PHENOMENOLOGICAL ANALYSIS OF SPRINKLING SPRAY EVAPORATION: THE AIR FRICTION EFFECT

Giulio Lorenzini, Daniele De Wrachien

1. INTRODUCTION

One of the most important problems that our near future will have to face is the raise of water cost due to its over-utilisation, which is ever more damaging the capacity of the ecosystem to renew its presence in the water tables and in every natural or artificial source available. In this sense it is fundamental to drive our habitudes in the way of a water waste reduction and this necessity cannot ignore that the biggest part of water resources (even up to 60 %) is used, in developed countries, for agricultural practices [1]. Irrigation is particularly due to be examined as it is often not performed following scientific criteria but just emotional ones by the technical operators. One of the causes for this behaviour is for sure a scarce agreement among scientists for what concerns a clear and univocal definition of the phenomena causing water loss during irrigation and of the parameters affecting its dynamics.

This is why talking about spray evaporation, that is water loss happening in the aerial path covered by a droplet exiting from a sprinkler nozzle before it reaches the soil surface, a wide and often discordant range of values has been in the last ten year periods provided in different papers, even if the same phenomenon was considered. So spray evaporation of water droplets in sprinkler practice was quantified with values changing from 2 per cent or less up to 40 per cent or more [2, 3]. Of course such a discrepancy has to be somehow understood or at least interpreted. Four hypotheses will be here formulated:

1) for the empirical/semi-analytical models available [4, 5, 6, 7, 8, 9]: the phenomenon faced is so difficult and affected by so complicate non linearities among the parameters involved in the process that every researcher, adopting his specific simplifications and empiricism in the impossibility of a full description, obtains results which are due to be seriously tested before deciding them as reliable and which are anyway often not easily comparable to one another because of the special conditions superimposed;
2) for experimental works obtaining low percentages of spray evaporation [2, 10, 11]: the results obtained are of the same order of magnitude of the experimental error typical of every measurement and so they might result to be more affected by an error of measurement rather than by the actual dynamics of the phenomenon;

Memoria presentata il 2.2.2003; accettata il 4.4.2003

Ing. GIULIO LORENZINI, Confirmed Researcher, Department of Agricultural Economics and Engineering, Alma Mater Studiorum-University of Bologna (Italy); Prof. DANIELE DE WRACHIEN, Full Professor, Head of the Institute of Agricultural Hydraulics, Faculty of Agriculture, University of Milan (Italy).

3) for experimental works obtaining high percentages of spray evaporation: the phenomenon is so strongly joint, because of the test conditions adopted, to the wind drift that a clear distinction between the two components could hardly be done on a scientific basis [12, 13];
4) in general: the phenomenon of aerial evaporation of water droplets exiting from a sprinkler has not been fully understood yet and something new has to be added to the description of the phenomenon in order to reach a better understanding of the evaporative effect analysed and to perform an easier description of the events.

It is possible to say that, apart from everyone's opinion, all the four hypotheses above provided make sense and this of course means that progress is needed in all of the four directions highlighted. The concerns expressed at point 1, that is the difficulty in modelling the problem, probably explain those at point 2 and 3: if the modellisation of the problem is affected by strong non-linearities, then the real matter is that every researcher who has performed experimental tests, without an actual separation of the significant parameters considered, has obtained results which are just valid for his particular conditions and not applicable to state a general principle. Further to this consideration and still for what concerns experimental, it would be interesting to try to "split the phenomenon" in all its components, that is trying to study separately (one at a time) all the parameters affecting spray evaporation (air temperature, relative humidity, solar radiation, nozzle height and diameter, droplet diameter, working pressure, etc.) to define the influence due to each of them. Such an approach has been chosen in [14] where just air temperature effect on spray evaporation in sprinkler irrigation was considered but of course future activity will have to use the same method to study the other significant parameters. Hypothesis 4 previously considered is instead the reason of this paper; it will be here, in fact, proposed the existence of a further parameter, air friction, which has been very little considered before to explain irrigation sprinkler spray evaporation but which instead, as it will be demonstrated, is not due to be aprioristically discarded. To do that it will be utilised an analytical model, defined and validated in [15], here implemented in its solution with the code Mathematica®.

2. METHOD

The analytical model utilised for this paper is that defined and validated in [15]. It defines a sprinkler droplet air path in terms of the Second Principle of dynamics: $\vec{F} = m\vec{a}$, where $\vec{F}$ is the total force acting on the droplet and equal to the vectorial sum of the weight of the droplet of mass m diminished by its buoyancy force and of the friction force acting during the flight on the droplet of acceleration $\vec{a}$. The friction factor f used in the model is that according to Fanning's definition [16]:

$$f = \frac{24}{Re} \qquad \text{for Reynolds number } Re < 0.1;$$

$$f = \frac{18.5}{Re^{\frac{3}{5}}} \qquad \text{for } 2 < Re < 500;$$

$$f = 0.44 \qquad \text{for } 500 < Re < 200000;$$

The model was defined in the following hypotheses:

1) each droplet is generated exactly in correspondence to the nozzle outlet;
2) the forces applied to the system are: weight, buoyancy, friction;
3) the droplet has a spherical shape all the way down;
4) the volume of the droplet is invariant during the flight;
5) friction has the same direction of the droplet velocity but opposite sense for all the path;
6) there is no wind disturbing the flight.

A few operative parameters are due to be inserted in the computations to achieve the results:

- the nozzle is high *h* with respect to ground level;
- the droplet exits from the nozzle with a velocity *v0* inclined of an angle α with respect to the horizontal direction.

If *n* is the weight of the droplet accounting for its buoyancy component and $k = \frac{f\rho A}{2}$ (where ρ is air density, so depending on temperature, and *A* is the cross section of the droplet) is the coefficient which defines the action of the friction force, then the final equations in the horizontal and vertical directions are as follows:

$$m\ddot{x} = -k\dot{x}^2$$
$$m\ddot{y} = -k\dot{y}^2 - ng$$

where $\dot{x}$, $\dot{y}$, $\ddot{x}$, $\ddot{y}$ respectively are velocities and accelerations in the horizontal and vertical direction. The initial conditions defined are:

$$x(t=0) = 0$$
$$\dot{x}(t=0) = v_{0x}$$

for the first equation and:

$$y(t=0) = h$$
$$\dot{y}(t=0) = v_{0y}$$

for the second one, where *t* is time and v_{0x}, v_{0y} are the horizontal and vertical velocity components, respectively, at the entrance. Integrating the system of differential equations gives the full analytical solution of the problem in the form of parametric equations of position (*x(t), y(t)*), velocity ($\dot{x}(t)$, $\dot{y}(t)$) and time of flight. The solutions provided by this model are analytical and so they can be widely applied to practical cases, in the hypotheses formulated. Attention though is needed for the parameter *k* as it is strongly affected by the flow state of the droplet.

3. A NEW HYPOTHESIS FOR SPRINKLING SPRAY EVAPORATION

Scientific literature related to sprinkler irrigation has up to now treated the problem of spray evaporation as a minor relevance one [2, 7, 9], mainly attributing to wind drift the global water mass reduction determined during the air path of the droplet. The experimental tests carried on in [14] have instead clearly demonstrated, because of the way through which any reasonable possibility of wind drift was eliminated, that aerial spray evaporation is a relevant water sink cause, generally not at all negligible: this consideration is due to the fact that just the evaporation gradient was to be attributed to air temperature, which was the main aim of that research. The challenge this paper is invested with is so that of giving an explanation to such a discrepancy. The consequent analysis has to be performed by means of a physical interpretation of the phenomenon. As, in fact, highlighted by hypothesis 4 in the Introduction, it is here believed that a full understanding of sprinkling spray evaporation has not been reached yet, especially for what concerns the definition of all the factors affecting it and all the relations among the factors. Up to now, in fact, spray evaporation from a droplet has been mainly attributed to air relative humidity [7, 8], to water vapour density gradient with respect to the droplet outer surface [17] or to vapour pressure gradient [9] with respect to the droplet outer surface: in practise focusing has almost ever been put, in a concept, to water vapour concentration and/or gradient in the air with respect to the location occupied by the droplet. This assumption is here just partially shared, as water vapour presence has for sure to be taken in consideration but the hypothesis here formulated is that the whole phenomenon cannot be attributed entirely to it, otherwise the results obtained by all the researches previously examined would not be in a so evident contrast from one another. The evidence of such a contrast means, physically speaking, that the phenomenon analysed has not been comprehended properly, in the sense that something could be missing in its schematisation. Following this idea it was utilised the analytical description of a flying sprinkler droplet defined by a mathematical model in [15] to make a few considerations about droplets aerial evaporation. In particular the analytical description performed there was based on a dynamic description of the phenomenon focusing on the concept of air friction force opposing to the motion of the water droplet. The model was validated and gave results in relevant agreement, with respect to those found in literature, for what concerns the kinematic part of the phenomenon, that is travel distance and time of flight: but as the results proved to be consistent, then it means that the physical model adopted, on which the mathematical one was based, makes sense. Following this evidence it results to be well posed the hypothesis that the friction encountered, due to air, by a sprinkler droplet in its aerial path is the "new" parameter that could enrich the description of the phenomenon aiding for the reaching of a better comprehension and so of a better description of spray evaporation, without of course here willing to diminish the already demonstrated relevance of the other parameters, especially air relative humidity, affecting the process studied. This approach has not been found elsewhere, probably because air friction was considered as a factor of minor relevance in affecting spray evaporation, moreover not at all easy to be calculated unless a few precise criteria are defined. Anyway, before stating its applicability to practical cases, that is before stating that it drives one to sensible results, it is necessary to implement the mathematical model in [15] calculating which part of a droplet spray evaporation can be attributed to the friction force opposing to its motion.

4. COMPUTATION OF SPRAY EVAPORATION

The computation of spray evaporation due to air friction will be based on the same hypotheses for which the model in [15] was arrived at, reported in Section 2 of this paper. As highlighted in the previous section, a few precise criteria are due to be defined to perform the requested calculations:

a) evaporation is obtained by the total work, entirely converted in thermal energy, done by the resultant force (sum of weight, buoyancy and friction force) acting on the droplet for its whole air path;
b) droplet evaporation, which actually happens continuously during the whole flight, is instead schematised as happening just at the end being computed all at one by the total work of which at point a;
c) for the computation of the total work, the droplet is considered as a material point;
d) air relative humidity is not a study parameter, in order to put in evidence just air friction effect.

Point b is the source of a practical limitation to this study due to the fact that the final kinetic energy of the droplet is calculated by means of its initial mass: this choice implies that the evaporative process determined is somehow over-estimated. A sort of "upper limit", in the proper mathematical meaning, for the effect of air friction in spray evaporation will so be found through this analysis but this will help anyway a deepening of the physical insight of the evaporative process related to sprinkler irrigation practice. What stated at point c allows for a simplified way of calculating the total work, which otherwise would request a very complicate computation of the integral of the variable local force multiplied by the consequent step: one can resort in fact to the general theorem of the Physics saying that: "the work done by the resulting force acting on a material point when it moves from one position to another, is equal to the difference between final and initial kinetic energy of the point itself". Point d clarifies, instead, the field of application of this research: the effect of air friction is investigated holding constant the other parameters (and in particular air humidity, which certainly is a relevant variable to consider) affecting the phenomenon and so, in this sense, the results obtained do not have an absolute meaning but just a relative one. Calculations are performed implementing the model utilised by means of the code Mathematica®, version 4.2, a powerful help in finding numerical but, in this case, also analytical solutions to many mathematical equations. A code for the determination of aerial spray evaporation in sprinkler irrigation was so realised.

5. RESULTS AND DISCUSSION

To ease the comprehension of the results a comparative form has been here chosen. In particular the code was tested referring to the data provided in [7] and [8]. This, of course, implies the introduction in the code of the parameters used for those reference works. It has to be underlined that those studies took into account a range of parameters, including among the others air humidity, while here the upper limit of air friction effect was the only study parameter. This of course makes, in principle, not comparable the results here obtained with those in [7] and [8], but it is here felt that, as the purpose of this paper is that of testing, by means of its maximum level just theoretically reachable, the role phenomenologically played by air friction in spray evaporation, a "visual" comparison with some established data could add something important in that of quantifying the achievements of this first stage of investigation, aiding a future development of the project.

Edling's settings were [7]:

- flow rate exiting from the sprinkler: 0.73 dm^3/s;
- nozzle diameter: 7.14 mm;
- jet inclination with respect to horizontal: 0°;
- nozzle height: 3.66 m;
- air temperature: 21.11°C;
- air relative humidity: 20%;
- no wind.

Thompson et al.'s settings, instead, were [8]:

- flow rate exiting from the sprinkler: 0.55 dm^3/s;
- nozzle diameter: 4.76 mm;
- jet inclination with respect to horizontal: 25°;
- nozzle height: 4.5 m;
- air temperature: 38°C;
- air relative humidity: 20%;
- no wind.

Figure 1 shows the results obtained by Edling [7] compared to those obtained by the authors of the present paper having inserted Edling's data in the code created, described in the previous section. Figure 2, instead, shows the results obtained by Thompson et al. [8] compared to those obtained by the authors of the present paper having inserted Thompson et al.'s data in the code created, described in the previous section. It appears a difference, apart from what referred for both cases to the smallest droplet diameter, between the reference values and those computed by means of the code here utilised. What, moreover, is due to be carefully noticed is that an opposite trend of the functions interpolating the data is found between reference values and computed one, again for both cases. In the droplet diameter interval considered for this study, the analytical trends of the evaporation functions for the cases study are reported in Table 1, where D is the variable droplet diameter: in particular the functions reported in the table are those interpolating the data represented in the histograms of Figures 1 and 2 and so referable to Edling [7], Thompson et al. [8] and to the results obtained in this work having inserted their parameters and their same physical conditions in the model just defined. It is important to remember that the interpolating functions quoted are to be retained as physically descriptive just in the droplet diameter interval investigated in the above presented comparative analysis, as the fitting process optimises its results just locally but may be far wrong in a wider field of application. The discrepancy showed in Figure 1, 2 and in Table 1 was anyway fully expected because of the different nature between the reference tests and those here performed, multiparametric the formers and monoparametric the latters, but it is important to remember that the aim of this work was that of verifying if the hypothesis of air friction relevance in spray evaporation, keeping as constant the other variables, was to be taken as reasonable: in this sense the comparisons achieved, even if do not have an absolute meaning because comparing two partially different physical situations, still are relevant because they open a window on a new perspective in the field of aerial evaporation of water droplets in sprinkler irrigation practice. This becomes possible exactly thanks to the overestimation of air friction effect by its upper limit: otherwise the implications of this parameters could have be hardly interpreted because of the predominance of the other variables affecting the process. Apart from these considerations it has to be put in evidence how qualitatively the results obtained, showed in both the figures, are correct as actual air friction effect on spray evaporation follows for sure those trends, because of the dependence of the friction force from the cross sectional area of the droplet, but of course not in the values represented which anyway have been clearly defined as upper limits.

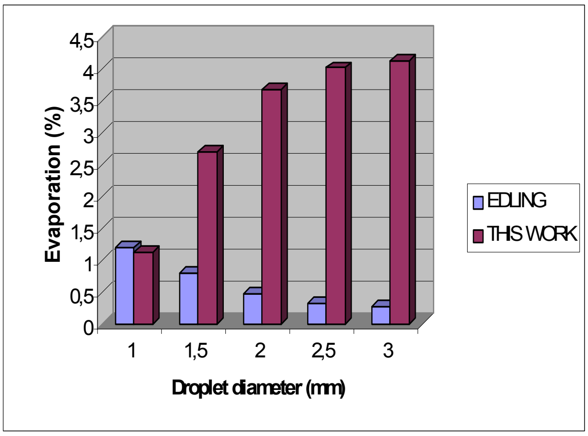

Figure 1. Spray evaporation results: Edling vs. this work

The actual values will so be described by smaller numbers (if the droplet diameter is held as variable during the flight). Saying that the trends presented are correct does not contradict the possibility of a trend of the global phenomenon so as presented in [7] and in [8] and consisting in the fact that droplet evaporation percentage diminishes while augmenting droplet diameter: in fact the actual trend of evaporation in the process is determined by all the parameters involved and nothing forbids that on the whole the final effect acts in the sense of an inverse dependence by the droplet diameter, even if air friction effect varies oppositely.

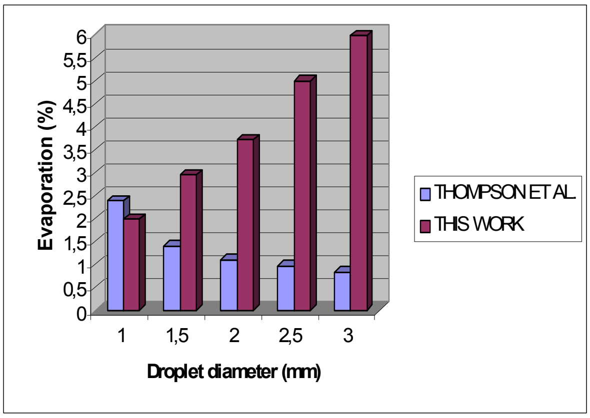

Figure 2. Spray evaporation results: Thompson et al. vs. this work

By the way, considering the results experimentally provided by a few of the papers in literature mentioned above, one has not to forget hypothesis 2 in the Introduction, relative to the dangerous closeness of measured data to experimental error limits.

6. CONCLUSIONS

Evaporation of water droplets during their aerial path in sprinkler irrigation practice, keeping as constant all the phenomenological variables but air friction effect, was investigated in this paper. A mathematical model elsewhere defined was here implemented by means of a numerical-analytical code named Mathematica®. The challenge of this analysis was that of

EVAPORATION FUNCTION (DROPLET DIAMETER INTERVAL 1 ÷ 3 mm)	
EDLING	**THIS WORK (WITH EDLING'S DATA)**
$2.416 - 1.46029\,D + 0.248571\,D^2$	$-3.352 + 5.51371\,D - 1.01143\,D^2$
THOMPSON ET AL.	**THIS WORK (WITH THOMPSON ET AL.'S DATA)**
$4.7 - 2.91143\,D + 0.542857\,D^2$	$0.478 + 1.364\,D + 0.16\,D^2$

Table 1. Interpolating functions of evaporation data by Edling and by Thompson et al. compared to those calculated in this work

verifying a new parametric description of the phenomenon investigated by hypothesising the role played by air friction, up to now widely neglected in literature, because retained as not relevant to the final result. Comparative results, even if not an agreement, show anyway that the hypothesis which this investigation was based on, that is that spray evaporation is also affected by air friction, proves to be well posed and that it is worth of a further theoretical deepening, possibly associated to careful experimental activity and to a full parametrical description of the process, including also the other factors affecting spray evaporation in agriculture.

REFERENCES

[1] Maidment D. R. (ed.), *Handbook of Hydrology*, (1993), McGraw-Hill, New York.

[2] James L. G., *Farm irrigation system design* (Chapter by Heermann, D. F., and Kohl, R. A.), (1996), Wiley, London.

[3] Tarjuelo J. M., Ortega J. F., Montero J., De Juan J. A., *Modelling evaporation and drift losses in irrigation with medium size impact sprinklers under semi-arid conditions*, Agricultural Water Management, (200) 43, 263 – 284.

[4] Seginer I., *Water loss during sprinkling*, Transactions of the ASAE, (1971) 14(4), 656-659, 664.

[5] Seginer I., Kantz D., Nir D., *The distortion by wind of the distribution patterns of single sprinklers*, Agricultural Water Management, (1991) 19, 341-359.

[6] Chernyak V., *The kinetic theory of droplet evaporation*, Journal of Aerosol Science, (1995) 26(6), 873-885.

[7] Edling R. J., *Kinetic Energy, Evaporation and Wind Drift of Droplets from Low Pressure Irrigation Nozzles*, Transactions of the ASAE, (1985) 28(5), September-October, 1543-1550.

[8] Thompson A. L., Gilley J. R., Norman J. M., *A sprinkler water droplet evaporation and plant canopy model: II. Model application*, Transcations of the ASAE, (1993) 36(3), 743-750, May-June.

[9] Kincaid D. C., Longley C., *A water droplet evaporation and temperature model*, Transactions of the ASAE, (1989) 32(2), 457-463, March-April.

[10] Yazar A., *Evaporation and drift losses from sprinkler irrigation systems under various operating conditions*, Agricultural Water Management, (1984) 8, 439-449.

[11] Kohl K. D., Kohl R. A., De Boer D. W., *Measurement of low pressure sprinkler evaporation loss*, Transactions of the ASAE, (1987) 30, 1071-1074.

[12] Ali S. M. A., Barefoot A. D., *Low trajectory sprinkler patterns and evaporation loss*, (1981) ASAE Paper No. 81-2085, ASAE.

[13] Clark R. N., Finley W. W., *Sprinkler evaporation losses in southern plains*, (1975) ASAE Paper No. 75-2573, ASAE.

[14] Lorenzini G., *Air temperature effect on spray evaporation in sprinkler irrigation*, Irrigation and Drainage, (2002) 51 (4), 301 – 309.

[15] Lorenzini G., *Simplified Modelling of Sprinkler Droplets Air Flow*, (2003) in publication.

[16] Bird R. B., Steward W. E., Lightfoot E. N., *Transport phenomena*, (1960) Wiley &Sons, New York.

[17] Kinzer G. D., Gunn R., *The evaporation, temperature and thermal relaxation-time of freely falling waterdrops*, Journal of Meteorology, (1951) 8 (2), 71 – 83.

NOTATION

$\vec{a}$, acceleration, m s^{-2}
A, cross sectional area of the droplet, m^2
D, droplet diameter, m
f, friction factor according to Fanning
$\vec{F}$, total force acting on the system, N
g, acceleration of gravity, m s^{-2}
h, nozzle height, m
m, mass of the droplet, kg
n, actual mass of the droplet because of the presence of buoyancy in air, kg
k, friction parameter, kg m^{-1}
Re, Reynolds number
t, time, s
v_{0x}, inlet horizontal velocity, m s^{-1}
v_{0y}, inlet vertical velocity, m s^{-1}
$v0$, velocity vector of the droplet exiting from the nozzle, m s^{-1}
$\dot{x}$, $\ddot{x}$, velocities and accelerations in the horizontal direction, m s^{-1}, m s^{-2}
$\dot{y}$, $\ddot{y}$, velocities and accelerations in the vertical direction, m s^{-1}, m s^{-2}
α, angle of the exiting droplet trajectory with respect to the horizontal direction, °
ρ, air density, kg m^{-3}

SUMMARY

An analytical model for irrigation droplets flight description is utilised to deepen a challenging topic, that is which role plays air friction in sprinkling spray evaporation while keeping as constant all the other parameters affecting the phenomenon. Results, focused on the computation of the upper limits of evaporation which could be attributed to an air friction effect, show that an hypothesis of air friction relevance in spray evaporation, up to now neglected by other authors, could make sense so advising for further attention to better achieve a full understanding of the phenomenon.

Key words:
sprinkler irrigation, spray evaporation, air friction effect, upper limit.