Monopoly, decreasing returns, and incentives to cost-reducing R&D

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Abstract

The present paper shows that it is possible to define cost innovations for which a monopolist has a higher incentive to invest than a social planner. This unveils the limits of the general claim, based on Arrow (1959), that a monopoly has a lower incentive to innovate than a social planner and therefore than socially desirable. In particular, exceptions to the rule are shown to arise only under decreasing returns. Further, it follows from the analysis, that the direction of the inequality in the comparison of incentives to invest also depends upon the shape of the demand function. Finally, only under a restricted domain of analysis, a rule for determining whether a monopoly has lower or higher incentives to invest than a social planner is derived.

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1 Introduction

This paper briefly reconsiders the result that a monopolist has lower incentives to spend in cost reducing R&D than socially optimal (Arrow 1959). While largely confirming that result, an analysis of cases without constant returns to scale reveals the existence of exceptions. As is well known, Arrow, contrasting the Schumpeterian hypothesis (Schumpeter 1942), finds that a monopoly has a lower incentive to introduce a non-drastic innovation than a competitive firm—in what is known as the "static" setting. The crucial results for a non-drastic innovation can be resumed by the inequality, $V^m < V^c < V^*$, where V^m is the value of a cost-reducing innovation to a monopoly, V^c that to a firm under perfect competition, and V^* that to an ideal "social planner" maximizing social welfare[1]. The intuition why V^m is less than V^* is basically (Tirole (1988)) that the monopolist cost reduction "pertains to a smaller number of units" than for a social planner—other interpretations often found in textbooks are erroneous. Obviously, if a monopolist is able to perfectly price discriminate output under monopoly and output under the social planner solution coincide, and $V^m = V$. As for the relation $V^m < V^c$, the monopolist's incentive is also ususally said to be hampered, with respect to that of a firm under perfect competition, by the so called "replacement effect", namely the existence of pre- innovation positive profits that are zero for a competitive firm (Gilbert 2006). The main purpose of the present analysis is to show that in the theory for the static setting the inequality $V^m < V^*$ can be reversed. Also, the relevance for some real world situations is briefly discussed.

The traditional proof of Arrow's inequalities, namely $V^m \leq V^*$, is associated with the claim that this is true for all demand functions. Actually, this is indeed the case when constant returns to scale are assumed. However, under general conditions the traditional technique of proving the inequality cannot be used, and a condition governing the direction of the inequality relating V^* and V^m, which is also independent of the shape of the demand

[1] For instance, Tirole (1988). For an analysis of various oligopoly settings that are not considered here, Vives (2008). Incentives when a monopolist is threatened by entry are first treated in Gilbert and Newbery (1982) and Reinganum (1983).

function, is obtained only in a restricted domain of analysis.

Another, incidental, implication of abandoning the ground of constant returns is that the ordering of innovations as to wether they are or not socially desirable is not obvious. While a lower constant cost is socially desirable under any industry configuration, a switch from one technology to another under decreasing returns, with associated cost functions, may be desirable under monopoly but not under the social planner solution.

The general conclusion that, beyond the other distortions introduced, there is an additional market failure under monopoly—the "pace of innovation" is too slow with respect to the social optimum—and the support for policies that redress this problem, find here a qualification. If technologies exhibit decreasing returns there can be much less of an underinvestment problem than otherwise believed, and in some cases no such a problem at all.

2 Non-increasing returns to scale

Consider a market for a good with demand function, $D(p)$ where $p \geq 0$ is the market price, satisfying the following conditions.

Assumption (R): (R.1) $D(p)$ is single-valued, continuous; (R.2) $D(0) = q^+$, with $q^+ > 0$ finite; further, there exists a price $p^+ > 0$ such that $D(p) = 0$ for all $p \geq p^+$ and $D(p) > 0$ otherwise. (R.3) for p', p'' with $p'' > p'$and such that $D(p'') \geq 0$, $D(p\prime) > 0$, the inequality $D(p'') < D(p')$ is verified. The notation $P(q)$ is used for the inverse function of $D(p)$.[2]

Two cost functions are to be compared, $C_0(q)$ and $C_1(q)$, stemming from two different technologies, where $C_1(q)$ stems from a (costly) innovation and $C_0(q)$ stems from the technology currently adopted. It shall be assumed that $C_i(q)$ be finite for $0 \leq q \leq q^+$.

Assumption 1. (Nonincreasing returns to scale) the cost functions $C_1(q)$

[2] Under the assumptions about $D(p)$ the inverse function $P(q)$, in its usual graphical representation, displays no vertical (nor horizontal) portions, and, if $a < b$, with $a, b \in [0, p^+]$ the integral $\int_a^b P(q)dq$ exists and is finite.

and $C_0(q)$ share the the property that if $q' < q''$, then for any $\eta > 0$ one has that $0 \leq C_i(q'+\eta) - C(q') \leq C_i(q''+\eta) - C_i(q'')$.

Assumption 2. (a) $C_i(0) = 0$, for $i = 0, 1$; (b) $C_1(q)$ and $C_0(q)$ possess first derivatives, denoted $C_i'(q)$, for $i = 0, 1$ with well defined (Riemann) integral values $\int_a^b C_1'(x)dx$ for $a, b \in [0, q^+]$ and $b > a$.

Assumption 3. If $C_1'(q') = C_0'(q')$ for some $q' \in [0, q^+]$, then $C_1'(q) \leq C_0'(q)$ for $q \in [0, q']$, and $C_1'(q) \geq C_0'(q)$ for $q \in [q', q^+]$.

Clearly, assumption 2(a) is only introduced to simplify exposition. Assumption 2(b) implies some reasonable restriction on the cost functions, without imposing continuous derivatives. Assumption 3 implies that the post-innovation marginal cost function may eventually cross the pre-innovation one only from below. In other words, it implies that *if* some cost efficiency is lost in going from the old to the new technology, this loss occurs above a given production scale but not below it. Otherwise, the new technology dominates the old one for all output levels.

The time horizon is assumed to be of only one period, so that no discounting is needed (this is immaterial to the comparisons of incentives to innovate). Consider now the case when the new technology can be obtained for free. Define the function $\psi(x_0, x_1)$ as the change in costs induced by a change in output from x_0 to x_1 and a simultaneous change in costs (technology) from $C_0(.)$ to $C_1(.)$. Clearly, under A2,

$$\psi(x_0, x_1) = C_1(x_1) - C_0(x_0) = \int_0^{x_1} C_1'(x)dx - \int_0^{x_0} C_0'(x)dx. \qquad (1)$$

Further, let the function $\omega(x_0, x_1)$ be defined as:

$$\omega(x_0, x_1) = \int_{x_0}^{x_1} P(x)dx - \psi(x_0, x_1), \qquad (2)$$

where $\omega(x_0, x_1)$ is the social gain or loss from changing output level from x_0 to x_1, while at the same time changing cost function from $C_0(q)$ to $C_1(q)$. Let $W(\bar{x})$ denote social surplus in the industry. Let x_i^* denote the solution to the social planner maximization problem $\max_{\bar{x}} W(\bar{x})$, under technology

i, for $i \in \{0, 1\}$. The incentive to innovate, gross of the innovation costs, for the social planner is given by:

$$\omega(x_0^*, x_1^*) = \int_{x_0^*}^{x_1^*} P(x)dx - \psi(x_0^*, x_1^*). \tag{3}$$

Let $F \geq 0$ denote the exogenously given cost required to "discover" the new technology.

Definition 1 *A technology leading to cost function $C_1(q)$ and costing F is a* socially desirable innovation when the cost function $C_0(q)$ is in place, under the social planner solution *if $\omega(x_0^*, x_1^*) - F > 0$. The set of such cost functions (innovations) is denoted by $\mathcal{I}_s(C_0)$. A technology leading to cost function $C_1(q)$ is a* potentially socially desirable innovation under the social planner solution *if $\omega(x_0^*, x_1^*) > 0$. The set $\mathcal{A}(C_0)$ is defined as the set containing all cost functions, $C_j(q)$, such that $\omega(x_0^*, x_j^*) \geq 0$.*

Assumption 4. Given the cost function $C_0(q)$, it is assumed that $C_1(q)$ belongs to $\mathcal{A}(C_0)$.

Assumption 4 is warranted because under A3 the admissible cost functions may cross each other, or more formally may be such that for some value $q < q^+$ one has $C_1(q) > C_0(q)$.

Lemma 1 *(a) If $C_0(x)$ and $C_1(x)$ satisfy A1-A3, then the socially optimal output after a potentially desirable innovation can be larger, equal, or smaller than the original output. (b) If $x_1^* < x_0^*$ and $C_1(.) \in \mathcal{A}(C_0)$, then $C_1(x_1^*) < C_0(x_0^*)$.*

Proof. (a) The case where $C_1'(q) \leq C_0'(q)$ for $q \in [0, q^+]$ clearly implies $x_1^* > x_0^*$. Consider, as an instance, the following case: $C_1(x) = \max\{0, C_0(x) - \varepsilon\}$ for $0 \leq x \leq x'$, and $C_1(x) = C_0(x) + \varepsilon$ for $x > x'$, where $0 < x' < q^+$. By suitable choice of x' one can obtain that $\omega(x_0^*, x_1^*) > 0$ while $x_0^* > x_1^*$. (b) Since $x_1^* < x_0^*$ implies $\int_{x_0^*}^{x_1^*} P(x)dx < 0$, then $\omega(x_0^*, x_1^*) > 0$ can possibly hold only if $\psi(x_0^*, x_1^*) < 0$. ■

Under assumption 3, the analysis is restricted to potentially socially desirable innovations, but not to innovations that increase net consumer surplus. Obviously, an innovation that is desirable from the social point of view may or may not be so for a monopolist.

Monopolist vs. Social Planner

Let $\pi_i(x) = P(x)x - C_i(x)$, for $i = 0, 1$. Let x_i^m denote the profit maximizing level of output under technology i (under Assumptions 1, 2 and 3 on cost functions, it is not granted hat x_i^m be increasing in i). Then, the monopolist's incentive to innovate, $\Delta\pi(C_0, C_1)$, gross of innovation costs, writes as $\Delta\pi(C_0, C_1) = P(x_1^m)x_1^m - P(x_0^m)x_0^m - \psi(x_0^m, x_1^m)$.

Definition 2 *Given a cost function $C_0(q)$, a technology leading to cost function $C_1(q)$, for which an amount F must be paid, is an implementable innovation under the monopoly solution if $\Delta\pi(C_0, C_1) - F > 0$. The set of all such cost functions (innovations) is denoted by $\mathcal{I}_m(C_0)$, as it clearly depends upon the technology in use.*

The term "profitable" here could be used instead of "implementable", as a shift to the new technology is actually going to be implemented by the monopolist only if it is profitable.

Recall that (Assumption 4) the restriction $C_1(q) \in \mathcal{A}(C_0)$ applies. Define the net consumer surplus, $s(x_i)$ as $s(x_i) = \int_0^{x_i} [P(x) - P(x_i)]\, dx$. Then, since fixed costs have been assumed to be nil, one can write:

$$\begin{aligned} \Delta\pi(C_0, C_1) &= \int_{x_0}^{x_1^m} P(x)dx - \psi(x_0^m, x_1^m) - [s(x_1^m) - s(x_0^m)] \\ &= \omega(x_0^m, x_1^m) - [s(x_1^m) - s(x_0^m)]\,. \end{aligned}$$

Hence, $\Delta\pi > \omega(x_0^*, x_1^*)$ if and only if

$$\omega(x_0^m, x_1^m) - \omega(x_0^*, x_1^*) - [s(x_1^m) - s(x_0^m)] > 0. \tag{4}$$

The terms in the last term in square brackets can easily be recognized as the part of additional social surplus that a monopolist is unable to appropriate under uniform pricing.

Let use the standard notation V^m for the value of the innovation to the monopolist, as a perpetual constant flow, discounted at the constant rate r, and gross of innovation costs. Let use the similar notation V^* and V^c for that value to the social planner and to a competitive innovator respectively. Then the following result can be stated.

Proposition 1 *Under Assumptions 1-3, (a) The inequality $V^m > V^*$ (respectively, $V^m \leq V^*$) holds if and only if inequality (4) is satisfied (respectively, violated); (b) furthermore, if output under monopoly is increased (respectively, decreased) after the innovation, then the condition $\omega(x_0^m, x_1^m) - \omega(x_0^*, x_1^*) > 0$ is sufficient (respectively, necessary) for the inequality $V^m > V^*$ to hold.*

Proof. The inequality $\Delta\pi > \omega^*$ obtains if the inequality in (4) determines the sign of $\Delta\pi - \omega(x_1^*, x_0^*)$; it is sufficient by construction (this proves part (a). (b) $[s(x_1^m) - s(x_0^m)]$ is positive whenever $x_1^m > x_0^m$ and negative otherwise; whence part (b) follows. ■

This result states that a potentially socially desirable innovation (as it is under Assumption 3) may not be realized under the social planner solution and be instead realized by a monopolist. Hence a monopolist will invest to "discover" some socially desirable technologies where the social planner would not invest. Also, it is possible that the monopolist invests in technologies that do not increase welfare, as shall be discussed below.

Proposition 2 (1) *Sufficient conditions for $V^m \leq V^*$ are that $C_1'(q)$ and $C_0'(q)$ be both constant, and that $D(p)$ satisfy the regularity conditions.*

Proposition 2 is the Arrow (1959) result restated for completeness[3].

Before continuing it is worth recalling here that $\mathcal{I}_m(C_0)$ has not be assumed to be a subset of $\mathcal{I}_s(C_0)$.

Remark 1 *The cost reducing innovations introduced by a monopolist, namely all those belonging to the set $\mathcal{I}_m(C_0)$, may lead to an increase or to a de-*

[3]In the case of constant returns, $C_i(q) = c_i(q)$, with $c_0 > c_1$. Then, treating c as a continuous variable, one has that $\pi^m(c) = D(p^m(c))p^m - cD(p^m(c))$, so that under the regularity conditions for $D(p)$ it is possible to apply the envelope theorem in order to obtain that $\partial\pi^m(c)/\partial c = -D(p^m)$, and $rV^m = \int_{c_1}^{c_0} D(p(c))dc$. Also, $rV^*(c_0, c_1) = \int_{c_1}^{c_0} D(c)dc$. Then, $V^*(c_0, c_1) \geq V^m(c_0, c_1)$ follows because $p(c) > c$ for all c values in $[c_0, c_1]$.

crease in social welfare in the industry. In particular, an increase in welfare necessarily obtains if $x_1^m > x_0^m$*.*

Remark 2 *There exist demand and cost functions* $C_0(x)$ *such that the set of innovations which are socially desirable under the social planner solution,* $\mathcal{I}_s(C_0)$*, and the set of innovations that a monopolist will introduce,* $\mathcal{I}_m^0 \equiv \mathcal{I}_m(C_0) \cap A(C_0)$*, can satisfy the following relations:* $\mathcal{I}_m^0 \nsubseteq \mathcal{I}_s(C_0)$ *and* $\mathcal{I}_s(C_0) \nsubseteq \mathcal{I}_m^0$*. Under constant returns the relation* $\mathcal{I}_m(C_0) \subset \mathcal{I}_s(C_0)$ *necessarily holds.*

The two remarks emphasize the difference between the social desirability of innovations, for a given market structure—in this case, monopoly—as distinct from the social desirability of a given market structure. In particular, one may have that $\omega(x_1^m, x_0^m) > 0$ for some innovations that belong to $\mathcal{I}_m(C_0)$ but not to $\mathcal{I}_s(C_0)$.

The second remark, related to the first, underlines the difference with the implications of the assumptions of constant returns to scale: in that case it is true that $\mathcal{I}_m(C_0) \subset \mathcal{I}_s(C_0)$, as shown in Arrow (1959).

An Example

Let $0 < c_0 < 1$, further, let $C_0(x) = c_0 x$, and $C_1(x) = (\delta/2)x^2$. Assume $D(p) = 1 - p$. Under these specifications of demand and cost functions, one has that $x_0^* = 1 - c_0$, $x_1^* = 1/(1+\delta)$, while $x_0^m = (1-c_0)/2$ and $x_1^m = 1/(2+\delta)$. The welfare change under the social planner solution is equal to $\omega(x_0^*, x_1^*) = (1/2)\left[1/(1+\delta) - (1-c_0)^2\right]$. Therefore, the innovation C_1 belongs to $\mathcal{A}(C_0)$ if $\omega(x_0^*, x_1^*) > 0$, or: (i) $(1-c_0)^2(1+\delta) < 1$. Similarly, $\pi_1^m - \pi_0^m = (1/2)\left[1/(2+\delta)^2 \; - (1-c_0)^2/2\right]$. Then, $\pi_1^m - \pi_0^m$ is larger than $\omega_1^* - \omega_0^*$ if : (ii) $(1-c_0)^2(1+\delta) > 2(4+\delta)/(2+\delta)^2$. The two conditions, (i) and (ii) define a non empty set of values for c_0 for each δ such that $\delta > 2$. So that there exist a non empty set of vectors (c_0, δ) such that the innovation C_1 is potentially socially desirable and such that for that innovation the incentive to innovate for the monopolist is higher than for the social planner.

3 A restricted domain of analysis

Assume now that total cost $C(x, I)$ is a function of quantity and of expenditures in R&D, I. with $I \geq 0$. Further, denote by $C'(x, I)$ the marginal cost in the usual sense, namely $C'(x, I) = \partial C(x, I)/\partial x$. Further, assume that $C(x, I)$ has a continuous partial derivative with respect to I, denoted $\partial C(x, I)/\partial I$, such that $\partial C(x, I)/\partial I < 0$ for $I > 0$. Then, by definition, $\omega^* = \int_0^{x^*(I)} (p(x) - C'(x, I))\, dx$, and hence $\frac{\partial \omega^*}{\partial I} = -\frac{\partial C(x^*(I), I)}{\partial I}$.

Similarly, $\pi^m = P(x^m(I))x^m(I) - C(x^m(I), I)$ and, $\frac{\partial \pi^m}{\partial I} = -\frac{\partial C(x^m(I), I)}{\partial I}$. Hence, the (traditional) inequality $V^m < V^*$ holds true if and only if

$$-\frac{\partial C(x^m(I), I)}{\partial I} < -\frac{\partial C(x^*(I), I)}{\partial I}.$$

Namely, since $x^* > x^m$, the monopolist's incentive to innovate is less than that of the social planner only if the (negative) effect of a marginal increase in I on total production costs is larger for larger output levels. This insight, which is also true in the traditional constant returns context, however, cannot be generalized to all contexts, as argued above.

4 Conclusion

The idea that a monopoly has a lower incentive to innovate than socially desirable is by now rooted in the tradition (this does not refer to innovation in quality as shown in Spence (1975)). As far as cost reducing R&D is concerned, so far there is no exception to the theory, first advanced by Arrow (1959), that a monopoly will invest short of the socially desirable level or, otherwise stated, that there exist socially desirable innovations that are foregone under monopoly. The analysis above illustrates the case where exceptions arise. Decreasing returns to scale appear to be necessary (but not sufficient) for the reversal of the inequality $V^m < V^*$ to arise. With increasing returns, as well as with constant returns, the standard result is confirmed.

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