to β_L fraction of people.
- only high-skilled individuals can gain from migration. It will bring to a lower reduction in redistribution, by "punishing" M-agents.

Since the utility abroad is, by assumption, increasing in type, $U_{H,B}$ is at least equal to $U_{M,B}$. Consequently, it is never the case that, ex ante, only medium skilled agents can credibly threat to migrate.

2.3.1 First-Best in a static open economy

If the Social Planner knows citizens' labor productivity, the only inevitable limits to redistribution it has to deal with are the Budget Constraint and the threat of H-type individuals to migrate to country B. Depending on $c_{M,B}$ and $k_{M,B}$, it can be the case that also medium-skilled participation constraint binds at the optimal solution.

Therefore, A's government must solve the following problem:

$$\max_{x_{H,A},y_{H,A},x_{M,A},y_{M,A},x_{L,A},y_{L,A}} W_{f.b.open}\left(U_{H,A},U_{M,A},U_{L,A}\right) = \sum_{i=H,M,L} \beta_i \left[u(x_{i,A}) - v\left(l_{i,A}\right)\right] \quad (2.10)$$

s.t. the Budget Constraint

$$\sum_{i=H,M,L} \beta_i \left(\omega_i l_{i,A} - x_{i,A}\right) \geq 0, \ \ (\lambda) \tag{2.11}$$

H-agents' participation constraint

$$u\left(x_{H,A}\right) - v\left(l_{H,A}\right) \geq \widehat{U}_{H,B}, \ \ (\mu_H) \tag{2.12}$$

and, finally, M-agents' participation constraint

$$u\left(x_{M,A}\right) - v\left(l_{M,A}\right) \geq \widehat{U}_{M,B}, \ \ (\mu_M) \tag{2.13}$$

Solving this problem, gives the following:

Proposition 2.3.1 *At the optimum, $\lambda > 0$.*

- *If both the participation constraints are binding (i.e., $\mu_H > 0$ and $\mu_M > 0$) , using the F.O.Cs of problem* (2.3.1) *and rearranging, gives:*

$$\left(1 + \frac{\mu_H}{\beta_H}\right) u'\left(x^*_{H,A}\right) = \left(1 + \frac{\mu_M}{\beta_M}\right) u'\left(x^*_{M,A}\right) = u'\left(x^*_{L,A}\right) \tag{2.14}$$

and

$$\frac{\left(1 + \frac{\mu_H}{\beta_H}\right) v'\left(l^*_{H,A}\right)}{\omega_L \left(1+\alpha\right)\left(1+\rho\right)} = \frac{\left(1 + \frac{\mu_M}{\beta_H}\right) v'\left(l^*_{M,A}\right)}{\omega_L \left(1+\alpha\right)} = \frac{v'\left(l^*_{L,A}\right)}{\omega_L} \tag{2.15}$$

- *If only* (1.21) *and* (2.12) *constraints are binding at the optimum, while* (2.13) *constraint is not (i.e., $\lambda > 0$, $\mu_H > 0$ and $\mu_M = 0$), we have:*

$$\left(1 + \frac{\mu_H}{\beta_H}\right) u'\left(x^*_{H,A}\right) = u'\left(x^*_{M,A}\right) = u'\left(x^*_{L,A}\right) \tag{2.16}$$

and

$$\frac{\left(1 + \frac{\mu_H}{\beta_H}\right) v'\left(l^*_{H,A}\right)}{\omega_L \left(1+\alpha\right)\left(1+\rho\right)} = \frac{v'\left(l^*_{M,A}\right)}{\omega_L \left(1+\alpha\right)} = \frac{v'\left(l^*_{L,A}\right)}{\omega_L} \tag{2.17}$$

When both high-type and median-type individuals' participation constraints are binding at the optimum, they receive exactly their reservation utility, $\widehat{U}_{H,B}$ and $\widehat{U}_{M,B}$, respectively; H-skilled agents pay a lump-sum tax equal to $\left(c_{H,B} + k_{H,B}\right)$, while M-skilled ones pay a lump-sum tax corresponding to $\left(c_{M,B} + k_{M,B}\right)$.

Since, at the optimum, constraint (2.11) is binding, each low-skilled citizen is subsidized by a lump-sum amount equal to

$$\frac{\beta_H \left(c_{H,B} + k_{H,B}\right) + \beta_M \left(c_{M,B} + k_{M,B}\right)}{\beta_L}.$$

Moreover, no marginal tax rate is levied on labor income. Compared to the *First-best closed economy* setting, both H-type and M-type agents are better off (and, of course, L-type are worse off).

Corollary 2.3.1 *Since, at the optimum, H-type and M-type individuals are net tax payers (paying a type-specific lump-sum tax) and the* (2.11) *is binding, L-agents are subsidized. Therefore, given also that latter's indifference curve is always the steepest in the* $\langle y, x\rangle$ *space, we can say that both high and medium types produce and consume more than low type citizens. Also, we have:*

$$\begin{cases} \omega_H l^*_{H,B} - (c_{H,B} + k_{H,B}) = x^*_{H,B} = \omega_H l^*_{H,A} - T^*_{H,A} = x^*_{H,A} > x^*_{H,cl.ec.} \\ l^*_{H,B} = l^*_{H,A} < l^*_{H,cl.ec.} \end{cases}$$

$$\begin{cases} \omega_M l^*_{M,B} - (c_{M,B} + k_{M,B}) = x^*_{M,B} = \omega_M l^*_{M,A} - T^*_{M,A} = x^*_{M,A} > x^*_{M,cl.ec.} \\ l^*_{M,A} < l^*_{M,cl.ec.} \end{cases}$$

where

$$\begin{cases} T^*_{H,A} = c_{H,B} + k_{H,B} \\ T^*_{M,A} = c_{M,B} + k_{M,B} \end{cases}$$

and

$$\begin{cases} x^*_{L,A} > x^*_{L,cl.ec.} \\ l^*_{L,A} < l^*_{L,cl.ec.} \end{cases}$$

Conversely, if (2.13) is not binding, we have what is called "*the curse*" of medium productive citizens. We can think that M-type's global cost of migration (i.e., $c_{M,B} + k_{M,B}$) is so relevant, that M-citizens strictly prefer to stay in country A, than moving abroad. Compared to the previous case, they are worse off, since they optimally consume as much as low-labor productive agents, while working more. Conversely, high-skilled agents are unaffected, since they get the same bundle as the above-mentioned case ($\mu_H > 0$).

Corollary 2.3.2 *Therefore, we have*

$$\begin{cases} \omega_H l^*_{H,B} - (c_{H,B} + k_{H,B}) = x^*_{H,B} = \omega_H l^*_{H,A} - T^*_{H,A} = x^*_{H,A} > x^*_{H,cl.ec.} \\ l^*_{H,B} = l^*_{H,A} < l^*_{H,cl.ec.} \end{cases}$$

with

$$\begin{cases} T^*_{H,A} = c_{H,B} + k_{H,B} \\ T^*_{M,A} < c_{M,B} + k_{M,B} \end{cases}$$

and

$$\begin{cases} x^*_{M,A} = x^*_{L,A} \\ l^*_{M,A} > l^*_{L,A} \end{cases}$$

2.3.2 Second-Best in a static open economy

As usual, we now move to the second-best environment, by assuming that the Social Planner is unable to observe individual productivity. When maximizing the social welfare, in principle, it has to deal with the budget constraint, six incentive constraints (two for each type) and two participation constraints (one for high- and one for medium-types). However, we can neglect low-type incentive constraints, since, given the assumptions on migration costs (i.e., $(c_{i,B} + k_{i,B}) > 0$, $i = L, M, H$), we guess that it is never optimal for L-agents to mimic other types' optimal productions[2]. Therefore, the tax authority will face the following maximization problem:

$$\max W_{s.b.open}\left(U_{H,A};U_{M,A};U_{L,A}\right)=\sum_{i=H,M,L}\beta_i\left[u(x_{i,A})-v\left(\frac{y_{i,A}}{\omega_i}\right)\right] \tag{2.18}$$

s.t.

$$\sum_{i=H,M,L}\beta_i\left(y_{i,A}-x_{i,A}\right)\geq 0,\ \ (\lambda) \tag{2.19}$$

$$u(x_{H,A})-v\left(\frac{y_{H,A}}{\omega_H}\right)\geq u(x_{M,A})-v\left(\frac{y_{M,A}}{\omega_H}\right),\ \ (\gamma_{HM}) \tag{2.20}$$

$$u(x_{H,A})-v\left(\frac{y_{H,A}}{\omega_H}\right)\geq u(x_{L,A})-v\left(\frac{y_{L,A}}{\omega_H}\right),(\gamma_{HL}) \tag{2.21}$$

$$u(x_{M,A})-v\left(\frac{y_{M,A}}{\omega_M}\right)\geq u(x_{H,A})-v\left(\frac{y_{H,A}}{\omega_M}\right),\ \ (\gamma_{MH}) \tag{2.22}$$

$$u(x_{M,A})-v\left(\frac{y_{M,A}}{\omega_M}\right)\geq u(x_{L,A})-v\left(\frac{y_{L,A}}{\omega_M}\right),\ \ (\gamma_{ML}) \tag{2.23}$$

$$u\left(x_{H,A}\right)-v\left(\frac{y_{H,A}}{\omega_H}\right)\geq \widehat{U}_{H,B},(\mu_H) \tag{2.24}$$

$$u\left(x_{M,A}\right)-v\left(\frac{y_{M,A}}{\omega_M}\right)\geq \widehat{U}_{M,B},(\mu_M) \tag{2.25}$$

From the solution of Social Planner's problem (2.3.2) , we can derive some Lemmas.

Lemma 2.3.1 (2.19) *constraint is always binding at the optimum.*

If it wasn't the case, it would have been possible to raise low-agents' consumption, therefore increasing the Social Welfare level, up to the equality between LHS and RHS of the social budget constraint.

Lemma 2.3.2 *If* (2.20) *constraint is binding,* (2.22) *constraint is not (and vice versa).*

[2]We will give the intuition, by assuming ex-ante it is possible for L-agents to pretend to be medium-productive citizens (the same argument is valid for L-type mimicking H-type). First, if, at the constrained optimum, M-workers strictly prefer (or are indifferent between the two options) to mimic L-ones, rather than to emigrate to B, it is straightforward that L-agents are strictly better off when revealing their true type, than mimicking M-citizens.

Therefore, only if $\widehat{U}_{M,B}>u(x_{L,A})-v\left(\frac{y_{L,A}}{\omega_M}\right)$, it may be the case that the local upward incentive constraint

$$u\left(x_{L,A}\right)-v\left(\frac{y_{L,A}}{\omega_L}\right)\geq u\left(x_{M,A}\right)-v\left(\frac{y_{M,A}}{\omega_L}\right)$$

is binding at the optimum. However, when receiving $\widehat{U}_{M,B}$, if high-type's local downward incentive compatibility constraint is slack, $\tau_M^{*op.sb}$ is not positive. Therefore, M-agents pay, at least, a lump-sum tax equal to $c_{M,B}+k_{M,B}$ to subsidize L-ones. Since, in $\langle y,c\rangle$ space, L-indifference curve is always steeper, than M-indifference curve, given L-type lump-sum subsidy and M-type lump sum tax, L-workers' local upward incentive compatibility constraint is strictly satisfied.

Since, in $\langle y, x \rangle$ space, M-citizens' indifference curve is always steeper than H-agents' one, it is not possible that the two curves cross each other twice. Therefore, (2.3.2) is a direct consequence of single-crossing condition.

Lemma 2.3.3 *If* (2.24) *constraint is not binding at the optimum,* (2.20) *is.*

If H-type agents' migration opportunity does not provide them an utility level higher than what they get at the optimum, in the second-best closed economy scenario, the latter result remains valid. Even if migration is not a credible threat, in order to limit redistribution from them to low-skilled agents, high-skilled workers can still threaten to mimic M-skilled ones. Therefore, they are levied a lump-sum tax until (2.20) constraint is binding.

Lemma 2.3.4 *If* (2.25) *constraint is slack at the optimum,* (2.23)*, or both* (2.23) *and* (2.22) *are binding.*

If M-type agents' migration is not a credible threat, depending on high- and low-skilled's level of utility, they can mimic L-type agents, just like in second-best scenario, or both low- and high-skilled agents. This latter case may arise, if H-citizens' reservation utility, i.e., $\widehat{U}_{H,B}$, is sufficiently high to make M-agents better off when mimicking H-ones.

Lemma 2.3.5 (2.21) *is always slack at the optimum.*

Depending on both high- and medium-skilled's outside opportunity, i.e., $\widehat{U}_{H,B}$ and $\widehat{U}_{M,B}$, different equilibria may arise. In order to present all possible solutions, we start assuming that neither (2.24), nor (2.25) constraints are binding at the optimum. Then, we let only $\widehat{U}_{H,B}$ raise, to stress the negative effect of high-skilled's increased mobility on social welfare. Given globalization, these are the most likely, and, therefore, interesting, equilibria to be faced in OECD countries, since, as shown in OECD data (OECD, 2002, 2008), many developed countries' government are worried by high-skilled's migration to tax havens and almost laissez-faire countries. Finally, in order to cover also different possible scenarios, we let $\widehat{U}_{M,B}$ arises, too. Clearly, such scenarios are not so common for two simple reasons: i) globalization has multiplied OECD highly-skilled citizens' migration opportunity, rather than medium- and low-skilled agents' ones; ii) often the median voter is a M-type agent. Therefore, an elected democratic government will favor him. However, in very poor countries, or in economies with a strong redistributive target, it may be the case that, as well as high-skilled agents, medium skilled citizens are attracted by foreign opportunities, too.

We will not analyze the case where (2.25) is binding at the optimum, while (2.24) is not. It arises when $\widehat{U}_{M,B}$ gives M-type a higher utility level, than the constrained optimal *Second-Best closed economy* allocation, while high-skilled agents strictly prefer their second-best closed economy optimal allocation, rather than $\widehat{U}_{H,B}$ (even if, by assumption, $\widehat{U}_{H,B} > \widehat{U}_{M,B}$). In such a scenario, (2.20) is tightened, by the increased M-utility level; while (2.23) is relaxed; however, the latter constraint may be still binding, or slack, if $\widehat{U}_{M,B}$ is high enough.

Case 2.3.1 (2.19)*,* (2.20) *and* (2.23) *are binding at the optimum.*

This case arises when both high- and medium-skilled agents' migration opportunities are slack at the second-best closed economy optimal allocation. Therefore, the Social Planner's problem in open economy, coincides with the problem in closed economy.

Let us assume that $\widehat{U}_{H,B}$ is such that *high-skilled workers are indifferent between moving abroad and mimicking medium agents* (i.e., both (2.24) and (2.20) are binding at the constrained optimum). Depending on M-agents' outside opportunity, we can face three different scenarios.

Case 2.3.2 (2.19), (2.24), (2.20) *and* (2.23) *are binding at the optimum.*

Since (2.24) is binding, H-agents are better off than in closed economy setting. Since high-type's incentive compatibility constraint (2.20) is binding, too, by Lemma (2.3.2), M-agents are not attracted by mimicking high-skilled ones. Hence no negative marginal tax rate is levied on H-types. They only pay a lower lump-sum tax, equal to $(c_{H,B} + k_{H,B})$. Therefore, the Social Planner is able to redistribute less consumption good to low-agents. If possible, it would offset this loss, increasing M-citizens' tax burden. However, this solution is unfeasible, since medium agents can credibly threat to mimic low ones. Therefore, both M- and L-skilled workers are worse off and (2.23) is binding.

Under case (2.3.2), Social Planner's problem is to maximize (2.18), subject to (2.19), (2.24), (2.20) and (2.23) constraints.

Proposition 2.3.2 *At the optimal allocation, all the relevant constraints are binding. Moreover,*

$$MRS_{y_{H,A},x_{H,A}} = \frac{v'\left(\frac{y^{*op.sb}_{H,A}}{\omega_H}\right)}{(1+\alpha)(1+\rho)\,\omega_L u'\left(x^{*op.sb}_{H,A}\right)} = 1, \tag{2.26}$$

$$\begin{aligned} MRS_{y_{M,A},x_{M,A}} &= 1 - \frac{\gamma_{HM}}{\beta_M + \mu_M - \gamma_{HM}}\left[\frac{v'\left(\frac{y^{*op.sb}_{M,A}}{\omega_M}\right)}{(1+\alpha)\,\omega_L u'\left(x^{*op.sb}_{M,A}\right)} - \frac{v'\left(\frac{y^{*op.sb}_{M,A}}{\omega_H}\right)}{(1+\alpha)(1+\rho)\,\omega_L u'\left(x^{*op.sb}_{M,A}\right)}\right] \\ &< 1, \end{aligned} \tag{2.27}$$

$$MRS_{y_{L,A},x_{L,A}} = 1 - \frac{\gamma_{ML}}{\beta_L - \gamma_{ML}}\left[\frac{v'\left(\frac{y^{*op.sb}_{L,A}}{\omega_L}\right)}{\omega_L u'\left(x^{*op.sb}_{L,A}\right)} - \frac{v'\left(\frac{y^{*op.sb}_{L,A}}{\omega_M}\right)}{(1+\rho)\,\omega_L u'\left(x^{*op.sb}_{L,A}\right)}\right] < 1, \tag{2.28}$$

where

$$\tau^{*op.sb}_{L,A} = \frac{\gamma_{ML}}{\beta_L - \gamma_{ML}}\left[\frac{v'\left(\frac{y^{*sb}_{L,A}}{\omega_L}\right)}{\omega_L u'\left(x^{*sb}_{L,A}\right)} - \frac{v'\left(\frac{y^{*sb}_{L,A}}{\omega_M}\right)}{(1+\rho)\,\omega_L u'\left(x^{*sb}_{L,A}\right)}\right] > 0,$$

$$\begin{aligned} \tau^{*sb}_{M,A} &> \tau^{*op.sb}_{M,A} = \frac{\gamma_{HM}}{\beta_M + \mu_M - \gamma_{HM}}\left[\frac{v'\left(\frac{y^{*op.sb}_{M,A}}{\omega_M}\right)}{(1+\alpha)\,\omega_L u'\left(x^{*op.sb}_{M,A}\right)} - \frac{v'\left(\frac{y^{*op.sb}_{M,A}}{\omega_H}\right)}{(1+\alpha)(1+\rho)\,\omega_L u'\left(x^{*op.sb}_{M,A}\right)}\right] \\ &> 0. \end{aligned}$$

H-agents are better off compared to the second best closed economy scenario, since they get exactly $\widehat{U}_{H,B}$. Moreover, $\tau^{*}_{H,A} = 0$; they only pay a (reduced) lump sum tax on $y^{*op.sb}_{H,A}$, equal to $(c_{H,B} + k_{H,B})$. Both M- and L-agents are worse off, compared to the second best closed economy scenario. However, optimal marginal tax rate on M-income is lower, than in closed setting, since high-type's participation constraint allows Social Planner to afford less distortion in M-citizens' production. L-agents are worse off, too, since the increased transfer from medium-skilled individuals can only partially offset the decrease of transfer from high-skilled ones.(the bigger is β_M compared to β_H, the lower will be low-skilled citizens' loss). Moreover, since (2.23) is binding at the constrained optimum, they are downward distorted at the margin, facing a positive marginal tax rate on their labor income $\tau^{*op.sb}_{L,A}$.

Case 2.3.3 (2.19), (2.24), (2.25), (2.20) *and* (2.23) *are binding at the optimum.*

This case arises when high-skilled and medium-skilled agents' outside opportunities, i.e., $\widehat{U}_{H,B}$ and $\widehat{U}_{M,B}$, respectively, are such that, at the optimum, these former are indifferent between emigrating to country B, or mimicking M-agents, while these latter are indifferent between moving abroad, or pretending to be L-agents.

At scenario (2.3.3) the Social Planner maximize (2.18) under (2.19), (2.24), (2.25), (2.20) and (2.23) constraints.

Proposition 2.3.3 *At case* (2.3.3) *constrained optimum,* $\lambda > 0$, $\gamma_{HM} > 0$, $\gamma_{ML} > 0$, $\mu_H > 0$ *and* $\mu_M > 0$. *Furthermore,*

$$MRS_{y_{H,A},x_{H,A}} = 1,$$

$$MRS_{y_{M,A},x_{M,A}} < 1,$$

$$MRS_{y_{L,A},x_{L,A}} < 1.$$

Under case (2.3.3), H-agents are levied only a lump-sum tax equal to their total migration costs. Conversely, both M- and L-citizens' productions are marginally downward distorted by type-depedent marginal tax rates, $\tau^{*op.sb}_{M,A} > 0$ and $\tau^{*op.sb}_{L,A} > 0$. However, both high-skilled and medium-skilled are better off, than in second-best closed case, since they receive exactly $\widehat{U}_{H,B}$ and $\widehat{U}_{M,B}$, respectively. Hence, low-skilled are worse off (otherwise, second-best closed economy optimal allocation would not optimal), even if M-type's increased outside option allows the Government to levy a low-type's marginal tax rate $\tau^{*op.sb}_{L,A}$ lower than in closed setting.

Case 2.3.4 (2.19), (2.24), (2.25) *and* (2.20) *are binding at the optimum.*

$\widehat{U}_{M,B}$ is sufficiently high, to make high-skilled agents being indifferent between the utility they can gain by mimicking M-ones, or by emigrating to B. Moreover, $\widehat{U}_{M,B}$ is such that intermediate-citizens cannot credibly threaten to mimic both high-skilled and low skilled fractions of people.

Under case (2.3.4), the tax authority maximizes (2.18), subject to (2.19), (2.24), (2.25) and (2.20) constraints.

Proposition 2.3.4 *At the constrained optimum,* (2.19), (2.24), (2.25) *and* (2.20) *constraints are binding. Moreover,*

$$MRS_{y_{H,A},x_{H,A}} = 1 \tag{2.29}$$

$$MRS_{y_{M,A},x_{M,A}} < 1 \tag{2.30}$$

$$MRS_{y_{L,A},x_{L,A}} = \frac{v'\left(\frac{y_{L,A}^{*sb}}{\omega_H}\right)}{\omega_L u'\left(x_{L,A}^{*sb}\right)} = 1 \tag{2.31}$$

Under Proposition (2.3.4), H-agents' utility level coincides with $\widehat{U}_{H,B}$. Moreover, $\tau_{H,A}^{*} = 0$; they only pay a (even more reduced) lump sum tax on $y_{H,A}^{*op.sb}$, equal to $(c_{H,B} + k_{H,B})$. M-agents receive exactly $\widehat{U}_{M,B}$. However, being constraint (2.20) binding at the optimal allocation, they middle-class citizens face a positive marginal tax rate on their labor income, $\tau_{M,A}^{*op.sb} > 0$. Finally, since both H- and M- types are, thanks to migration opportunity, better off, and being (2.19) binding at the optimal solution, just like under case (2.3.3) L-individuals are worse off, compared to closed setting. Low-skilled agents face no marginal tax rate, since (2.23) is strictly satisfied at the optimal allocation, and receive a, limited but still positive, lump-sum subsidy.

Let us now assume that $\widehat{U}_{H,B}$ is high enough, and, therefore, is such that *high-skilled workers are strictly better off by emigrating to laissez-faire country B*, rather than *by pretending to be medium agents* (i.e., (2.24) constraint is binding at the optimum, while (2.20) is not). Furthermore, we assume that (2.25) is strictly satisfied at the optimum. Two possible scenarios may arise.

Case 2.3.5 (2.19), (2.24) *and* (2.23) *are binding at the optimum.*

This case is likely to arise in developed countries with redistribution as a main political goal. High-skilled agents' must receive their abroad utility level, i.e., $\widehat{U}_{H,B}$, while neither L-type nor M-type citizens have credible outside options. Hence, since H-agents only pay a small lump-sum tax, the Social Planner would desire to offset this loss, by levying a relevant tax burden on M-income. As a consequence, a worker belonging to middle-class can credibly mimicking a low-skilled one.

Therefore, Social Planner's problem is to maximize (2.18), subject to (2.19), (2.24) and (2.23).

Proposition 2.3.5 *At the optimal allocation under scenario* (2.3.5), (2.19), (2.24) *and* (2.23) *are all binding. Moreover,*

$$MRS_{y_{H,A},x_{H,A}} = 1 \tag{2.32}$$

$$MRS_{y_{M,A},x_{M,A}} = 1 \tag{2.33}$$

$$MRS_{y_{L,A},x_{L,A}} < 1 \tag{2.34}$$

At the optimal solution of the social welfare maximization problem, under case (2.3.5), we have that H−agents are better off compared with the second best closed economy scenario, since the Social Planner, in order to keep them in country A, has to give exactly what they could get abroad, i.e., $\widehat{U}_{H,B}$. Moreover, $\tau_{H,A}^{*} = 0$; they only pay a (reduced) lump sum tax on $y_{H,A}^{*op.sb}$, equal to $(c_{H,B} + k_{H,B})$, since (2.22) constraint is strictly satisfied at the optimal allocation. More, even if $\tau_{M,A}^{*op.sb} = 0$, medium-skilled agents are worse off, compared both with the second-best closed economy scenario and with (2.3.2) case, since, with respect to this latter setting, $\widehat{U}_{H,B}$, ceteris paribus, has to be higher. As a consequence, the increased high-productive agents' reservation utility reduces the redistribution to the lowest skilled ones. This fact relaxes (2.23); therefore, the tax authority can levy additional lump-sum tax on medium-type citizens in order to subsidize β_L fraction of population (until (2.23) is binding) , under (2.19) constraint. Last, with respect to both closed economy and (2.3.2) cases, L-agents are worse off, too, since the transfer from medium-skilled individuals can only partially offset the decrease of subsidies from high-productive workers.(the bigger is $\frac{\beta_M}{\beta_H}$ fraction, the lower is low-skilled citizens' loss). Moreover, they are downward distorted at the margin, facing a positive marginal tax rate on their labor income, i.e. $\tau_{L,A}^{*op.sb} > 0$.

Case 2.3.6 *(2.19), (2.24), (2.22) and (2.23) are binding at the optimum.*

Starting from (2.3.5), assume high-skilled's outside opportunity $\widehat{U}_{H,B}$ further increases, (i.e., H-agents' migration costs are further lower) This may be seen as a consequence of increasing globalization. Hence, it may be the case that M-individuals can have an incentive to mimic H-ones, i.e., *countervailing incentive is a credible issue* for the Social Planner. In order to avoid pooling at the top, most skilled agents' production will be optimally upward distorted at the margin. Moreover, medium-skilled agents still have an incentive to mimic L-ones, since, under this scenario, $\widehat{U}_{M,B}$ is strictly lower than the utility M-agents can get by preteding to be low-skilled ones. As usual, in order to avoid pooling at the bottom, least-skilled agents' production will be downward distorted at the margin.

The Social Planner will maximize (2.18), subject to (2.19), (2.24), (2.22) and (2.23).

Proposition 2.3.6 *At the constrained optimum, the relevant constraints are all binding, i.e.,* $\lambda > 0$, $\gamma_{HM} > 0$, $\mu_H > 0$. *Moreover,*

$$MRS_{y_{H,A},x_{H,A}} = 1 + \frac{\gamma_{MH}}{\beta_L + \mu_H - \gamma_{MH}} \left[\frac{v'\left(\frac{y_{H,A}^{*op.sb}}{\omega_M}\right)}{\omega_L (1+\rho) u'\left(x_{H,A}^{*sb}\right)} - \frac{v'\left(\frac{y_{H,A}^{*op.sb}}{\omega_H}\right)}{(1+\alpha)(1+\rho)\omega_L u'\left(x_{H,A}^{*op.sb}\right)} \right] > 1, \quad (2.35)$$

where

$$\tau_{H,A}^{*op.sb} = \frac{\gamma_{MH}}{\beta_L + \mu_H - \gamma_{MH}} \left[\frac{v'\left(\frac{y_{H,A}^{*op.sb}}{\omega_M}\right)}{\omega_L (1+\rho) u'\left(x_{H,A}^{*op.sb}\right)} - \frac{v'\left(\frac{y_{H,A}^{*op.sb}}{\omega_H}\right)}{(1+\alpha)(1+\rho)\omega_L u'\left(x_{H,A}^{*op.sb}\right)} \right] > 0,$$

$$MRS_{y_{M,A},x_{M,A}} = 1, \quad (2.36)$$

$$MRS_{y_{L,A},x_{L,A}} < 1. \quad (2.37)$$

High-skilled agents receive exactly $\widehat{U}_{H,B}$. However, since, under case (2.3.6) we assumed that H-type's migration opportunity $\widehat{U}_{H,B}$ is higher, because of M-agents' threat of mimicking high-skilled workers, i.e., (2.22) is binding,most skilled individuals' production is upward distorted at the margin through an implicit negative marginal tax rate $\tau_{H,A}^{*op.sb} < 0$. This tax schedule will reduce (up to indifference) the incentive of M-citizens of pretending to be H-ones. Hence, since these latter are marginally subsidized on their labor income, they, must pay a lump-sum tax higher than $(c_{H,B} + k_{H,B})$. Conversely, M-agents are not distorted at the margin, i.e., $t_{M,A}^{*op.sb} = 0$. It is reasonable to say that they are worse off, with respect to (2.3.6) case. Indeed, the increased high-type's outside opportunity level $\widehat{U}_{H,B}$ reduces the redistribution to low-skilled individuals.

This relaxes the incentive of medium-type population to mimic them, i.e., relaxes (2.23) constraint. Therefore, Social Planner can increase M-agents' lump-sum tax (or, depending on parameters, to further reduce their lump-sum subsidy) until (2.23) constraint is, again, binding. Finally, low-skilled agents are worse off, too, since the increased $\widehat{U}_{H,B}$ level affects them negatively. Moreover, in order to avoid a partial pooling of medium- and low-types, these latter are downward distorted at the margin, facing a positive marginal tax rate on their labor income $\tau^{*op.sb}_{L,A} > 0$.

Again, let us assume that *high-skilled workers are strictly better off by emigrating to laissez-faire country B*, rather than *by pretending to be medium agents,* and that medium-types' migration threat is credible. This means that $\widehat{U}_{M,B}$ is strictly higher than the utility M-agents receive at the second-best closed economy optimum.

Case 2.3.7 (2.19), (2.24)*,* (2.25) *and* (2.23) *are binding at the optimum.*

This scenario arises when both M- and H-agents have a credible outside option. Their participation constraints are binding at the optimal second-best closed economy setting. Moreover, $\widehat{U}_{H,B}$ is strictly higher than the RHS of (2.20); therefore, there is no need of a downward distortion of intermediate agents. Conversely, even if these latter are better off, they are still indifferent between moving abroad and pretending to be low-type agents. Hence, a positive marginal tax rate on L-type's income is optimal. Just like the following (2.3.8) case, this scenario is common in country with significant redistributional goals, where moving abroad is a really profitable option for H-type (such that (2.20) constraint is slack, but not enough to make M-agents, at least, indifferent between emigrating and mimicking high-skilled people) and is a credible threat for M-agents, too.

Therefore, the Social Planner maximizes (2.18), subject to (2.19), (2.24), (2.25) and (2.23).

Proposition 2.3.7 *At the optimal allocation, with all the relevant constraints, i.e.,* (2.19), (2.24)*,* (2.25) *and* (2.23), *being binding, we have*

$$MRS_{y_{H,A},x_{H,A}} = 1,$$

$$MRS_{y_{M,A},x_{M,A}} = 1,$$

and

$$MRS_{y_{L,A},x_{L,A}} < 1.$$

Qualitatively, this case is similar to (2.3.5) case. However, under case (2.3.7) M-agents are better off, since they get exactly $\widehat{U}_{M,B}$. Hence, it is optimal to reduce $\tau^{*op.sb}_{L,A}$, until (2.23) is binding. Even if L-agents' optimal income is less downward distorted, assuming $\widehat{U}_{H,B}$ coincident with the utility level high-skilled workers obtain in (2.3.5) case (i.e., $c_{H,B} + k_{H,B}$ unchanged), since M-agents are better off, low-skilled are worse off. Both M- and H-citizens are levied a personalized lump-sum tax, equal to type-specific migration costs, while low-type ones received a lump-sum subsidy and face a positive marginal tax rate on their labor income, i.e., $\tau^{*op.sb}_{L,A} > 0$.

Case 2.3.8 (2.19), (2.24) *and* (2.25) *are binding at the optimum.*

This case arises when both high-skilled and medium skilled agents' outside opportunity levels, i.e., $\widehat{U}_{H,B}$ and $\widehat{U}_{M,B}$, are sufficiently high to relax all their incentive compatibility constraints. Actually, a higher $\widehat{U}_{M,B}$ tightens (2.20), but not enough to make the most productive workers indifferent between threaten to moving abroad and mimicking medium-productive citizens.This scenario may be faced in developed countries with important redistributional target, when migration costs are relatively low (for example, migration within EU). Qualitatively, case (2.3.8) corresponds to the, already analyzed, *First-Best one-period open economy* scenario. As already shown, there is no need for distorting marginal tax rate on labor income. Consequently, high- and medium-type workers receive exactly what they would get abroad, respectively, $\widehat{U}_{H,B}$ and $\widehat{U}_{M,B}$, paying lump-sum taxes equal to their own migration costs (respectively, $(c_{H,B}+k_{H,B})$ and $(c_{M,B}+k_{M,B})$). each L-agent receives a lump-sum subsidy corresponding to $\frac{\beta_H(c_{H,B}+k_{H,B})+\beta_M(c_{M,B}+k_{M,B})}{\beta_L}$.

Case 2.3.9 (2.19), (2.24), (2.25) *and* (2.22) *are binding at the optimum.*

Whenever $\widehat{U}_{H,B}$ is so high, that, not only (2.20) constraint is no longer an issue, but M-agents can credibly mimick to be H-ones, countervailing incentives may, as in case (2.3.6), arise. The only novelty in this scenario is that M-agents can gain from moving abroad, too. Therefore, with respect to case (2.3.6), middle-class citizens are better off, since, at the optimum, they get, exactly, $\widehat{U}_{M,B}$. Since both M- and H-agents receive they what they would obtain if emigration to laissez-faire country B, low-skilled fraction of citizens are, now, worse off.

Since we assumed that medium-productive workers are indifferent (at the constrained optimum) between mimic high-agents and emigrating, in order to avoid pooling at the top, the government finds optimal to distort upward these latter' marginal production, up to indifference of M-agents.

Therefore, Social Planner's problem is to maximize (2.18), subject to (2.19), (2.24), (2.25) and (2.22).

Proposition 2.3.8 *At the optimal solution,*

$$MRS_{y_{H,A},x_{H,A}} > 1, \tag{2.38}$$

$$MRS_{y_{M,A},x_{M,A}} = 1, \tag{2.39}$$

$$MRS_{y_{L,A},x_{L,A}} = 1.$$

At the optimal solution, both H- and M-workers receive an utility equal to their outside opportunity. However, high-agents' production is marginally upward distorted, i.e., $\tau_{H,A}^{*op.sb} < 0$, since the Social Planner wants to avoid medium-skilled agents mimicking them. They also are levied a positive lump-sum tax, higher than $(c_{H,B}+k_{H,B})$, while M-agents pay a lump-sum amount coincident with their migration costs, i.e., $c_{M,B}+k_{M,B}$. Low-skilled agents face a reduction in their lump-sum subsidy; more, the higher $\widehat{U}_{H,B}$ and $\widehat{U}_{M,B}$, the lower is L-agent lump-sum subsidy. Finally, no marginal tax rate is levied on low-labor productive agents' output.

2.4 Conclusions

In Chapter 1, we have analyzed the effect of high skilled workers' migration threat on social welfare. As we have shown, in both a static and two-period framework, the redistribution from H-agents to L-type ones is even more reduced, if high type's participation constraints are binding. This second-best solution is far away from the first best one. However, the model in Chapter 1 lies on some simplifying assumptions, which affect our results. In particular, (i) the absence of country B's citizens as possible immigrants in country A and (ii) the existence of only two types of agents, i.e. H- and L-individuals. In this Chapter we generalized our analysis by introducing a third type of agents, endowed with intermediate skill. Such an extension allow us to better analyze the effects on redistribution of both adverse selection and migration opportunities arising simultaneously. Even in a static framework, the presence of middle class agents has a relevant effect on results. In particular, in a *first-best open economy* setting, we find the well known "*curse*" of middle-skilled workers, if H-type's outside option is not binding at the optimum, while H-type's one is. Otherwise, if both H- and M-agents can credibly threahen to moving abroad, the redistribution to low-skilled citizens is heavily bounded. In the second-best scenario, the number of possible solution has increased dramatically. Depending on the utility that both H- and M-agents can get by moving abroad, or by mimicking a different type of agents, we obtain different optimal allocations. In particular, when high-skilled citizens' utility from migration is so high that M-agents are better off when mimicking H-ones, rather than revealing their own type, the Social Planner deals with binding countervailing incentives. Given globalization, such scenarios are always more likely to be faced, since developed countries' redistributional goal conflicts with increasing high-skilled's migration opportunity. For this reason, we believe that this topic deserves attention and further investigation. In particular, we leave to future research the study of optimal tax policy in a dynamic context. It may be the case that for the Social Planner it is optimal to let a certain fraction of high and medium type individuals go abroad in period 1. The effects of this strategy are twofold. In period 1, such a policy could make a (partial) separating equilibrium less costly. It is worth noting that in such a framework, migrated agents may have an incentive to come back to country A. This is due to the fact that the Social Planner cannot distinguish their type. Differently, in the two-type case, the only agents who credibly threat to migrate are the high-skilled ones. Therefore, by moving abroad, indirectly they would reveal themselves. Another relevant extension is a two-country setting model, with possibility of emigration and immigration. As a consequence, a proper choice of the social welfare function to be used, becomes necessary. Indeed, an utilitarian social welfare function may imply that a huge low-skilled country is preferable. Conversely, a per-capita social welfare function may imply that it is optimal to let all low-skilled agents migrate.

Chapter 3

Optimal Taxation of Capital with asymmetric information

3.1 Introduction

In this Chapter we analyze the effects of optimal entrepreneurial capital taxation on social welfare when effort and individual labor productivity may be private information. In an economy populated by two continua of entrepreneurs, differing in their skills, if effort is private information, individuals are subject to dynamic moral hazard. Under this scenario, if the Social Planner aims to induce all agents to exert high effort, it is socially optimal that both types of citizens are partially insured. Moreover, in order to discourage savings, each fraction of population face a positive type-dependent intertemporal wedge. Then, we study what happens when adverse selection arises. Under the same assumptions about Social Planner's desires, we show that is optimal to full insure high-skilled agents and to partially insure low-skilled ones. Moreover, no high-type aggregate intertemporal wedge is implemented, while a positive L-aggregate intertemporal wedge is. Finally, we study the effects of both dynamic moral hazard and adverse selection on social welfare, finding that both type are partially insured and that the interaction between moral hazard and adverse selection amplifies each other.

There is a recent, but growing literature, called New Dynamic Public Finance, that applies Mirrlees (1971) approach to optimal taxation of labor extending it in a dynamic framework. The main goal is to analyze the trade-off between insurance and incentive. A specific branch of this research is related to optimal taxation of entrepreneurial activity. Golosov, Kocherlakota and Tsyvinsky (2003) show that, with additive separable utility function, if it optimal that future consumption is random given present information, distortions on consumption (wedges) path arise. Albanesi and Sleet (2006) use a similar utility function, finding a positive intertemporal wedge between the social optimum and the individual one. Furthermore, optimal marginal tax rate on capital income decreases in setting with labor risk. Albanesi (2006) studies optimal taxation of entrepreneurial capital with private information and multiple assets. In her setup, entrepreneurial activity is subject to a dynamic moral hazard problem and entrepreneurs face idiosyncratic capital risk. Given this framework she shows that differential asset taxation is optimal. Moreover marginal taxes on bonds are proven to depend on the correlation of their returns with idiosyncratic capital risk. In such a context, therefore, entrepreneurial capital always receives a subsidy relative to other assets in the bad states.

We focus, basically, on Albanesi (2006) work and we try to extend it. It is worth noting that Albanesi's analysis is based on the assumption that only one type of entrepreneur exists. However, entrepreneurial ideas and skill are not equally distributed among people. For this reason, in this Chapter we depart from the Albanesi framework by introducing both low- and high-skilled agents. As a consequence, we will also be able to study optimal taxation under adverse selection. If effort is private information, while labor productivity is common knowledge, we find results close to Albanesi (2006): both high-skilled agents and low-skilled agents are partially insured. Moreover, there are positive intertemporal wedges between both medium- and high-type's aggregate return of investment and their intertemporal marginal rate of substitution. Conversely, when only labor productivity is private information, high-skilled agents are fully insured, and no H-aggregate intertemporal wedge arises, while low-skilled are over-insured. In addition, a positive L-aggregate intertemporal wedge is optimal. Finally, we study the effects of both dynamic moral hazard and adverse selection on social welfare, finding that both type are partially insured and both H-aggregate intertemporal wedge and L-aggregate intertemporal wedge are positive.

In section 2 we describe the basic setup. In section 3, we analyze a model where effort is private information. Then, in section 4, we focus on adverse selection, assuming that labor productivity is private information, while effort is not. In section 5, we analyze a model where both effort and labor productivity are private information. Finally, in section 6, we give a brief conclusion. Proofs of propositions are in appendix.

3.2 The Set Up

Let us introduce a two-period economy, that is populated by two types of entrepreneurs: low- (L) and high-productivity (H) ones. Denoting their productivity by θ_i, $i = H, L$, we assume that the inequality $\theta_H > \theta_L$ always holds. Moreover, productivity parameters do no change over time.

We also assume a continuum of entrepreneurs. Normalizing to one their distribution, the fraction of high-skilled workers (H-agents, hereafter) then be $\alpha \in (0,1)$, and hence, $(1-\alpha) \in (0,1)$ will be the fraction of low-skilled ones (L-agents). By assumption, agents live for two periods and share identical additive and separable (across arguments) utility functions, given by:

$$U = u(c_0) + \beta u(c_1) - v(e),$$

where c_t is consumption at time t, with $t = 0, 1$, $e = \{0, 1\}$ is effort spent at time $t = 0$ and $\beta \in (0,1)$ is the relevant discount factor. As can be seen, effort is dichotomous and is optimally chosen after the agent sets the consumption level at time 0 (c_0).

As usual, $u(c_t)$, $t = 0, 1$ $(u' > 0,\ u'' < 0\ \lim_{c_t \to 0} u'(c_t) = \infty)$ is the utility of consumption in period 0 and period 1, while $v(e)$ $(v' > 0,\ v'' > 0)$ is the disutility of effort.

It is worth noting that agents choose not only their own consumption levels over time, but also decide how much to invest at time 0. In other words, given the initial endowment of resources $k_0 > 0$ (in terms of units of consumption good), the entrepreneur chooses k_1, with $0 \leq k_1 \leq k_0$. At time 1, agent i will receive a gross return equal to

$$R(k_{i1}, x; \theta_i) = k_{i1}(1 + x),$$

where x is a random variable. By assumption its realization of x will be either $\overline{x}$,with probability $\pi(\theta_i, e_i)$, or $\underline{x}$, with probability $1 - \pi(\theta_i, e_i)$, with $\overline{x} > \underline{x}$.

As usual, the probability $\pi(\theta_i, e_i)$ is a positive function of effort. Moreover, the inequality $\pi(1|\theta_i) > \pi(0|\theta_i)$, $i = H, L$, holds. This means that an agent endowed with type θ has a higher expected investment return by exerting $e = 1$, rather than $e = 0$ (i.e., $E_{(1|\theta_i)}[x] > E_{(0|\theta_i)}[x]$) and $\pi(\theta_H| e_H = \widetilde{e}) > \pi(\theta_L| e_L = \widetilde{e})$. So, given a certain level of exerted effort, high-skilled agents have higher expected investment return, than low-skilled ones (i.e., $E_{(\theta_H|e_H=\widetilde{e})}[x] > E_{(\theta_L|e_L=\widetilde{e})}[x]$).

Finally, notice that $\pi(\theta_H, 0) \gtreqless \pi(\theta_L, 1)$. This means that high-skilled agents' expected investment return, given low effort ($e_H = 0$), may be higher, equal, or lower than low-skilled citizens' one, when these latter exert high effort ($e_L = 1$). As a consequence, we have

$$E_{(\theta_H,1)}[x] > E_{(\theta_H,0)}[x] \gtreqless E_{(\theta_L,1)}[x] > E_{(\theta_L,0)}[x].$$

Notice that i's return depends on both individual productivity θ_i and effort e_i. Therefore, the amount of capital k_{i1} is agent-specific and capital risk is idiosyncratic.

It is worth noting that: i) consumption in both periods (i.e., c_0 and c_1), ii) investment , K_{i1}, $\forall i$, iii) the realized value of x and its distribution, and iv) the distribution of θ are common knowledge. Depending on some specific assumptions, which will be introduced later, agent i's labor productivity θ_i and effort e_i, $\forall i$, may be private information.

By assumption, the Social Planner is a benevolent utilitarian one. In order to maximize social welfare, it chooses a state-contingent level of consumption and effort allocation, conditional on k_0. Also, we assume that the Social Planner cannot redistribute the endowment k_0 across agents.

3.3 Pure Moral Hazard Model

Let start with a model, where only effort is private information. This framework is similar to that described in Albanesi (2006), with one important exception: unlike Albanesi, who assumes that agents have the same productivity, we let agents be heterogeneous: agents differ in individual productivity θ_i, although, for the moment, this is assumed to be common knowledge. Under these assumptions, since the expected investment return is stochastic and increasing in unobservable effort, the ability of entrepreneurs to save a positive amount of resources at time 0 reduces the dependence of future consumption on the realization of uncertainty and, therefore, on effort. This means that the Social Planner faces a (dynamic) moral hazard problem: higher risk-free wealth insures citizens against negative future outcome, thereby discouraging a high level of effort.

The Social Planner's problem is

$$\max_{e_i \in \{0,1\},\ k_{1i} \in [0,k_0],\ c_{0i}, c_{1i}(x) \geq 0,\ i=H,L} \left\{ \begin{array}{l} \alpha \left[u(c_{0H}) + \beta E_{(\theta_H,e_H)} u(c_{1H}(x)) - v(e_H)\right] \\ +(1-\alpha)\left[u(c_{0L}) + \beta E_{(\theta_L,e_L)} u(c_{1L}(x)) - v(e_L)\right] \end{array} \right. \quad (3.1)$$

subject to

$$c_{0H} + k_{1H} \leq k_0, \quad (3.2)$$

$$c_{0L} + k_{1L} \leq k_0, \quad (3.3)$$

$$\begin{array}{rl} (\lambda) & \alpha E_{(\theta_H,e_H)}[c_{1H}(x)] + (1-\alpha) E_{(\theta_L,e_L)}[c_{1L}(x)] \\ \leq & \alpha k_{1H} E_{(\theta_H,e_H)}[1+x] + (1-\alpha) k_{1L} E_{(\theta_L,e_L)}[1+x], \end{array} \quad (3.4)$$

$$(\mu_H) \ \ \beta E_{(\theta_H,1)}[u(c_{1H}(x))] - v(1) \geq \beta E_{(\theta_H,0)}[u(c_{1H}(x))] - v(0), \quad (3.5)$$

$$(\mu_L) \;\; \beta E_{(\theta_L,1)}\left[u\left(c_{1L}\left(x\right)\right)\right]-v\left(1\right)\geq\beta E_{(\theta_L,0)}\left[u\left(c_{1L}\left(x\right)\right)\right]-v\left(0\right), \tag{3.6}$$

where (3.2), (3.3) and (3.4) are the first-period high-skilled and low-skilled agents' budget constraints (since the Social Planner cannot make any transfer across agent during $t=0$) and second-period social budget constraint, respectively. Moreover inequalities (3.5) and (3.6) denote high-type and low-type incentive compatibility constraint (ICC), respectively, for any level of effort. This means that we are implicitly assuming that the Government finds optimal that all agents exert high effort[1]. Given this problem, we can write the following:

Proposition 3.3.1 *At the constrained optimal allocation*

$$\left\{e_H^*, e_L^*, k_{H1}^{*m.h.}, k_{L1}^{*m.h.}, c_{H0}^{*m.h.}, c_{L0}^{*m.h.}, c_{H1}^{*m.h.}\left(\overline{x}\right), c_{H1}^{*m.h.}\left(\underline{x}\right), c_{L1}^{*m.h.}\left(\overline{x}\right), c_{L1}^{*m.h.}\left(\underline{x}\right)\right\},$$

when $e_H^*=e_L^*=1$, $\lambda>0$, $\mu_H>0$ *and* $\mu_L>0$. *Moreover,*

$$k_{1H}^{*m.h.} > k_{1L}^{*m.h.}, \tag{3.7}$$

$$\frac{u'\left(c_{1H}^{*m.h.}\left(\underline{x}\right)\right)}{u'\left(c_{1H}^{*m.h.}\left(\overline{x}\right)\right)}=\frac{\left[1+\frac{\mu_H\Delta\pi(\theta_H)}{\alpha\pi(\theta_H,1)}\right]}{\left[1-\frac{\mu_H\Delta\pi(\theta_H)}{\alpha(1-\pi(\theta_H,1))}\right]}>1, \tag{3.8}$$

$$u'\left(c_{0H}^{*m.h.}\right)E_{(\theta_H,1)}\left[\frac{1}{u'\left(c_{1H}^{*m.h.}\left(x\right)\right)}\right]=\beta E_{(\theta_H,1)}\left(1+x\right), \tag{3.9}$$

$$\frac{u'\left(c_{1L}^{*m.h.}\left(\underline{x}\right)\right)}{u'\left(c_{1L}^{*m.h.}\left(\overline{x}\right)\right)}=\frac{\left[1+\frac{\mu_L\Delta\pi(\theta_L)}{(1-a)\pi(\theta_L,1)}\right]}{\left[1-\frac{\mu_L\Delta\pi(\theta_L)}{(1-a)(1-\pi(\theta_L,1))}\right]}>1, \tag{3.10}$$

$$u'\left(c_{0L}^{*m.h.}\right)E_{(\theta_L,1)}\left[\frac{1}{u'\left(c_{1L}^{*m.h.}\left(x\right)\right)}\right]=\beta E_{(\theta_L,1)}\left(1+x\right), \tag{3.11}$$

where $\Delta\pi\left(\theta_i\right)=\pi\left(\theta_i,1\right)-\pi\left(\theta_i,0\right).$ *Moreover, since both* (3.5) *and* (3.6) *are binding at the optimum, if*

$$\Delta\pi\left(\theta_H\right)>\Delta\pi\left(\theta_L\right), \tag{3.12}$$

we derive

$$u\left(c_{1L}^{*m.h.}\left(\overline{x}\right)\right)-u\left(c_{1L}^{*m.h.}\left(\underline{x}\right)\right)>u\left(c_{1H}^{*m.h.}\left(\overline{x}\right)\right)-u\left(c_{1H}^{*m.h.}\left(\underline{x}\right)\right). \tag{3.13}$$

Proposition (3.3.1) deserves some comment. Inequality (3.7) means that the Social Planner finds it optimal to induce H-agents to invest more, than L-agents (and therefore, $c_{H0}^{*m.h.} < c_{L0}^{*m.h.}$). Since the return on investment is linear in capital and since at $e_H^*=e_L^*=1$, $E_{(\theta_H,1)}\left[1+x\right] > E_{(\theta_L,1)}\left[1+x\right]$, the Government can increase transfer across agents at time $t=1$, after the realization of x.

[1] Depending on parameters, It may also be the case that the Social Planner finds optimal to incentivate only one type of agents to exert high effort, usually H-type one. In a model without adverse selection, at the constrained-efficient allocation H-type agents are partially insured, with $e_H^*=1$, while L-type ones are fully insured, with $e_L^*=0$. We leave this analisys (both in pure moral hazard setting and in *mixed s*etting) to future research.

Inequality (3.8) implies that high-skilled agents are partially insured. Since e_H is private information, partial insurance incentives them to exert high effort, by increasing the difference

$$\left\{\beta E_{(\theta_H,1)}\left[u\left(c_{1H}\left(x\right)\right)\right]-\beta E_{(\theta_H,0)}\left[u\left(c_{1H}\left(x\right)\right)\right]\right\}.$$

By the same reasoning, inequality (3.10) implies partial insurance for L-agents.

From (3.9), by Jensen's inequality, we derive

$$\frac{u'\left(c_{0H}^{*m.h.}\right)}{E_{(\theta_H,1)}\left[u'\left(c_{1H}^{*m.h.}\left(x\right)\right)\right]}<\beta E_{(\theta_H,1)}\left(1+x\right). \tag{3.14}$$

In other words, the intertemporal marginal rate of substitution H-agents is strictly lower than the discounted expected *H-agents' aggregate* intertemporal rate of transformation. This means that there is a positive wedge in the intertemporal Euler equation, due to limit H-agents ' saving in $t=0$, in order to increase their expected consumption in $t=1$. H-agents' aggregate positive intertemporal wedge is:

$$\begin{aligned} IW_H^{m.h.} &= \beta E_{(\theta_H,1)}\left(1+x\right)E_{(\theta_H,1)}\left[u'\left(c_{1H}^{*m.h.}\left(x\right)\right)\right]-u'\left(c_{0H}^{*m.h.}\right) \\ &= \frac{\beta}{\alpha}E_{(\theta_H,1)}\left(1+x\right)\mu_H\Delta\pi\left(\theta_H\right)\left[u'\left(c_{1H}^{*m.h.}\left(\underline{x}\right)\right)-u'\left(c_{1H}^{*m.h.}\left(\overline{x}\right)\right)\right]>0. \end{aligned} \tag{3.15}$$

Similarly, from (3.11), we have

$$\frac{u'\left(c_{0L}^{*m.h.}\right)}{E_{(\theta_L,1)}\left[u'\left(c_{1L}^{*m.h.}\left(x\right)\right)\right]}<\beta E_{(\theta_L,1)}\left(1+x\right). \tag{3.16}$$

Then, L-aggregate positive intertemporal wedge is therefore equal to

$$\begin{aligned} IW_L^{m.h.} &= \beta E_{(\theta_L,1)}\left(1+x\right)E_{(\theta_L,1)}\left[u'\left(c_{1L}^{*m.h.}\left(x\right)\right)\right]-u'\left(c_{0L}^{*m.h.}\right) \\ &= \frac{\beta}{1-\alpha}E_{(\theta_L,1)}\left(1+x\right)\mu_L\Delta\pi\left(\theta_L\right)\left[u'\left(c_{1L}^{*m.h.}\left(\underline{x}\right)\right)-u'\left(c_{1L}^{*m.h.}\left(\overline{x}\right)\right)\right]>0. \end{aligned} \tag{3.17}$$

Clearly, when α is equal to either 0 or 1, there is only one type of agent and, therefore, we find the same results as Albanesi (2006).

3.4 Pure Adverse Selection

Let us now assume that the distribution of θ is common knowledge, although individual productivity is private information. Unlike Albanesi (2006), effort is now public information. Again, we assume that the Social Planner finds optimal that both types of agents exert $e_i=1, i=H,L$. In this case, a high-effort level can be obtained with no cost, since variable e is observable. So, it can force them. According to the standard literature on optimal taxation with adverse selection, agent i's production function is deterministic, i.e.,

$$R_i=\theta_i e_i k_{i1}.$$

Therefore, if agent i wants to mimic agent j,he has to invest k_{j1} unit of capital (assuming this variable is common knowledge) and exert an effort $\widetilde{e_i}\neq e_i^*$ (usually, i's effort is private information) such that $\theta_i \widetilde{e_i} = \theta_j e_j^*$. In our framework, the per-unit of capital investment return is stochastic, with only two possible values, i.e., $(1+\overline{x})$, or $(1+\underline{x})$. Therefore, from the realization of uncertainty, even if each agent's effort is public information, the Social planner cannot infer his labor productivity. The only way to separate its citizens (if it is optimal), is to induce them to invest different amount of money, i.e., $k_{H1}^{*a.s.} \neq k_{L1}^{*a.s.}$. From inequality (3.7), we see that the Social Planner finds optimal to induce high-skilled workers to save more, than low-skilled ones. As a consequence, we guess that high-skilled agents may choose to mimic low-type agents' $k_{L1}^{*a.s.}$, therefore increasing their optimal $t=0$ consumption $c_{H0}^{*a.s.}$. As a result, $t=1$ social budget constraint is tightened, with respect to pure moral-hazard case, and, therefore, transfer across agents is reduced.

Hence, the Social Planner, when maximizing the social welfare function, in order to avoid pooling, has to take into account the following high-type's incentive compatibility constraint

$$u(c_{H0}) - v(1) + E_{(\theta_H,1)}[u(c_{H1}(x))] \geq u(c_{L0}) - v(1) + E_{(\theta_H,1)}[u(c_{1L}(x))].$$

Finally, we neglect the low-type's incentive compatibility constraint, since we guessed that the Social Planner's problem arises from high-type ability of claiming to be low-skilled, in order to limit redistribution at his expenses. We check ex-post if the constrained optimal allocation $(e_L^*, k_{L1}^{*a.s.}, c_{L0}^{*a.s.}, c_{L1}^{*a.s.}(\overline{x}), c_{L1}^{*a.s.}(\underline{x}))$ strictly satisfies it.

Therefore, the Social Planner's problem is:

$$\max_{e_i\in\{0,1\},\ k_{1i}\in[0,k_0],\ c_{0i},c_{1i}(x)\geq 0,\ i=H,L} \left\{ \begin{array}{c} \alpha\left[u(c_{0H}) + \beta E_{(\theta_H,e_H)} u(c_{1H}(x)) - v(e_H)\right] \\ +(1-\alpha)\left[u(c_{0L}) + \beta E_{(\theta_L,e_L)} u(c_{1L}(x)) - v(e_L)\right] \end{array} \right. \tag{3.18}$$

subject to

$$c_{0H} + k_{1H} \leq k_0, \tag{3.19}$$

$$c_{0L} + k_{1L} \leq k_0, \tag{3.20}$$

$$\begin{array}{c} (\lambda)\ \ \alpha E_{(\theta_H,e_H)}[c_{1H}(x)] + (1-\alpha) E_{(\theta_L,e_L)}[c_{1L}(x)] \\ \leq \alpha k_{1H} E_{(\theta_H,e_H)}[1+x] + (1-\alpha) k_{1L} E_{(\theta_L,e_L)}[1+x], \end{array} \tag{3.21}$$

$$(\gamma_H)\ \ u(c_{0H}) - v(1) + \beta E_{(\theta_H,1)}[u(c_{1H}(x))] \geq u(c_{0L}) - v(1) + \beta E_{(\theta_H,1)}[u(c_{1L}(x))]. \tag{3.22}$$

As can be seen, both period 0 and period 1 budget constraints are unchanged. Moreover, since, at the optimum, (3.19) and (3.20) constraints imply, respectively, $c_{0H}^{*a.s.} = k_0 - k_{1H}^{*a.s.}$ and $c_{OL}^{*a.s.} = k_0 - k_{1L}^{*a.s.}$, we can slightly modify (3.22) constraint:

$$(\gamma_H)\ \ \beta E_{(\theta_H,1)}[u(c_{1H}(x))] - \beta E_{(\theta_H,1)}[u(c_{1L}(x))] \geq u(k_0 - k_{1L}) - u(k_0 - k_{1H}). \tag{3.23}$$

Therefore, the effect of adverse selection is clear: only if rewarded in period 1, in expected terms, high-type citizens choose to consume less in period 0 (we have already shown that the Social Planner finds optimal to force high-skilled agents to save more, than low-skilled ones). Otherwise, constraint (3.21) is tightened, since high-skilled entrepreneurs invest $k_{1L}^* < k_{1H}^*$ and, therefore, expected redistribution across agents is reduced. Solving Problem 3.4 gives the following:

Proposition 3.4.1 *At the constrained optimal allocation*

$$\{e_H^*, e_L^*, k_{H1}^{*a.s.}, k_{L1}^{*a.s.}, c_{H0}^{*a.s.}, c_{L0}^{*a.s.}, c_{H1}^{*a.s.}(\overline{x}), c_{H1}^{*a.s.}(\underline{x}), c_{L1}^{*a.s.}(\overline{x}), c_{L1}^{*a.s.}(\underline{x})\},$$

when $e^*_H = e^*_L = 1$, both $\lambda > 0$ and $\gamma_H > 0$, while the neglected low-skilled's incentive constraint is slack. Moreover, we have

$$k_{1H}^{*a.s} < k_{1H}^{*m.h.}, \tag{3.24}$$

$$k_{1L}^{*a.s.} > k_{1L}^{*m.h.}, \tag{3.25}$$

$$\frac{u'\left(c_{1H}^{*a.s.}\left(\underline{x}\right)\right)}{u'\left(c_{1H}^{*a.s.}\left(\overline{x}\right)\right)} = \frac{\left[1+\frac{\gamma_H}{\alpha}\right]}{\left[1+\frac{\gamma_H}{\alpha}\right]} = 1, \tag{3.26}$$

$$u'\left(c_{0H}^{*a.s.}\right) = u'\left(c_{1H}^{*a.s.}\right)\beta E_{(\theta_H,1)}\left(1+x\right), \tag{3.27}$$

$$\frac{u'\left(c_{1L}^{*a.s.}\left(\underline{x}\right)\right)}{u'\left(c_{1L}^{*a.s.}\left(\overline{x}\right)\right)} = \frac{\left[1-\frac{\gamma_H}{(1-\alpha)}\frac{\pi(\theta_H,1)}{\pi(\theta_L,1)}\right]}{\left[1-\frac{\gamma_H}{(1-\alpha)}\frac{(1-\pi(\theta_H,1))}{(1-\pi(\theta_L,1))}\right]} < 1, \tag{3.28}$$

$$u'\left(c_{0L}^{*a.s.}\right)E_{(\theta_L,1)}\left[\frac{1}{u'\left(c_{1L}^{*a.s.}\left(x\right)\right)}\right] = \beta E_{(\theta_L,1)}\left(1+x\right). \tag{3.29}$$

It is worth noting that the existence of inequalities (3.24) and (3.25) is due to the incentive compatibility constraint of high-skilled agents. The reasoning is as follows: in order to induce H-type agents to reveal their type, $c_{0H}^{*a.s.}$ must be higher and $c_{0L}^{*a.s.}$ must be lower than under a pure moral-hazard context. Also, from equations (3.63) and (3.64) we can see that there is full insurance for H-type agents (i.e., $c_{1H}^{*a.s.}\left(\underline{x}\right) = c_{1H}^{*a.s.}\left(\overline{x}\right)$). Moreover, from equations (3.63), (3.65) and (3.66) is straightforward show that $c_{1H}^{*a.s.}$ is higher than both $c_{1L}^{*a.s.}\left(\overline{x}\right)$ and $c_{1L}^{*a.s.}\left(\underline{x}\right)$. This means that $c_{1H}^{*a.s.}$ is higher than $E_{(\theta_H,1)}\left[c_{1L}^{*a.s.}\left(x\right)\right]$. Since high-skilled agents' incentive compatibility constraint (3.23) is binding at the optimum, i.e.,

$$\beta u\left(c_{1H}^{*a.s.}\right) - \beta E_{(\theta_H,1)}\left[u\left(c_{1L}^{*a.s.}\left(x\right)\right)\right] = u\left(k_0 - k_{1L}^{*a.s.}\right) - u\left(k_0 - k_{1H}^{*a.s.}\right),$$

and, given $u' > 0$, we have

$$\beta u\left(c_{1H}^{*a.s.}\right) - \beta E_{(\theta_H,1)} u\left(c_{1L}^{*a.s.}\left(x\right)\right) > 0.$$

Hence, we have

$$\left[u\left(k_0 - k_{1L}^{*a.s.}\right) - u\left(k_0 - k_{1H}^{*a.s.}\right)\right] > 0,$$

or equivalently,

$$k_{1H}^{*a.s.} > k_{1L}^{*a.s.}. \tag{3.30}$$

Given these results, we can therefore say that, at the constrained optimum, the spread $\left(k_{1H}^{*a.s.} - k_{1L}^{*a.s.}\right)$ is lower than the one under moral hazard, i.e., $\left(k_{1H}^{*m.h.} - k_{1L}^{*m.h.}\right)$. However, high-skilled agents are still induced to save more than low-skilled ones.

Equality (3.26) implies that high-skilled agents are fully insured. High-skilled agents' optimal effort is common knowledge, hence the Social Planner can force them to exert $e^*_H = 1$. Moreover, at the optimum, low-skilled's incentive constraint is strictly satisfied. Therefore, there is no distortion at the top. Conversely, from inequality (3.28) a bit counterintuitive result arises: low-skilled agents are *over-insured* (rather than under-insured) at the optimum, i.e.,

$$c_{1L}^{*a.s.}\left(\underline{x}\right) > c_{1L}^{*a.s.}\left(\overline{x}\right).$$

This result is due to Social Planner's attempt to minimize H-agents' informational rent. Since, when mimicking L-agents, high-skilled agents have strictly higher probability to get $\overline{x}$ return (since $\pi(\theta_H, 1) > \pi(\theta_L, 1)$), if $c_{1L}^{*a.s.}(\underline{x}) > c_{1L}^{*a.s.}(\overline{x})$, we immediately derive $E_{(\theta_H,1)}u(c_{1L}^{*a.s.}(x)) < E_{(\theta_L,1)}u(c_{1L}^{*a.s.}(x))$. Therefore, the Social Planner relaxes high-type's incentive compatibility constraint. Hence, the Social Planner can decrease c_{0H} (thereby increasing k_{1H}), increasing $t = 1$ expected output (and, consequently, expected $t = 1$ redistribution), until H-agents are indifferent between the optimal allocation designed for them and the one designed for L-agents. Clearly, in order to offer $c_{1L}^{*a.s.}(\underline{x}) > c_{1L}^{*a.s.}(\overline{x})$, the Social Planner should harshly tax L-agents' investment return, when $\overline{x}$ occurs, and subsidizes low-consumption $c_{1L}^{*a.s.}$, when $\underline{x}$ occurs.

Equality (3.27) shows that H-agents' intertemporal marginal rate of substitution coincides with the discounted expected *H-agents' aggregate* intertemporal rate of transformation. Since high-skilled agents are fully insured, they do not desire to save more than the social optimal level. Therefore, there is no need to introduce a wedge in the intertemporal Euler equation, i.e.,

$$IW_H^{a.s.} = 0. \tag{3.31}$$

Finally, from (3.29), by Jensen's inequality, we obtain

$$\frac{u'(c_{0L}^{*a.s.})}{E_{(\theta_L,1)}[u'(c_{1L}^{*a.s.}(x))]} < \beta E_{(\theta_L,1)}(1+x). \tag{3.32}$$

Like the pure moral-hazard case analyzed by Albanesi (2006), L-agents' intertemporal marginal rate of substitution is strictly lower than the discounted expected *L-agents' aggregate* intertemporal rate of transformation. Again, there is a positive wedge in L-agents' intertemporal Euler equation. Since, from high-skilled's incentive compatibility constraint, $c_{1L}^{*a.s.}(\underline{x}) > c_{1L}^{*a.s.}(\overline{x})$, low-skilled agents would prefer to exert $e_L = 0$, in order to minimize the probability of having a per-unit of k_{1L} investment return equal to $(1 + \overline{x})$. However, effort is public information. Therefore, low-skilled agents are forced by the Social Planner to exert $e_L^* = 1$. Hence, they would like to move consumption from $t = 0$ to $t = 1$, by saving more than the social optimal level. Under adverse selection, L-agents' aggregate positive intertemporal wedge is

$$\begin{aligned} IW_L^{a.s.} &= \beta E_{(\theta_L,1)}(1+x) E_{(\theta_L,1)}[u'(c_{1L}^{*a.s.}(x))] - u'(c_{0L}^{*a.s.}) \\ &= \beta \frac{\gamma_H}{(1-\alpha-\gamma_H)}(\pi(\theta_H,1) - \pi(\theta_L,1)) \\ &\quad \times E_{(\theta_L,1)}(1+x)[u'(c_{1L}^{*a.s.}(\overline{x})) - u'(c_{1L}^{*a.s.}(\underline{x}))]. \end{aligned} \tag{3.33}$$

Since $c_{1L}^{*a.s.}(\underline{x}) > c_{1L}^{*a.s.}(\overline{x})$, wedge $IW_L^{a.s.}$ is positive.

3.5 Moral Hazard and Adverse Selection

Let us finally introduce both assume moral hazard and adverse selection. This entails that the distribution of θ is common knowledge, although both θ_i and e_i are private information.

Again, we assume that the Social Planner finds optimal to induce both *continua* of individuals to exert high effort, i.e. $e_i^* = 1$, $i = H, L$. i's optimal decision on how much to invest (i.e., k_{i1} level) is agent-specific and reveals i's type. Since decision on k_{i1} occurs before exerting effort, adverse selection occurs before moral hazard. Hence, the Social Planner is aware that i-agent may choose both to mimic $j-$agent, where $\theta_i \neq \theta_j$, and exert high effort, i.e., $e_i = 1$, or to mimic j-agent and to exert low effort, i.e., $e_i = 0$. Therefore, from Social Planner's point of view, low-type's incentive compatibility constraints are

$$u(c_{0L}) - v(1) + \beta E_{(\theta_L,1)}[u(c_{1L}(x))] \geq \max \begin{cases} A_L : u(c_{0H}) - v(1) + \beta E_{(\theta_L,1)}[u(c_{1H}(x))], \\ B_L : u(c_{0H}) - v(0) + \beta E_{(\theta_L,0)}[u(c_{1H}(x))], \end{cases} \tag{3.34}$$

and

$$\beta E_{(\theta_L,1)}[u(c_{1L}(x))] - v(1) \geq \beta E_{(\theta_L,0)}[u(c_{1L}(x))] - v(0). \tag{3.35}$$

Similarly, high-skilled agents' incentive compatibility constraints are

$$u(c_{0H}) - v(1) + \beta E_{(\theta_H,1)}[u(c_{1H}(x))] \geq \max \begin{cases} A_H : u(c_{0L}) - v(1) + \beta E_{(\theta_H,1)}[u(c_{1L}(x))], \\ B_H : u(c_{0L}) - v(0) + \beta E_{(\theta_H,0)}[u(c_{1L}(x))], \end{cases} \tag{3.36}$$

and

$$\beta E_{(\theta_H,1)}[u(c_{1H}(x))] - v(1) \geq \beta E_{(\theta_H,0)}[u(c_{1H}(x))] - v(0). \tag{3.37}$$

Hereafter, we will refer to (3.34) and (3.36) as the low-skilled's and high skilled's *adverse selection constraints*, respectively. Accordingly, (3.35) and (3.37) constraints are defined as the low-skilled's and high-skilled's *moral hazard constraints*, respectively.

Let us focus now on constraint (3.34). Low-skilled agents strictly prefer A_L (i.e., to mimic high-skilled agents and to exert high effort), rather than B_L (i.e. to mimic high-skilled agents and to exert *zero* effort) if and only if

$$\beta E_{(\theta_L,1)}[u(c_{1H}(x))] - \beta E_{(\theta_L,0)}[u(c_{1H}(x))] > v(1) - v(0),$$

or equivalently,

$$\beta[u(c_{1H}(\overline{x})) - u(c_{1H}(\underline{x}))] > \frac{v(1) - v(0)}{\Delta\pi(\theta_L)}. \tag{3.38}$$

Rearranging (3.37), we immediately derive that, in order to stimulate high-skilled agents to exert high effort,

$$\beta[u(c_{1H}(\overline{x})) - u(c_{1H}(\underline{x}))] \geq \frac{v(1) - v(0)}{\Delta\pi(\theta_H)}. \tag{3.39}$$

Inequality (3.12) entails that the RHS of (3.38) is strictly higher than the RHS of (3.39). Therefore, if low-skilled agents strictly prefer A_L to B_L, the moral hazard constraint of high-skilled citizens, i.e., (3.39), is slack at the constrained optimum.

Notice that we assumed that the Social Planner finds it optimal to induce both types of citizens to exert high effort. This implies that the policy maker must offer an optimal allocation that induces agents to reveal their own type and, at the same time, discourage mimicking. This latter effect is ensured both types of agents strictly prefer to exert high effort, i.e., when both inequalities $A_L > B_L$ and $A_H > B_H$ hold. In order to make low-skilled better off when choosing A_L, instead of B_L, our Social Planner must offer an expected gain from effort, equal to

$$\{\Delta\pi(\theta_L)\beta[u(c_{1H}(\overline{x})) - u(c_{1H}(\underline{x}))]\},$$

that is higher than the additional disutility due to exerting high effort (i.e., $v(1) - v(0)$). Since we assumed $\Delta\pi(\theta_L) < \Delta\pi(\theta_H)$, we can show that

$$\{\Delta\pi(\theta_L)\beta[u(c_{1H}(\overline{x})) - u(c_{1H}(\underline{x}))]\} < \{\Delta\pi(\theta_H)\beta[u(c_{1H}(\overline{x})) - u(c_{1H}(\underline{x}))]\}.$$

This means that, given $A_L > B_L$, (3.39) is strictly satisfied at the optimal allocation. Namely, high-skilled agents strictly prefer high effort.

Moreover, from the RHS of (3.36), H-citizens strictly prefer A_H (i.e., to mimic low-skilled agents and to exert high effort), rather than B_H (i.e. to mimic low-skilled agents and to exert zero effort) iff

$$\beta E_{(\theta_H,1)}\left[u\left(c_{1L}\left(x\right)\right)\right] - \beta E_{(\theta_H,0)}\left[u\left(c_{1L}\left(x\right)\right)\right] > v\left(1\right) - v\left(0\right),$$

or

$$\beta\left[u\left(c_{1L}\left(\overline{x}\right)\right) - u\left(c_{1L}\left(\underline{x}\right)\right)\right] > \frac{v\left(1\right) - v\left(0\right)}{\Delta\pi\left(\theta_H\right)}. \tag{3.40}$$

Rearranging (3.35) and following the same reasoning, we can show that low-skilled agents weakly prefer high effort if

$$\beta\left[u\left(c_{1L}\left(\overline{x}\right)\right) - u\left(c_{1L}\left(\underline{x}\right)\right)\right] \geq \frac{v\left(1\right) - v\left(0\right)}{\Delta\pi\left(\theta_L\right)}. \tag{3.41}$$

Given $\Delta\pi\left(\theta_L\right) < \Delta\pi\left(\theta_H\right)$, the RHS of (3.40) is strictly lower than the RHS of (3.41). Since we assumed that the Social Planner aims to induce L-workers to exert high effort, constraint (3.41) implies (3.40). In other terms, H-workers strictly prefer A_H to B_H.

Given these results, the relevant constraints for the Social Planner are:

1. the time $t = 0$ and $t = 1$ budget constraints, and

2. the inequalities (3.38), (3.41), and, now,

$$u\left(c_{0L}\right) - u\left(c_{0H}\right) \geq \beta E_{(\theta_L,1)}\left[u\left(c_{1H}\left(x\right)\right)\right] - \beta E_{(\theta_L,1)}\left[u\left(c_{1L}\left(x\right)\right)\right], \tag{3.42}$$

$$u\left(c_{0L}\right) - u\left(c_{0H}\right) \leq \beta E_{(\theta_H,1)}\left[u\left(c_{1H}\left(x\right)\right)\right] - \beta E_{(\theta_H,1)}\left[u\left(c_{1L}\left(x\right)\right)\right]. \tag{3.43}$$

Constraints (3.42) and (3.43) are low-type's and high-type's adverse selection constraints, given that $A_L > B_L$ and $A_H > B_H$, respectively. Collecting these two constraints, we obtain

$$\beta\left[u\left(c_{1H}\left(\overline{x}\right)\right) - u\left(c_{1H}\left(\underline{x}\right)\right)\right] \geq \beta\left[u\left(c_{1L}\left(\overline{x}\right)\right) - u\left(c_{1L}\left(\underline{x}\right)\right)\right]. \tag{3.44}$$

From (3.38), we know that the LHS of (3.44), i.e., $\beta\left[u\left(c_{1H}\left(\overline{x}\right)\right) - u\left(c_{1H}\left(\underline{x}\right)\right)\right]$ has to be strictly higher than $\frac{v(1)-v(0)}{\Delta\pi(\theta_L)}$. Moreover, being (3.41) constraint binding at the constrained optimum (in order to minimize high-type informational rent), the RHS of (3.44) coincides with $\frac{v(1)-v(0)}{\Delta\pi(\theta_L)}$. Therefore, at the optimum, (3.44) is strictly satisfied. Hence, if both low-skilled's adverse selection and moral hazard constraints, (3.42) and (3.41), respectively, are binding at the optimum, both high skilled's constraints, (3.43) and (3.39), respectively, are slack[2].

Therefore, under both adverse selection and moral hazard, the Social Planner's problem is as follows:

$$\max_{e_i\in\{0,1\},\ k_{1i}\in[0,k_0],\ c_{0i},c_{1i}(x)\geq 0,\ i=H,L}\left\{\begin{array}{c}\alpha\left[u\left(c_{0H}\right) + \beta E_{(\theta_H,e_H)}u\left(c_{1H}\right) - v\left(e_H\right)\right]\\ +\left(1-\alpha\right)\left[u\left(c_{0L}\right) + \beta E_{(\theta_L,e_L)}u\left(c_{1L}\right) - v\left(e_L\right)\right].\end{array}\right. \tag{3.45}$$

[2] An other solution may be derived, when both low-type's moral hazard constraint and high-type's adverse selection constraint are binding. In this case, both low-type adverse selection constraint and high-type incentive compatibility constraint are strictly satisfied at the optimal solution. We leave this analisys to future research.

subject to

$$c_{0H} + k_{1H} \leq k_0,$$

$$c_{0L} + k_{1L} \leq k_0,$$

$$\begin{aligned} &(\lambda) \;\; \alpha E_{(\theta_H, e_H)} [c_{1H}(x)] + (1-\alpha) E_{(\theta_L, e_L)} [c_{1L}(x)] \\ \leq \;\; & \alpha k_{1H} E_{(\theta_H, e_H)} [1+x] + (1-\alpha) k_{1L} E_{(\theta_L, e_L)} [1+x], \end{aligned}$$

$$(\gamma_L) \;\; u(c_{0L}) - u(c_{0H}) \geq \beta E_{(\theta_L, 1)} [u(c_{1H}(x))] - \beta E_{(\theta_L, 1)} [u(c_{1L}(x))],$$

$$(\mu_L) \;\; \beta [u(c_{1L}(\overline{x})) - u(c_{1L}(\underline{x}))] \geq \frac{v(1) - v(0)}{\Delta\pi(\theta_L)},$$

where γ_L and μ_L are the Lagrangian multipliers of low-skilled agent's adverse selection and moral hazard constraints, respectively.

The solution of problem 3.5 allows us to write the following:

Proposition 3.5.1 *At the constrained optimal allocation, when $e_L^* = e_H^* = 1$, all the Lagrangian multipliers are strictly positive, i.e., $\lambda > 0$, $\gamma_L > 0$ and $\mu_L > 0$. Both the neglected high-skilled's incentive compatibility constraints, i.e., (3.43) and (3.39), are slack at the optimum. Furthermore,*

$$k_{1H}^{*mh.as} > k_{1L}^{*mh.as} \tag{3.46}$$

and

$$\left(k_{1H}^{*mh.as} - k_{1L}^{*mh.as}\right) > \left(k_{1H}^{*m.h.} - k_{1L}^{*m.h.}\right) > \left(k_{1H}^{*a.s.} - k_{1L}^{*a.s}\right), \tag{3.47}$$

$$\frac{u'\left(c_{1H}^{*mh.as}(\underline{x})\right)}{u'\left(c_{1H}^{*mh.as}(\overline{x})\right)} = \frac{\left[1 - \frac{\gamma_L}{\alpha}\frac{\pi(\theta_L,1)}{\pi(\theta_H,1)}\right]}{\left[1 - \frac{\gamma_L}{\alpha}\frac{(1-\pi(\theta_L,1))}{(1-\pi(\theta_H,1))}\right]} > 1, \tag{3.48}$$

$$u'\left(c_{0H}^{*mh.as}\right) E_{(\theta_H,1)} \left[\frac{1}{u'\left(c_{1H}^{*mh.as}(x)\right)}\right] = \beta E_{(\theta_H,1)} (1+x), \tag{3.49}$$

$$\frac{u'\left(c_{1L}^{*}(\underline{x})\right)}{u'\left(c_{1L}^{*}(\overline{x})\right)} = \frac{\left[1 + \frac{\gamma_L}{(1-\alpha)} + \frac{\mu_L}{(1-\alpha)}\frac{\Delta\pi(\theta_L)}{\pi(\theta_L,1)}\right]}{\left[1 + \frac{\gamma_L}{(1-\alpha)} - \frac{\mu_L}{(1-\alpha)}\frac{\Delta\pi(\theta_L)}{(1-\pi(\theta_L,1))}\right]} > 1, \tag{3.50}$$

$$u'\left(c_{0L}^{*mh.as}\right) E_{(\theta_L,1)} \left[\frac{1}{u'\left(c_{1L}^{*mh.as}(x)\right)}\right] = \beta E_{(\theta_L,1)} (1+x). \tag{3.51}$$

Proposition deserves some comments. First of all, we can say that inequalities (3.47) are due to the existence of the low-skilled's adverse selection constraint: by increasing $\left(k_{1H}^{*mh.as} - k_{1L}^{*mh.as}\right)$, the Social Planner increases the LHS of (3.42) constraint and, therefore, relaxes it. Even if, at the optimum, high-skilled's moral hazard constraint is strictly satisfied, inequality (3.48) shows partial insurance for H-type. This is driven from L-type agents' adverse selection constraint: in order to induce low-skilled agents to choose to exert high effort, when mimicking high-skilled citizens, it is necessary that $\left\{\beta E_{(\theta_L,1)} [u(c_{1H}(x))] - \beta E_{(\theta_L,0)} [u(c_{1H}(x))]\right\}$ difference is strictly positive (actually, bigger than $\{v(1) - v(0)\} > 0$ difference), as shown by inequality (3.38). If high-skilled agents are fully insured, $\left\{\beta E_{(\theta_L,1)}\left[u\left(c_{1H}\left(x\right)\right)\right]-\beta E_{(\theta_L,0)}\left[u\left(c_{1H}\left(x\right)\right)\right]\right\}=0$. Therefore, the Social Planner finds optimal to insure H-agents only partially.

A direct consequence of H-type's partial insurance is, like the pure moral-hazard case, the result in (3.49) ; by Jensen's inequality, it can be shown that there exists a positive wedge in the intertemporal Euler equation, in order to limit k_{1H} savings in $t=0$. For the same argument, we have (3.51) .

Inequality (3.50) entails that low-skilled agents are partially insured. This is due to the fact that the Social Planner aims at inducing them to exert high effort at the optimum.

Using (3.49), we can calculate the H-type aggregate intertemporal wedge:

$$\begin{aligned}
IW_H^{mh.as} &= \beta E_{(\theta_H,1)}\left(1+x\right)E_{(\theta_H,1)}\left[u'\left(c_{1H}^{*mh.as}\left(x\right)\right)\right]-u'\left(c_{0H}^{*mh.as}\right) &(3.52)\\
&= \beta\frac{\gamma_L}{\alpha-\gamma_L}E_{(\theta_L,1)}\left(1+x\right)\left(\pi\left(\theta_H,1\right)-\pi\left(\theta_L,1\right)\right)\left[u'\left(c_{1H}^{*mh.as}\left(\underline{x}\right)\right)-u'\left(c_{1H}^{*mh.as}\left(\overline{x}\right)\right)\right]\\
&> 0. &(3.53)
\end{aligned}$$

Inequality (3.52) shows that the H-aggregate intertemporal wedge is a consequence of low-skilled's adverse selection constraint. If it was slack at optimum, i.e., $\gamma_L=0$, no intertemporal wedge should be imposed. By $IW_H^{mh,as}>0$, the Social Planner limits high-skilled's savings.(even if, as shown by inequality (3.47) , the spread between two types' optimal saving is higher, than in previous scenarios); hence inducing them to exert high effort and, therefore, making low-skilled agents better off when exerting positive effort, than when exerting zero effort (both if mimicking H-agents, or revealing their low-type).

Finally, from (3.51), L-aggregate intertemporal wedge is

$$\begin{aligned}
IW_L^{mh.as} &= \beta E_{(\theta_L,1)}\left(1+x\right)E_{(\theta_L,1)}\left[u'\left(c_{1L}^{*mh.as}\left(x\right)\right)\right]-u'\left(c_{0L}^{*mh.as}\right) &(3.54)\\
&= \frac{\beta}{1-\alpha+\gamma_L}E_{(\theta_L,1)}\left(1+x\right)\mu_L\Delta\pi\left(\theta_L\right)\left[u'\left(c_{1L}^{*mh.as}\left(\underline{x}\right)\right)-u'\left(c_{1L}^{*mh.as}\left(\overline{x}\right)\right)\right]\\
&> 0.
\end{aligned}$$

3.6 Conclusions

In this Chapter we analyzed optimal taxation on entrepreneurs. Under dynamic moral hazard, it is socially optimal that high-skilled agents invest more, than low–skilled ones, since these former's expected return of investment is higher. Therefore, it is possible to maximize transfer across agents in time $t=1$. Both H- and L-agents are partially insured, since partial insurance incentives citizens to exert high effort. Moreover, both types of workers face a positive aggregate intertemporal wedge between their intertemporal marginal rate of substitution and their discounted expected *aggregate* intertemporal rate of transformation.

Conversely, if effort is public information, while skill is private information, adverse selection is Social Planner's issue. Under this scenario, high-skilled workers can pretend to be low-skilled ones. Therefore, in order to induce these former to reveal their own type, time $t=0$ optimal H-type's consumption must increase, while L-type's one decreases. However, high-type' optimal investment is higher, than low-type's one. Since effort is public information and low-skilled agents' incentive compatibility constraint is strictly satisfied at the optimal allocation, high-productive citizens are fully insured and H-aggregate intertemporal wedge is zero. Conversely, to minimize H-type's informational rent (i.e., to relax H-type's incentive compatibility constraint), low-type agents are overinsured. Hence, the Social Planner must harshly tax L-agents' investment return, when $\overline{x}$ occurs, and subsidizes low-consumption $c_{1L}^{*a.s.}$, when $\underline{x}$ occurs. Furthermore, in order to limit low-skilled's savings, L-aggregate intertemporal wedge is positive.

Finally, we analyzed a model, where both effort and skill are private information. Therefore, both dynamic moral hazard and adverse selection are relevant issue. Moreover, we are in a scenario, such that adverse selection occurs before moral hazard. Hence, the Social Planner knows that $i-$agent can both mimic $j-$agent, $i \neq j$ and exert high effort, or mimic $j-$agent and exert low effort. At a constrained equilibrium, both L-agents' moral hazard and adverse selection constraints are binding, while H-agents' constraints are slack. However, both agents are partially insured and both H-aggregate intertemporal wedges are strictly positive. Moreover, due to low-skilled's adverse selection constraint, with respect to previous scenarios, the spread between type-dependent optimal investment, i.e., $\left(k_{1H}^{*mh.as} - k_{1L}^{*mh.as}\right)$, is higher.

Various additional analysis are worthy to be studied in deep. Firstly, we should analyze at which conditions, under adverse selection, forcing only high-skilled workers to exert positive effort may be a more favorable equilibrium. Secondly, we must investigate the presence of other interesting separating equilibria under both moral hazard and adverse selection. Finally, after having characterized entrepreneur's intertemporal wedge, we should apply the model to different market structures.

3.A Appendix

Proof of Proposition 3.3.1

At $e_H^* = e_L^* = 1$, the first order conditions for Social Planner's problem (3.3) are:

$$\frac{\partial L}{\partial k_{1H}} : u'\left(k_0 - k_{1H}\right) = \lambda E_{(\theta_H,1)}\left[1+x\right], \tag{3.55}$$

$$\frac{\partial L}{\partial k_{1L}} : u'\left(k_0 - k_{1L}\right) = \lambda E_{(\theta_L,1)}\left[1+x\right], \tag{3.56}$$

$$\frac{\partial L}{\partial c_{1H(\overline{x})}} : \beta u'\left(c_{1H}\left(\overline{x}\right)\right)\left[1 + \frac{\mu_H \Delta\pi\left(\theta_H\right)}{\alpha\pi\left(\theta_H,1\right)}\right] = \lambda, \tag{3.57}$$

$$\frac{\partial L}{\partial c_{1H(\underline{x})}} : \beta u'\left(c_{1H}\left(\underline{x}\right)\right)\left[1 - \frac{\mu_H \Delta\pi\left(\theta_H\right)}{\alpha\left(1-\pi\left(\theta_H,1\right)\right)}\right] = \lambda, \tag{3.58}$$

$$\frac{\partial L}{\partial c_{1L(\overline{x})}} : \beta u'\left(c_{1L}\left(\overline{x}\right)\right)\left[1 + \frac{\mu_L \Delta\pi\left(\theta_L\right)}{\left(1-\alpha\right)\pi\left(\theta_L,1\right)}\right] = \lambda, \tag{3.59}$$

$$\frac{\partial L}{\partial c_{1L(\underline{x})}} : \beta u'\left(c_{1L}\left(\underline{x}\right)\right)\left[1 - \frac{\mu_L \Delta\pi\left(\theta_L\right)}{\left(1-\alpha\right)\left(1-\pi\left(\theta_L,1\right)\right)}\right] = \lambda. \tag{3.60}$$

Rearranging these conditions gives the conditions (3.7) to 3.13 of Proposition 3.3.1.

Proof of Proposition 3.4.1

At $e_H^* = e_L^* = 1$, the first order condition for Social Planner's problem (3.4) are:

$$\frac{\partial L}{\partial k_{1H}} : u'\left(k_0 - k_{1H}\right)\left(1 + \frac{\gamma_H}{\alpha}\right) = \lambda E_{(\theta_H,1)}\left[1+x\right], \tag{3.61}$$

$$\frac{\partial L}{\partial k_{1L}} : u'\left(k_0 - k_{1L}\right)\left(1 - \frac{\gamma_H}{1-\alpha}\right) = \lambda E_{(\theta_L,1)}\left[1+x\right], \tag{3.62}$$

$$\frac{\partial L}{\partial c_{1H(\overline{x})}} : \beta u'\left(c_{1H}\left(\overline{x}\right)\right)\left[1 + \frac{\gamma_H}{\alpha}\right] = \lambda, \tag{3.63}$$

$$\frac{\partial L}{\partial c_{1H(\underline{x})}} : \beta u'\left(c_{1H}\left(\underline{x}\right)\right)\left[1 + \frac{\gamma_H}{\alpha}\right] = \lambda, \tag{3.64}$$

$$\frac{\partial L}{\partial c_{1L(\overline{x})}} : \beta u'\left(c_{1L}\left(\overline{x}\right)\right)\left[1 - \frac{\gamma_H}{(1-\alpha)}\frac{\pi\left(\theta_H,1\right)}{\pi\left(\theta_L,1\right)}\right] = \lambda, \tag{3.65}$$

$$\frac{\partial L}{\partial c_{1L(\underline{x})}} : \beta u'\left(c_{1L}\left(\underline{x}\right)\right)\left[1 - \frac{\gamma_H}{(1-\alpha)}\frac{\left(1-\pi\left(\theta_H,1\right)\right)}{\left(1-\pi\left(\theta_L,1\right)\right)}\right] = \lambda, \tag{3.66}$$

where $\frac{\pi(\theta_H,1)}{\pi(\theta_L,1)} > 1 > \frac{(1-\pi(\theta_H,1))}{(1-\pi(\theta_L,1))}$. Rearranging gives the results of Proposition (3.4.1).

Proof of Proposition 3.5.1

At $e_H^* = e_L^* = 1$, the first order condition for Social Planner's problem (3.5) are:

$$\frac{\partial L}{\partial k_{1H}} : u'\left(k_0 - k_{1H}\right)\left(1 - \frac{\gamma_L}{\alpha}\right) = \lambda E_{(\theta_H,1)}\left[1+x\right], \tag{3.67}$$

$$\frac{\partial L}{\partial k_{1L}} : u'\left(k_0 - k_{1L}\right)\left(1 + \frac{\gamma_L}{1-\alpha}\right) = \lambda E_{(\theta_L,1)}\left[1+x\right], \tag{3.68}$$

$$\frac{\partial L}{\partial c_{1H(\overline{x})}} : \beta u'\left(c_{1H}\left(\overline{x}\right)\right)\left[1 - \frac{\gamma_L}{\alpha}\frac{\pi\left(\theta_L,1\right)}{\pi\left(\theta_{H,1}\right)}\right] = \lambda, \tag{3.69}$$

$$\frac{\partial L}{\partial c_{1H(\underline{x})}} : \beta u'\left(c_{1H}\left(\underline{x}\right)\right)\left[1 - \frac{\gamma_L}{\alpha}\frac{\left(1-\pi\left(\theta_L,1\right)\right)}{\left(1-\pi\left(\theta_{H,1}\right)\right)}\right] = \lambda, \tag{3.70}$$

$$\frac{\partial L}{\partial c_{1L(\overline{x})}} : \beta u'\left(c_{1L}\left(\overline{x}\right)\right)\left[1 + \frac{\gamma_L}{(1-\alpha)} + \frac{\mu_L}{(1-\alpha)}\frac{\Delta\pi\left(\theta_L\right)}{\pi\left(\theta_L,1\right)}\right] = \lambda, \tag{3.71}$$

$$\frac{\partial L}{\partial c_{1L(\underline{x})}} : \beta u'\left(c_{1L}\left(\underline{x}\right)\right)\left[1 + \frac{\gamma_L}{(1-\alpha)} - \frac{\mu_L}{(1-\alpha)}\frac{\Delta\pi\left(\theta_L\right)}{\left(1-\pi\left(\theta_L,1\right)\right)}\right] = \lambda, \tag{3.72}$$

where $\frac{\pi(\theta_H,1)}{\pi(\theta_L,1)} > 1 > \frac{(1-\pi(\theta_H,1))}{(1-\pi(\theta_L,1))}$. Rearranging gives the results of Proposition (3.5.1).

Bibliography

[1] Albanesi, S., (2007), "Optimal taxation of entrepreneurial capital with private information", NBER.

[2] Albanesi, S., Sleet, C., (2006), "Dynamic optimal taxation with private information", Review of Economic Studies, 73, pp. 1-30.

[3] Apps P., Rees R., (2008), "Two extensions to the theory of optimal income taxation", working paper.

[4] Bolton, P., Dewatripont, M., (2005), "Contract theory", The MIT Press.

[5] Brett, C., Weymark, J., (2008), "Optimal nonlinear taxation of income and savings without commitment", working paper.

[6] Dillén, M., Lundholm, M., (1996), "Dynamic income taxation, redistribution, and the ratchet effect", Journal of Public Economics, 59, pp. 69-93.

[7] Golosov, M., Kocherlakota, N., Tsyvinski, A., (2003), "Optimal indirect and capital taxation", Review of Economic Studies, Vol. 70, no. 3, pp. 569-587.

[8] Hamilton, J., Pestieau, P., (2005), "Optimal income taxation and the ability distribution: Implications for migration equilibria", International Tax and Public Finance, 12, pp. 29–45.

[9] Hindriks, J. (1999), "The consequences of labour mobility for redistribution: tax vs. transfer competition", Journal of Public Economics, 74, pp. 215–234.

[10] Jullien, B., (2000), "Partecipation constraint in adverse selection models", Journal of Economic Theory, 93, pp.1-47.

[11] Kocherlakota, N. R., (2005a), "Zero expected wealth taxes: a Mirrlees approach to dynamic optimal taxation", Econometrica, 73, pp. 1587-1621.

[12] Kocherlakota, N. R., (2005b), "Advances in dynamic optimal taxation", Ninth World Congress of the Econometric Society.

[13] Kocherlakota, N.R., (2010), " The new dynamic public finance", Princeton University Press.

[14] Krause, A. (2009), "Education and taxation policies in the presence of countervailing incentives", Economica, 76, pp 387-399.

[15] Laffont, J.J., Martimort, D., (2001), "The theory of incentives I: The principal-agent model", Princeton University Press.

[16] Lewis, T., Sappington, D., (1989), "Countervailing incentives in agency problems", Journal of Economic Theory, 49, pp. 294-313.

[17] Maggi, G, Rodriguez-Clare, A., (1995), "On countervailing incentives", Journal of Economic Theoty, 66, pp. 238-263.

[18] Mirrlees, J., (1971), " An exploration in the theory of income taxation", Review of Economic Studies, 38, pp. 175-208.

[19] Mirrlees, J., (1982), "Migration and optimal income taxes", Journal of Public Economics, 18, pp. 319-341.

[20] OECD, (2002), "Internation mobility of highly skilled", OECD, Paris.

[21] OECD, (2008), "OECD Economic surveys: Netherlands, vol. 1", OECD, Paris.

[22] Rogerson, W. (1985), "Repeated moral hazard", Econometrica, 53, pp. 69-77.

[23] Saez, E., (2001) "Using elasticities to derive optimal income tax rates", Review of Economic Studies, 68, pp. 205-229.

[24] Salanié, B., (2002), "The economics of taxation", The MIT Press.

[25] Simula, L., Trannoy, A., (2006), "Optimal non-linear income tax when highly skilled individuals vote with their feet", IDEP Working Papers 0610, Institut d'economie publique (IDEP), Marseille, France.

[26] Simula, L., Trannoy, A., (2010), "Optimal income tax under the threat of migration by top-income earners", Journal of Public Economics, 94, pp. 163-173.

[27] Stiglitz, J. E., (1987), "Pareto efficient and optimal taxation in the new new welfare economics", In Auerbach, A. J., Feldstein, M. (Eds.), Handbook of Public Economics Vol. 2. North-Holland, Amsterdam, pp. 991–1042.

[28] Weitzman, M. L., (1980), "The ratchet principle and performance incentives", Bell Journal of Economics, 11, pp. 302–308.

[29] Wilson, J., (1992), "Optimal income taxation and international personal mobility", American Economic Review, 82, pp. 191-196.