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Topics on General and Formal Ontology

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General and Formal Ontology

Paolo Valore

The word *ontology* derives from τὸ ὄν (*tò òn*), which is the neutral participle of εἰμί (*eimì* =I am) and whose genitive form is ὄντος (*òntos*), and from λόγος (*logos*). The literal meaning of this discipline is therefore "investigating that which is". Currently, 'ontology' is mainly used in two meanings: *a)* a philosophical discipline that studies being, that is as a part of metaphysics; *b)* a theory that deals with types of entities, specifically those types of abstract entities that are allowed in a language (ontology as specification of a conceptualization). Meaning *b)* has become widespread above all in areas connected with Artificial Intelligence and computer science. The ontology addressed in this collection of essays is predominantly that engaged in the study of being. Also included is the question of the types of entities that are allowed.

Research in ontology has often overlapped with research in metaphysics in general or the two have been viewed as identical. It is possible, however, to introduce at least one criterion for distinguishing between them. This is the meaning that we give to the two perspectives: what we call ontology can be considered as the study of being and existence in general, whereas the nature (essence) of being (its ultimate meaning, its known characteristics – if there are any) is the object under investigation by metaphysics. While we can include the existence of thoughts among the various objects permitted in our ontology, we are unable to enquire into the nature of "thought" without abandoning ontology for metaphysics. In some cases, moreover, it is not possible to draw a very clear dividing line between the two branches, the reasons being both intrinsic (the effective connection between them) and extrinsic (the common overlapping of ontology and traditional metaphysics). Nevertheless, it is clear that the field of ontology is traced out in relation to existence and being.

Is ontology today the same as the old traditional ontology? The field of study is evidently the same in certain respects: the problem of existence and being, of what there is and what is real, both in the sense of the question as to the single entities that can be acknowledged as real and in that of asking oneself what, in the final analysis, reality is. From a very general point of view, this can be understood

in at least two ways: either in relation to existence and being as such or to *real* existence and *real* being (as opposed to ordinary or phenomenal existence).

In the first case, one can follow two paths: either giving preference to common sense as the common denominator for the different theories and as the final criterion for the factual truth of the various levels of objectuality or acknowledging various ontologies in relation to different fields and theories, often with great elasticity even to the point of the coexistence of openly conflicting objectual fields.

In the second case, ontology is understood as reconstructing the domain of reality *in itself*, or even only that of *authentic* reality, beyond and often underlying the objective level of experience. This kind of ontological research is unlikely to speak of ontologies in the plural and proceeds very much like traditional metaphysics.

Hence, the object of ontological study is above all that which we are prepared to acknowledge as real, that which we will accept or posit. In this definition, no emphasis should be placed on any psychological connotation: we are forced to *posit* a certain area of objectuality (even the very level of reality *in the proper sense*) since this area must be acknowledged. An ontology thus includes everything that can be acknowledged as real (or as *actually* real). The emphasis is not on the act of experiencing or of acknowledging objects but on the content of experience or on the acknowledged object. Due to this characteristic, ontology cannot even be reduced to epistemology.

Contemporary ontology exploits the idea of logical form to tackle the problems of traditional ontological research. The logical-mathematical conversion of existential assumptions may be of interest also to the philosopher who is engaged in ontology in the "classical" sense of the term. The task of such a remodeled ontology is to promote a new conversion of traditional philosophical problems by freeing itself from the "literary methods" and making use of advances in scientific research, in the first place in logic. In this way, the "logical form" of our theories is displayed, which does not necessarily coincide with their linguistic-grammatical form. Therefore, current ontology differs from traditional ontology in the method of study, based essentially on the *logical form of existential statements*. But there is another aspect that can help us to circumscribe contemporary ontology: that of an ontology *of justification*. Instead of presenting a picture or series of pictures of what the world is like (or a simple list of what exists), it aims to find the reasons for which it is justifiable to claim existence.

I will now discuss briefly the meaning to be attributed to the expression "formal ontology". The formality of formal ontology may be understood in various ways. First, formal ontology should be thought of as a part of general ontology, distinct for example from a material or other ontology. In this sense, formal ontology deals with something that *is* formal whereas *material* ontology deals with something that *is* material. Along the same lines, formal ontology has developed as a formalized discipline that represents not so much a formal part of ontology as an "ontological" part of the varied world of formal disciplines. Formal is understood in the strong sense as in "formal logic" and ontology is constructed relying on axiomatic structures and logical-deductive procedures.

Another way to understand the "formal" restriction on "ontology" is to refer to the typical meaning of the phenomenology of formal ontology as a *categorial* ontology: in other words, there are categorial structures studied by ontology. Note, in passing, that we are free to choose this meaning also without identifying these categorial structures with those of intentional acts. What is understood here is close to this use of "formal", for which it is better not to assume, at least in advance, all the phenomenological implications in the sense of the school of thought. This use also comprises what was presented as a first hypothesis: the idea of subjecting the "forms" to investigation, which makes our ontology formal. However, this should not be interpreted as implying an alternative discipline called "material ontology": "formal" is not opposed to anything but, rather, it clarifies "ontology" by integrating its *object* in the definition. In other words, this does *not* mean that there are other objects of ontology that are neglected. Formal ontology in this sense *is* general ontology. At the same time, this addition also explains the instruments we can utilize for our research, instruments that are sometimes precisely "formal". In this way, the second meaning is in part recuperated as well, without however embracing the idea of a formal discipline within logic. To rely also on formal instruments does *not* mean that ontology is no longer a part of metaphysics, nor that it is an axiomatized and "artificial" system (in the sense in which we speak of "artificial languages").

To sum up, formal ontology is general ontology which, *as such*, deals with the formal structures of objectual levels, the formal preconditions for assuming objects in general and the formal categories of levels of reality. We thereby encompass also the concise definition of ontology as "specification of a conceptualization" (meaning *b*). In fact, an ontology is incomplete if it does not also include an indication of the basic categories of our level of objectuality, that is an indication of the ways in which we conceptualize something as "object" or "entity". Ontology in this sense is formal also because it does not spurn resorting to logic, in the broad sense, as the sometimes privileged path to achieving results that are not in themselves internal to logic.

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Constructing an Ontology

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Abstract. Empirical, logical, historical and mathematical inquiries may be able to tell us what it is ultimately right for us to believe in these various fields, and, in this pragmatic sense, they may be able to tell us what is true. But such inquiries cannot tell us what must exist in reality for them to be true, since they cannot tell us what their truthmakers are. To do this, we have to step back from the particular disciplines of science, mathematics, history, and so on, and seek to offer a general account of the nature of reality – one that is adequate to accommodate most of our empirical and theoretical knowledge of it. This is the task of ontology. It is not one that can be left to scientists, or other specialist inquirers, for at least two important reasons. The methodology of ontology is necessarily very different from that of any specialist field, and the account that it seeks to offer must be general. It cannot be limited to any one field of inquiry. In this paper, I shall describe and defend a strategy for arriving at an ontology that depends on keeping ontological and epistemological questions apart. The strategy is required, I shall argue, because these two sorts of questions are easily confused with one another.

1. Introduction

Ontology is the theory of the basic categories of existence. The postulated categories should be able to accommodate everything that we know to exist, and provide adequate truthmakers for the propositions that we know to be true. That is, for every proposition that we know to be true, there should be some presences or absences of things in these categories in virtue of which the proposition is true. The theory should be as economical as possible, yet be able to explain some of the general features of our best current account of reality -- how it is constituted, how it is structured, and how it works. It should also provide a sound basis for deciding what theoretical entities are genuine existents, as well as a way of understanding ourselves, and how we, as human beings, experience and relate to the world.

My starting point in constructing an ontology is what Sellars (1963) called the scientific image. I take this as my starting point, because I consider the scientific image to be the most rational picture of reality that we have. It is, after all, the

product of about three hundred years of collaborative research, which, for the most part, has been conducted according to the very highest standards of rational inquiry. So, plausibly, the scientific image of the world provides a more rational basis than any other for constructing an ontology. I reject the usual stategy of starting with a bold intuitively appealing hypothesis concerning the ultimate constituents of the world, and then working hard from there to show that it is sufficient. For such a strategy gives too much weight to our entrenched philosophical prejudices. If our ontology could eventually be simplified to such an extent that there remained only one kind of constituent of the universe (such as facts, tropes, or states of affairs[1]), then that would certainly be a bonus. But an ontology has a lot of explaining to do, and one should not be willing to pay too high a price for simplicity or intuitive appeal. In the early stages of constructing an ontology, we should err on the side of profligacy, rather than parsimony.

Accepting the scientific image, I start with the assumption that the world is fundamentally a physical world, i.e. a world of physical things, having only physical properties, standing in physical relations, and taking part in what are all, ultimately, only physical events or processes. By a physical thing, I mean anything that has energy, or consists of things having energy. By a physical event, I mean any change of energy distribution in the universe. By a physical process, I mean any causally or inertially connected sequence of events. By a physical property or relation, I mean any property or relation that anything may have that would make a difference to at least some physical processes involving that thing. The basic concept of physicalism is therefore that of energy, and in saying that it is a physical world, I am saying that it is fundamentally an energetic world. There are, of course, many difficulties with this assumption. Most importantly, there are the unsolved problems of the status of mathematical truths and of mental events and processes. I do not have any special insights into these problems, but I remain hopeful that both are soluble within a physicalist framework. If they are not, then this will spell deep trouble for the kind of ontology that I wish to propose.

These physicalist assumptions are roughly the ones that I had made and tried to defend in the 1960s and seventies (Ellis, 1976). They were the basis of my physicalist theory of mind, and my unrealism about forces. However, I no longer think that they are adequate for ontology, and would now make a number of additional assumptions. They are:

a. The fundamental things belonging to each of the categories of physical entities are all members of natural kinds

b. The natural kinds in each category are hierarchically structured by the species relationship

c. There is a global kind at the summit of each hierarchy of natural kinds that includes every kind of thing in the corresponding category.

[1] Russell (1918) had an ontology of basic facts, Campbell (1976) and Molnar (2004) have defended trope ontologies, and Armstrong (1997) argues for an ontology of states of affairs.

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These additional assumptions are of much more recent origin. The first is required to explain the vast numbers of exact similarities in nature. In a world without natural kinds, exact similarities would be cosmic coincidences, and in an unstructured world one would naturally expect them to be rare. But in fact there are enormous numbers of things in each of the principal categories of existence (i.e. of objects, processes and located properties, i.e. tropes) that are exactly similar. Therefore, the world is not unstructured. The second hypothesis is required to explain the vast numbers of partial exact similarities that are known to exist amongst the members of certain broad classes of things in each of these principal categories. If there were no generic natural kinds in the various categories, these partial exact similarities would also be inexplicable. Therefore, there are generic kinds in each category, as well as their infimic species. Finally, there are evidently some exact similarities (described in the genuinely global laws of nature) possessed by all things in their respective categories. To account for these, in the same kind of way, it is necessary to suppose that there are global natural kinds, i.e. natural kinds that include all of the more specific kinds of things in their respective categories.

In fact, I did not arrive at these natural kinds hypotheses by considering what suppositions we should have to make to account for the exact similarities that evidently exist in nature. In fact, they arose out of (a) my unsuccessful attempt in *Truth and Objectivity* (1990) to solve the problem of objective knowledge, and (b) as a result of wondering what the laws of nature were. In the final chapter of *Truth and Objectivity*, I speculated that the key to the problem of objective knowledge might lie in the fact that the world has a natural kind structure. For it seemed to me then that if it had such a structure, then there clearly would be a way that the world is that is independent of human information processing systems. If the world has such a structure, then it is classified by nature, implying that objective knowledge could simply be grounded in objective facts. Of course, there would still be the question of whether our best theories corresponded to the objective facts. But at least the hypothesis of a natural kinds structure would provide a metaphysical toehold for a correspondence theory that would have some semblance of respectability.

The hypothesis of a natural kinds structure of the world (i.e. the Basic Structural Hypothesis in Ellis 2001, pp. 22-3) also seemed promising for the development of a new theory of laws of nature. For the laws of nature are evidently hierarchically structured in the same sort of way. They range from global laws that apply directly to all events and processes (e.g. the law of conservation of energy) or to all physical systems (e.g. Lagrange's principle of least action) to highly specific laws that detail how specific kinds particles or chemical substances are intrinsically disposed to behave. But the implications of my Basic Structural Hypothesis turned out to be much more profound than I had originally thought. Not only did it provide a basis for thinking that there is a way that the world is that is independent of human classificatory systems, and lead to the development of a new theory of laws of nature, it also provided a basis for a new theory of causal necessity. John Bigelow, Caroline Lierse and I (1992) had speculated that the world itself might be a member of a natural kind (perhaps the only member), and

that the global laws might be of its essence, and so metaphysically necessary. But we had not yet arrived at the view that the causal laws, which describe circumstances and ways in which the causal powers of things are displayed, might also be metaphysically necessary. Indeed, this was not the only thing that came as a surprise: if the implications of the Basic Structuring Hypothesis are properly carried through, it leads to a radically new ontology that is incompatible with Humeanism, and has profound implications for philosophy almost everywhere. It changes our whole way of looking at things.

In this paper, I want to concentrate on the beginnings of this process, and say something about the origins of this change of perspective, and the methodological principles that made it possible. In particular, I want to talk about my efforts to solve the problem of objective knowledge, and the methodological principles that I have come to believe are required if one is to deal seriously with questions of ontology. Specifically, I shall argue that it is necessary to keep epistemological and semantic questions quite separate from ontological ones. My main criticism of the programme of scientific realism is precisely that it does not do this. For it seeks to derive a scientific ontology by means of a two-stage argument from the success of science, to the truth, or at least approximate truth, of its established laws and theories, to the reality most of the theoretical entities involved in these laws and theories.[2] By doing so, the programme throws the whole weight of deriving the ontology on to the correspondence theory of truth that is presupposed. For reasons I give elsewhere, I do not think that there is any version of the correspondence theory of truth that is up to this task.

2. The Problem of Objective Knowledge

In *Truth and Objectivity*, I argued against the correspondence theories of truth that were then currently in vogue. The only ones that were ontologically plausible were applicable only to the sentences of first order extensional languages, and were therefore incapable of analysing the modal truth claims of science, including most of its laws and theories. Consequently, if a correspondence theory was nevertheless to be accepted as adequate for truth in science, extensional semantics for all of the different kinds of modality would be required, and the only candidates then available were possible worlds semantics. But since the main motivation for trying to develop an adequate correspondence theory of truth in the first place was to defend scientific realism, commitment to realism about possible worlds seemed much too heavy a price to pay. So, I argued against accepting any form of correspondence theory, and, after much agonising, opted for what I called 'the evaluative theory of truth', i.e. the theory that truth is an ideal of epistemic rightness. Quine and Smart took a different stand. Confronted with precisely the same difficulty, they argued that an extensional language really *is* adequate for science. Smart advocated a Humean theory of causation, and followed Quine in

[2] As argued in Psillos (1999).

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rejecting all modal talk as second grade discourse that could ultimately be dispensed with.

Their reasons for doing so were conservative. An evaluative theory of truth, such as the one I advocated in *Truth and Objectivity* would not only undermine the standard argument for scientific realism, it would also invalidate much of the work done in the twentieth century on foundations of logic, and in philosophical analysis. And that would have worried them a great deal. I, personally, was not worried about preserving objectivist foundations of logic, because I had already demonstrated (in *Rational Belief Systems,* 1979) that such foundations are not needed. I was able to prove there that adequate foundations could be provided for all of the standard logical systems using only a subjectivist concept of truth. Therefore, objective truth was no more needed for the foundations of standard logics than objective probability was needed for the foundations of standard probability theory. Both could be grounded in a theory of rationality, which is where I thought logic properly belonged anyway. Nor was I worried too much by the impact of accepting an evaluative theory of truth on the programme of philosophical analysis. I thought it was a mind-numbingly dull programme, and I was not persuaded that the world is fundamentally a Humean world, as this programme seemed to presuppose. Rather, I was inclined to think that it wasn't a Humean world. But I did not then have a viable alternative to offer.

My main worry about the evaluative theory of truth that I developed in *Truth and Objectivity* was that it did not lie easily with scientific realism. For a while, in the 1980s, I toyed with internal realism, and earned myself a bit of a reputation for ratbaggery, as Putnam did on very different grounds. He had come to his position independently through formal semantics, and theorising about the philosophical implications of the Lowenheim-Skolem theorem. But at least he caused a bit of a stir in philosophical circles. The very different argument for internal realism that I presented in the 1980s was, perhaps fortunately, largely ignored. In the simplest cases, the correspondence theory of truth seemed so obviously right to most philosophers that it was beyond reasonable doubt. 'The cat is on the mat' is true, they said in Australia, iff the bloody cat is on the bloody mat. It has nothing to do with what anyone would be justified in believing, however anyone may wish to qualify this. Truth is a correspondence relationship, not a concept of epistemic rightness.

But whatever knowledge scientists may be seeking, it is not knowledge of propositions like 'The cat is on the mat' or 'All men are mortal'. They do not adequately represent the laws or theories of which scientists are rightly proud. They are just crude commonsense statements with little theoretical involvement, and the ways in which they correspond to reality would seem to be perfectly obvious. With regard to the significant claims of science, however, the theory of truth as epistemic rightness seemed to be much more promising. For it, and the redundancy theory, seemed to be the only plausible theories of truth that could possibly explain our use of the word 'true' in fields as diverse as physics, mathematics, semantics, economics and metaphysics. The correspondence theory of truth could hardly begin to come to grips with this wide range of usages. To me, it seemed much more plausible to say, as the pragmatists do, that the truth is what

we should finally come to believe about the world in the ideal limit of rational inquiry.

Moreover, truth is manifestly not value neutral. If I say 'It is true, but I don't believe it', or 'I believe it, but it isn't true', then I am involved in a pragmatic contradiction. For in saying that it is true, but I don't believe it, I am endorsing a proposition that I say I disbelieve, and in saying that I believe it, but it isn't true, I am saying that I believe a proposition that I would not endorse. So the word 'true' certainly behaves in epistemology in ways similar to the word 'right' in ethics. For 'This is the right thing to do morally, but I morally disapprove of doing it', and 'This is not the right thing to do morally, but I morally approve of doing it', are also pragmatic contradictions, and for a similar reason. So 'true' seems fairly clearly to function in language, not as a name of a relation between words and the world, but as a term of epistemic approval, just as 'right' functions in ethics as a term of moral approval. If 'true' named an unknown correspondence relationship, these pragmatic contradictions would be inexplicable. If ϕ is the unknown relationship between the sentence S and the world W such that S is true at W iff $\phi(S,W)$, why would it be a pragmatic contradiction to assert that $\phi(S,W)$, but I don't believe S, or that I believe S, even though $\sim\phi(S,W)$? How can asserting that a certain unknown relationship holds between a sentence S and the world W be tantamount to endorsing that sentence? The connection between 'S is true' and '$\phi(S,W)$' cannot be that of analytic equivalence, because we do not know what ϕ is. Nor can the holding of ϕ be the unknown essence of truth, because 'truth' is presumably not a natural kind term, and there is, therefore, no reason to suppose that truth has an essence. Moreover, neither kind of connection would explain the pragmatic contradictions between the relevant statements of belief and various truth and falsity claims. The only solution to the problem that I could see is to allow that truth is an epistemic endorsement operator, just as moral rightness is a moral endorsement operator.

It was mainly for these reasons that I concluded in *Truth and Objectivity* that truth just is epistemic rightness. But I now see that this was a mistake. If truth has a role in language as an epistemic endorsement operator, this is not its only role. If someone says that even the ideally best possible theory could be false, they are not involved in anything like a pragmatic contradiction. Certainly, it would be epistemically right to believe the ideally best possible theory. Nevertheless, ultimate deception or epistemic failure would seem to be possible. For surely it is possible that the world is not as the ideally best theory says it is. So there must be metaphysical implications of truth claims that go beyond mere epistemic endorsement. The trouble, I think, is that truth is really a hybrid notion. The truth operator certainly has the role of expressing epistemic endorsement. But there is a separate metaphysical question to be asked. Do the statements that we would endorse tell us how the world really is?

My present way of dealing with the hybrid nature of truth is to keep the issues of epistemology and ontology apart. Science aims to increase our knowledge and understanding of the world as much as possible. Therefore, it aims at epistemic perfection. It seeks to develop theories about reality that are as empirically

adequate, simple, comprehensive, explanatory and conceptually integrated as it can make them. But it cannot go beyond these aims to consider how the theoretical structures it has developed relate to the world. That, ultimately, is the task of metaphysics. The question whether the theories that best satisfy the epistemological criteria for acceptance can be regarded straightforwardly as descriptive of reality, as 'The cat is on the mat' most plausibly is, is a metaphysical question, not a scientific one. My answer is that many of our best scientific theories are not straightforwardly descriptive of reality. There are things in the world, mostly things that exist independently of our scientific knowledge, that make them 'true', in the sense of being epistemically right to believe. But there are many different kinds of truthmakers for the laws and theories of science, and most of them are neither superficial nor apparent. The truthmakers for causal conditionals, for example, are intrinsic causal powers. But until recently most philosophers would have denied their existence.

3. Keeping Ontological and Epistemological Issues Apart

In attempting to construct an ontology, it is important to separate questions about the nature of reality from those concerning epistemic rightness. Science can tell us what it is epistemically right to believe. But it does not tell us how those beliefs are related to reality. For the latter is not a question of science, but one of metaphysics. Yet there is a tradition that seeks to derive ontology directly from the theories that we take to be true. I refer to the tradition of scientific realism, in which it is argued that, given the extraordinary empirical success of science, it would be miraculous if most of its laws and theories were not objectively true (or at least approximately so), i.e. true in the metaphysical sense of correspondence with reality. Therefore, say the scientific realists, the scientific quest to discover the pragmatically best theories about the world must actually have resulted (for the most part) in the discovery of objectively true ones. This is Putnam's famous 'No Miracles Argument'. But if truth is a relationship of correspondence with reality that holds independently of anything that we may think we know, as most scientific realists also believe, then the conclusion that science has yielded objective knowledge should appear to be something of a surprise. For, they say, objective knowledge of anything that is not immediately apparent is essentially beyond us, and even the best theories that could be constructed by ideal observers about what is not immediately given may not be objectively true. Indeed, if truth really does designate a metaphysical relationship, and is not just a matter of what it is epistemically right to believe, then there is no reason to suppose that the best theories are even objectively probable.

But science aims to *understand* reality, not just to describe it, and its mission is to *explain* its processes, dynamics and structure. Its aims thus involve much more than description. For explaining is essentially an epistemic endeavour, since to explain something is at least to embed it in some wider context of knowledge,

thus displaying its connectedness to other things that we think we know. We can do this minimally with a covering law, e.g. by showing something to be an instance or species of a generalisation. But there are many other ways of establishing connectivity. We might, for example, be able to show that different things are intrinsically similar, even though they are manifestly dissimilar (cf. burning and rusting). Or, we might discover an unsuspected group structure at the heart of some cluster of phenomena, (e.g. M. Gell-Mann's Eightfold Way). Or, we might construct an ideal theory that seeks to explain how highly idealised things would behave in the absence of disturbing influences, as Carnot did, for example, in his theory of heat engines. Or, we might make a mathematical discovery. Or, we might find, as Einstein did, that the laws of nature are the same with respect to all inertial systems. Consequently, there appear to be many different kinds of theories and explanations in science – causal process theories, abstract model theories, formal structural theories, mathematical theories, space-time theories, meta-theories, and so on. So, if the laws and theories of the mature sciences are mostly true, as I think they are, there are many different kinds of significant truths in science. But it is not at all obvious that there is any correspondence theory of truth that will account for all of them.

My complaint about the programme of scientific realism is not that the laws and theories of science are not mostly true, or at least approximately so. It is, rather, that the metaphysical implications of this are unknowable in the absence of a high-powered correspondence theory of truth that will apply across the board in science. I am not much interested in a correspondence theory for 'The cat is on the mat', or 'All men are mortal', or any of the other popular examples of true statements. I think these are ontologically fairly transparent. I would be much more interested in one that offered a sound metaphysical explanation of the truth of:

1. It is impossible to construct a perpetual motion machine of the second kind' (the Second Law of Thermodynamics),

2. For any ideal incompressible fluid in free flow, the sum of the pressure head, plus velocity head, plus height is a constant' (Bournilli's theorem on free flow), and

3. $e^{i\pi} = -1$.

Existing correspondence theories of truth are not up to this task. The programme of scientific realism puts far more metaphysical weight on existing correspondence theories of truth than they are able to bear.

4. Ontological Questions are not Semantic Either

For similar reasons, questions of ontology should also be kept separate from those of reference and satisfaction. For such questions of semantics are about how words

relate to the world, and necessarily presuppose theories about the nature of the reality that the language is used to describe. The standard theories, when you examine them, all turn out to postulate Humean worlds of logically independent particulars. The set of all such particulars constitutes what is called 'the domain', within which the names occurring in the language are to be interpreted. The domain is then divided into so many different sets of objects, ordered pairs of objects, and so on, which can be interpreted as the extensions of the predicates of the language. The formal language that is supposed to be adequate for science is thus interpreted as describing a Hume world. If one's ontology is as crudely Humean as this, then, of course, such a semantic theory can be acceptable. But if you have a rich ontology, such as I have, then, of course, it is not. In my view, metaphysicians should just forget about the implications of their ontology for semantic theory. That is a question for semanticists to worry about.

Of course, I have to speak a language in order to talk about the world. But I manifestly do not have to speak *about* language in order to do so, even though the language I use has to be related to it in various ways. Thus, I utterly reject what is known as 'the linguistic turn' in philosophy. In my two recent books (Ellis, 2001 and 2002), I have eschewed all arguments that depend on concepts that belong properly to the philosophy of language, such as those of meaning, reference and objective truth. I talk about objects or events, rather than the names or definite descriptions that are used to refer to them. And I certainly do not identify properties with predicates or relations, or suppose that to each significant predicate of the language there is a corresponding property or relation that it designates. I talk of properties and relations, only where I think that the predicates we use to describe the world designate genuine universals. I do not use the apparatus of possible worlds semantics, even though I talk quite a lot about the modalities of necessity and possibility. I avoid using the term 'rigid designator' like the plague, because it wrongly suggests that the rigidity of the designation depends on the language we use, rather than the nature of the reality we are describing. I avoid using this term, despite Kripke's great pioneering work on essentialism, and even though I have quite a lot to say about the properties and natural kinds that rigid designators are supposed to designate. You will not find a single argument in either of these two recent books of mine that depends on our theories of reference, meaning or truth.

What we think exists must, of course, depend on what we think is true about the world. But it is a mistake to think that the ontology to which we should be committed follows simply from what we are rationally committed to believing. If I am rationally committed to believing something, then, of course, I must also believe it to be true. This we may take as given. But there are two separate questions that need to be asked:

1. What must exist if it is to be true?

2. Why is it right to believe that it is true?

The first is the ontological question concerning its truthmakers. The second is, or should be, a purely epistemic question concerning its ultimate rational justifiability. But, unfortunately, these two questions are easily conflated. For the concepts of truth that are almost universally accepted in philosophical circles are mixed. They are not just designations of relationships supposedly holding between bearers of truth (sentences, propositions, beliefs, or whatever they may be supposed to be) and the world, as the semantic and correspondence concepts all suppose. Nor is truth just a matter of what it is ultimately right for us to believe, as the pragmatists generally maintain. The fact is that truth has a dual role -- as the concept of epistemic rightness that is manifest in the pragmatic paradoxes, and as the metaphysical predicate of analysis. Normally, these two roles are not seen as conflicting with one another. We say what we think it is right to believe, but commonly suppose ourselves to be talking about what exists in reality independently of our knowledge or understanding of it. Mostly, this duality of roles causes no trouble, for the two concepts of truth only come apart here and there in special circumstances. They do so, for example, if an otherwise rational belief system on our language is thought to be incompletable (so that T(pvq) does not entail : Either Tp or Tq, even at the limit of inquiry). They also come apart at the limit of inquiry, if we contemplate the possibility of ultimate epistemic failure, (e.g. where we speculate that the world is really different from the way it would ultimately be right for us to believe it is).

I believe now that the best way to deal with the problem of truth is to separate these two concepts, and put the objective one into the too hard basket. For the problem of giving substance to this notion of objective truth is one of both metaphysics and semantics, and therefore not one that can be solved at this stage. The epistemic concept of truth is demonstrably adequate for epistemology, logic and for the theory of rationality. The objective concept of truth will have to wait until we have developed a theory of reality that is adequate to explain our epistemic truth judgements.

5. Truthmakers for the Current Scientific Image of the World

To develop the required concept of objective truth, we need a theory of language, and a theory of reality. For such a theory is concerned with the relationship between words and the world. According to Wittgenstein, this relationship is ineffable. And perhaps in the end this will prove to be so. But it does not seem impossible to develop a theory of reality to account for some of the general features of the world that our past efforts to know and understand it have revealed. Nor, given such a theory, would it seem to be impossible to develop a satisfactory semantics for the sort of language we use to describe the world. Given a Humean ontology, philosophers were able to construct a semantic theory adequate for extensional languages, and, given a Lewisian ontology of possible Hume worlds, they were able to construct semantic theories for modal and conditional languages.

There is no obvious reason why they should find it impossible to construct an acceptable semantics for a physically realist ontology, such as that developed in *Scientific Essentialism*. However, as a metaphysician, this is not my task. I saw my task as being to elaborate an ontology that is adequate for the current scientific image.

No one seriously believes that what we count as true is independent of what exists in the world. But plausibly, what we count as true also depends on language, on what we value epistemically, and on how we process information. An ontologist must seek to discover what exists in reality independently of such factors. So the strategy of keeping ontological questions separate from epistemological and semantic ones is essential to the ontologists' task. One approach to ontology that is careful to focus on questions of existence is truthmaker theory (Fox, 1987), which has deep roots in Aristotelian metaphysics. Fox defines a possible truthmaker for a given proposition p as anything the existence of which would necessitate p. It is plausible to suppose that every true proposition has at least one truthmaker (Fox's Truthmaker Axiom), and that in some cases it has a unique truthmaker (i.e. a fact or state of affairs). My approach to ontology has been strongly influenced by truthmaker theory. For in attempting to construct an ontology, I have sought to find truthmakers for the current scientific image of the world. I did so, because I think that this is really what has to be explained ontologically. We have to identify its broad features, and the central concepts used in its construction, and ask what assumptions we should make about the nature of reality to accommodate them.

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Variables, Generality and Existence: Considerations on the Notion of a Concept-Script

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> 'We persist in breaking reality down somehow into a multiplicity of identifiable and discriminable objects, to be referred to by singular and general terms'. W. V. O. Quine, 'Speaking of Objects'.

> 'Seldom if ever does Nature operate in closed and separate compartments, and she has not done so in distributing the earth's water supply. Rain, falling on the land, settles down through pores and cracks in soil and rock, penetrating deeper and deeper until it reaches a zone where all the pores of rock are filled with water, a dark, subsurface sea, rising under the hills, sinking beneath valleys. This groundwater is always on the move, sometimes at a pace so slow that it travels no more than 50 feet a year, sometimes rapidly, by comparison, so that it moves nearly a tenth of a mile in a day. It travels by unseen waterways, until here and there it comes to the surface as a spring, or perhaps is tapped to feed a well. But mostly it contributes to streams and so to rivers. Except for what enters streams directly as rain or surface runoff, all the running water of the earth's surface was at one time groundwater. And so, in a very real and frightening sense, pollution of the groundwater is pollution of water everywhere'. Rachel Carson, *Silent Spring*

1. Ontology and concept-script

In that semantic tradition of which Frege and Russell are among the most distinguished members, the project of formalising natural-language sentences is not simply a matter of developing smooth and effective techniques for the

representation of reasoning. Over and above the representation of valid inference as valid, and invalid inference as invalid, there is a further objective. Logic in this tradition is what Frege himself famously calls a *concept-script*, the import of the notion being chiefly that in *natural* languages, as Frege emphasizes, 'the connection of words corresponds only partially to the structure of concepts', thereby compelling the logician to 'conduct an ongoing struggle against language and grammar, insofar as they fail to give clear expression to the logical'.[1] In the more recent past, a kindred overall approach is forcefully expounded in the work of Quine, who writes, albeit with a positivistic slant, that

> the simplification and clarification of logical theory to which a canonical logical notation contributes is not only algorithmic, it is also conceptual... each elimination of obscure constructions or notions that we manage to achieve, by paraphrase into more lucid elements, is a clarification of the conceptual scheme of science.[2]

The approach is one with which I find myself in general sympathy, and the contrast between clear and less-than-clear 'expressions of the logical' is fundamental to the framework of this piece. Though it has not always received the understanding and respect which it deserves, the ideal of a logically transparent language represents no merely interesting episode in the history of ideas. It embodies, rather, a permanently valid insight, an enduringly valuable ideal for any analytical approach to philosophy. The remarks which follow constitute the summary outline for an elucidation and defense of this ideal — and also for an exploration of its limits.[3]

Now Frege's clear expression of the logical is clarity as to the form of what is *said*; and since clarity concerning what is said must call for clarity concerning what one talks *about*, or what is said to *be*, clear expression of the logical is also clear expression of the *ontological*. Indeed, the two concerns may well be seen as more or less identical; hence Quine, again, in stressing the relationship of logic to ontology, insists that 'the quest of a simplest, clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate categories, a limning of the most general traits of reality'.[4] Or as he also writes, to 'paraphrase a sentence

[1] ___'Logik', quoted in J. A. Coffa, *The Semantic Tradition from Kant to Carnap* (Cambridge: Cambridge University Press, 1991), 64.

[2] *Word and Object*, 161.

[3] See e.g. the otherwise excellent recent book by Scott Soames, *Philosophical Analysis in the Twentieth Century*, for an illustration of this point. Russell however, and more generally those with a positivist agenda, are prone to believe that common sense itself is contaminated by language. Speaking perhaps from personal experience, Russell writes that 'common sense is influenced by the existence of the word, and tends to suppose that one word must stand for one object... the influence of vocabulary is towards a kind of platonic pluralism...' (331). Russell's remark may be ambiguous; it might be construed in such a way as to imply that it is our spontaneous *reflective* tendency to use an over-simple model in thinking about the significance of our language; or it might be construed more broadly to apply also to everyday non-reflective thought. The former interpretation is the one I would here recommend.

[4] *Ibid*.,161.

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into the canonical notation of quantification is, *first and foremost*, to make its ontic content explicit'.[5] This emphasis upon the ontic content and its explicit or transparent formulation is to be found also in Russell, who writes of what he calls a 'logically perfect language' that it must be such as to '*show at a glance* the logical structure of the facts asserted or denied'.[6]

At the end of the day, the notation which 'thus confronts us as a scheme for systems of the world', Quine tells us, is precisely

> that structure so well understood by present-day logicians, the logic of quantification or calculus of predicates... all traits of reality worthy of the name can be set down in an idiom of this austere form if in any idiom.[7]

And let us, in this connection, agree to put entirely to one side all issues relating to extensionalism in metaphysics, issues which in any case it might perhaps be possible to address in *some* 'idiom of this austere form, if in any'. There neverthless remains, as it seems to me, a quite specific question as to whether and in what sense Quine's assertion is correct. The question focusses upon, but does not concern exclusively, a category of nouns which are sometimes characterized as *mass* nouns, but which, for reasons I shall not pursue, I here describe as nouns which are *non-count*. Non-count nouns (*NCNs* for short) are one of two great categories into which common nouns may be exhaustively divided, the other, evidently, being that of count nouns (*CNs*).

2. Two great semantic categories

CNs include words like 'number', 'sheep' and 'car', NCNs such words as 'water', 'gold' and 'trash'. With CNs we may ask, almost truistically, 'How many...?', whereas with NCNs, whether abstract or concrete, we may only ask 'How much...?'.[8] In the nature of the case, CNs alone accept numerical adjectives ('one',

[5] *Ibid*., 242, my emphasis.

[6] Russell's 'facts', of course, have *objects* of various sorts as their 'constituents'.

[7] *W&O* 228. The notion of an ideal language typically carries the implication that it is not only *transparent*, in a sense to be explored, but also that it is *complete*, in the sense of being capable of expressing or representing everything which is 'worth expressing'. Quine's remark embodies the notion of an ideal language in this 'full-fledged' sense, as one which is not just transparent but also complete. Here however my focus is chiefly on the first of these two parameters. A sensitive more recent examination of these issues occurs in Paul Grice, *Studies in the way of words*, (Cambridge, Mass: Harvard University Press, 1989), 'Retrospective Epilogue' 372-385.

[8] '*Almost* truistically', since the criteria for identifying CNs, and for distinguishing between CNs and NCNs, are far from clear. And in particular, there is a diverse assortment of syntactically plural nouns, including e.g. 'ashes', 'clouds' and 'groceries', which do not (always or ever) come with determinate criteria for counting that which they are true of. Natural language — perhaps reflecting reality in this regard — can be a pretty messy business. And while my focus here is upon concrete nouns, both CNs and NCNs may be either abstract or concrete (the appellation 'mass noun', as it happens, is typically applied to concrete nouns exclusively). Thus the former group includes such terms as 'hill', 'house', 'word', 'number', 'atom', 'piece of clothing', 'planet', 'attribute' and 'cat', while the latter includes such terms as 'wine', 'wool', 'tension',

'two', etc.) along with the quantifiers 'every', 'each', 'a number of', 'few' and 'many' ('so few', 'too few', 'so many', 'too many').[9] NCNs by contrast characteristically accept either 'a degree of' or 'an amount of', as well as 'much' and 'little' ('so much', 'too much', 'so little', 'too little').[10]

'furniture', 'xenon', 'leisure', 'refinement', 'clothing', 'beer', and 'food'. I focus upon *concrete* nouns, or uses or occurrences of nouns, in part to mark a contrast with those contexts in which nouns are used *generically*, or as so-called 'abstract' nouns. For the fact is that the very words we class as NCNs, in such contexts, may themselves be used for counting — for counting kinds or types — and phrases like 'a wine', 'one wine' and 'several wines' are perfectly in order. And it seems appropriate to speak of *uses* or occurrences of nouns, in part because on one view of word individuation, some words are used concretely both as NCNs and as CNs. Not only do we have 'less beer', 'less cheese', and so forth, we also have the *non*-generic 'fewer beers' and 'fewer cheeses'. There are numerous expressions which, like 'cheese' and 'hair', can figure as both CNs and NCNs; e.g. 'apple' has a non-count use. (However that of which 'apple' as an NCN is true is the result of doing certain things to apples — e.g. chopping or pulping them — whereas nothing need be done to hairs to justify the application of the non-count 'hair' to them).

[9] To echo and expand on the previous note, these remarks are hardly sufficient to precisely demarcate the categories; the categories themselves are far from being neat and tidy. For one thing, it is plainly not the case that all CNs take 'one'. There are various kinds of irregular nouns — plural invariable nouns, among others, nouns such as 'riches', 'goods', 'baked goods', 'goods and chattels', 'hops', 'groceries', 'wares', 'housewares', 'clothes', 'cattle', 'droppings' and so on — which have no singular, hence do not fit the paradigm. Indeed though these particular nouns all have a syntactically plural form, it is not even clear that they are all semantically CNs. Somewhat arbitrarily, perhaps — the issue is both theoretical and insufficiently explored — I shall take it to be necessary and sufficient for a noun to be classed as semantically count that it allows talk of *few*, *some* and *many* items of the type, even if the assignment of specific numerical adjectives, e.g. 'seven clothes', is not standard English. By the same token, if a term 'P' is to be counted as semantically plural, then whatever its syntactic stripe, it seems to be essential that such forms of words as '*one of* the P' and '*each of* the P' should make sense; and this is evidently not the case with *bona fide* NCNs. (Again, where 'one of the P' makes sense, there must also be at least the possibility of some singular CN 'S' such that 'one of the P' counts also as 'one S'. This does not, naturally, preclude the typographical identity of 'P' and 'S'). On this view of the matter, 'riches', for example, would probably not be classed as a count noun. Furthermore, the boundaries between CNs and NCNs are far from clear. For example, the contrast between 'ash' and 'ashes', in the sense of what, for instance, burning wood results in, *looks* as if it is that between an NCN and a plural invariable CN. But do we or can we speak of few or many ashes? It is possible that the apparent plural captures the quasi-granular or particulate character of certain residues, e.g. of burnt organic matter — though in contrast with such cases as that of clothing, the idea that there is a plurality of discrete and demarcated objects which *are* individual ashes seems implausible. In other words, it looks as if the fact that clothes *are* discrete and demarcated objects, in spite of the peculiarity of counting procedures in this context, legitimates the thought that 'clothes' may be included in the class of nouns which are semantically count whereas 'ashes' may not. And finally, some nouns which seem to be semantically non-count can take syntactically plural forms: 'snows', 'sands', 'waters', 'molasses' and the like. It may of course turn out that intuitions of what is semantically a *bona fide* plural fail us at the borders, and that for the purposes of a neatly regimented theoretical account, some such condition as the one I have suggested may have to be simply stipulated as criterial.

[10] This feature of the entire class of NCNs extends beyond the class of concrete NCNs, but it nonetheless remains of fundamental interest when we focus *only* upon NCNs which are concrete. Work is evidently called for on distinguishing *bona fide* abstract nouns (which correspond to concrete *adjectives*) from the generic uses of what are otherwise concrete *nouns*. The contrast is that of 'humility', as in 'Humility is a virtue', and 'water', as in 'Water is a liquid'. As against Kripke, Putnam, *et. al.*, it is my working hypothesis that 'abstract' or generic uses of nouns in general, and non-count nouns in particular, are best approached by way of their concrete or specific cognates, and not, Platonistically, vice-versa. In this, I note support from the views of Chomsky in 'Language and Nature', *Mind* 104 (1995), 1-61., who challenges the essentialist semantics of Putnam and Kripke, and of Barbara Malt, 'Water is not H20', *Cognitive Psychology* 27 (1994), 41-70. See also Barbara Abbott, 'A Note on the Nature of "water"' *Mind* 106 (1997), 311-319.

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NCNs typically receive no significant examination, perhaps not even a single mention, in standard logic texts. They do not figure in such texts, because they simply do not figure in our logico-semantic canon.[11] Here I mean to raise the question of just what their logico-semantic status is, in relation to the concepts and the apparatus of the canon. The general import of the arguments which follow is that the 'facts which are asserted or denied' through using NCNs are at least semantically distinct from those involving CNs; and more particularly, that while a formal system of quantifiers and variables is reasonably well-suited to the intrinsic logic of CNs, the same cannot be said for that of NCNs.

Now *the* most fundamental feature of NCNs consists precisely in the fact that they are *non-count*. What then is it to be *count*? CNs, or their occurrences, are semantically *either* singular ('thing', 'apple', 'piece of clothing') *or* plural ('things', 'apples', 'clothes'). To be non-count ('stuff', 'water', 'clothing') is therefore to be *neither* singular *nor* plural. NCNs are then semantically *non-singular*, simply in virtue of being non-count; and it is this which underlies their often noted kinship with the plural — plural nouns themselves, self-evidently, are non-singular. The relationships between the semantics of CNs and NCNs may then be briefly represented in the following tableau (and here, of course, the equation 'Non-singular + non-plural = non-count' is tacitly affirmed):

[11] It is then hardly surprising that there are no standard recipes for the formal representation of such sentences as 'All water contains impurities' and 'Some snow is black.' There are, to be sure, various proposals and suggestions in the literature; but these are largely matters of dispute. Regarding the sentence 'All water is wet' (numbered as 7.5.4a) James D. McCawley considers a possible formalisation as '$(x)(W_a x \mid W_e x)$'. McCawley then suggestively observes that

> The problem with this formalisation is that it is far from clear what must be allowed as values of the bound variables for it to make sense. The values must include things of which 'is water' can be predicated, and while there are many entitites of which 'is water' can innocuously be predicated (puddles, pools, drops), it is not clear that any such set of entities would provide enough values for the bound variable.... Example 7.5.4a is valid not only for a believer in the modern atomic and molecular composition of matter but also for someone of 1700 AD who believed that matter is continuous and infinitely divisible, and an adequate account of mass terms must be as consistent with the latter view as with the former, since the logic of quantifiers cannot by itself establish or refute any theory of matter... this makes for a whopping big universe of discourse, especially for states of affairs in which a pre-atomic conception of matter holds and all physical objects will have uncountably many parts... *Everything that linguists have always wanted to know about logic*, 235.

	1. **Singular** ('one')	2. **Non-singular** ('not-one')
3. **Plural** ('many')	X X X	'things' 'apples' 'clothes'
4. **Non-plural** ('not-many')	'thing' 'apple' 'piece of clothing'	'stuff' 'water' 'clothing'

Table I

The singular / non-singular contrasts are epitomized in Table I by those between 'thing' on the one hand, and 'things' and 'stuff' on the other; and the inclusion of the contrast between 'clothes' and 'clothing', alongside that of 'apples' and 'water', serves to emphasize the point that these contrasts are first and foremost *semantic* or quasi-semantic, as opposed to *metaphysical* or ontic contrasts (it being assumed that the 'clothes' / 'clothing' contrast itself is essentially a semantic one).[12]

What has to be the most obviously significant dimension of the contrast of CNs and NCNs consists in its correspondence with the distinction between what is and is not *countable*. At bottom, what this contrast embodies are distinct modalities of *quantity* or of *amount* — modalities which I propose to call 'discrete' and 'continuous' quantity. And though at first blush this contrast of discrete and continuous quantity might *sound* ontological, I have suggested that this is not, or not immediately, the case. The contrast is related to, though it does not coincide with, one of distinct modes for the *determination* and *specification* of quantity or amount — modes I shall call 'counting' and 'measuring'.

3. Continuity and discreteness

'Counting', as I intend to use the term, denotes the determination or specification of quantity through the use of natural number-related expressions — 'one horse', 'two shirts', 'so many things', 'too few clothes', 'a dozen eggs', 'a single professor', etc. In this preferred and intuitive sense, it is a truism that counting is applicable to the denotata of CNs *exclusively*. Measurement by contrast, while typically involving the use of numerals, is applicable to the denotata of both CNs and NCNs alike: we may speak both of '75 ccs of poppy seeds' and of '75 ccs of

[12] This table first appears, albeit in a slightly different form, in my 'Words without objects', *Principia*, vol. 2 no. 2 (1998), 147-182. The general claim, to be precise, is that the category-contrasts here at issue — those of singular and non-singular, plural and non-plural, are *all* essentially semantic and not ontological.

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water', both of '2.5 kilos of apples' and of '50 kilos of clothing'. In contrast with counting, *any* real number can in principle be assigned to the measure of an amount of something. The concept of weight, for instance, is such that it is intelligible to assign a weight of n kilos (where 'n' represents an integer), or of n x π kilos, to a quantity of snow (rice, apples, clothing, underwear, water, etc.). And though measurement is applicable to the denotata of both NCNs and CNs, the denotata of NCNs, as such, may be *only* measured and not also counted. It is the latter which I call 'continuous' quantity; that which can be counted I refer to as 'discrete'.

The non-ontic nature of this contrast is particularly obvious in the opposition of such words as the CN 'clothes' and its cognate 'collective' NCN 'clothing'. Though 'clothing' ('furniture', 'footware') is continuous and 'clothes' discrete, to say that *there is* clothing here or there is to say no more than that *there are* clothes here or there — it is to commit one to the existence merely of individual pieces. In this respect there is good sense in Quine's remark: 'The contrast lies in the terms and not in the stuff they name... "shoe"... and "footwear" range over exactly the same scattered stuff'.[13] For this reason, the semantic or quasi-semantic contrast between the two modalities of quantity seems best understood via the contrast of the two modalities for the determination and / or specification of quantity, rather than *vice-versa*. Though *pieces* of clothing may be counted, clothing *as such* is measured but not counted (such-and-such a volume, such-and-such a weight; any real number might be assigned to a measure of an amount of clothing). Though there are *units* of clothing, and in spite of the *ontological* equivalence of collective nouns like 'clothing' with CNs, such NCNs are no less *semantically* non-count than 'pure' NCNs like 'water' and 'mashed potato'. While there is a clear sense to talk of the smallest number of clothes — a single item of clothing — there is no clear sense to talk of the smallest amount of clothing; is one woollen winter coat the same amount of clothing as a single nylon stocking? It is false to call individual items of clothing, furniture, etc. *smallest amounts* of clothing, furniture, etc. In fact the very idea of *the* amount of clothing (or of cotton, snow, or sand) in a given region, as such and without qualification, seems lacking in significance. There is no unique measure of the amount of clothing in a warehouse; this might be specified, e.g., by volume, or by weight, or indeed by counting the number of bales; and these different measures cannot be expected to be correlated in any uniquely determinate way.[14] The notion of continuous quantity seems relative to some specific *measure* of amount. Discrete quantity, by contrast, is a privileged modality of quantity and counting seems a privileged modality for the determination of quantity; there is exactly *one* non-relative way of determining the quantity of eggs or pieces of clothing in a box, which is precisely to count them.[15]

[13] *Word and Object*, (Cambridge, Mass.: MIT, 1960). 91.

[14] Since bales may be of indefinitely many different sizes, counting the bales is strictly a measure of the *bales* alone, and only indirectly a measure of the *clothing*; hence this is a case in which counting 'goes proxy' for measuring, rather than one of measurement *per se*.

[15] There are indeterminate forms for the specification of continuous quantity — 'so much stuff', 'too much cotton', 'so little water', etc. — which are parallel with forms for the specification of discrete quantity — 'so many things', 'so few birds', 'too many cars'.

While both 'clothes' and 'clothing' apply to what they are true of *en masse*, they do so *via* crucially distinct modalities for the specification of quantity.

Though not directly ontological, the contrast of discrete and continuous quantity is linked to certain ontic contrasts. Counting does of course involve discrete units; and though what is measured may consist of discrete units, measurement as such does not require it, and there are ontic category-differences *within* the semantic category of NCNs. Thus, contrast the two groups of NCNs (a) 'furniture', 'footwear' and 'clothing' and (b) 'rubble', 'sand' and 'snow', with the 'pure' NCNs of group (c) 'mashed potato', 'wine' and 'water'. The collective nouns of group (a) may be said to be 'object-involving' in that they are semantically 'atomic' — there are units of furniture, clothing, etc. (individual pieces of furniture, pieces of clothing, etc.) standardly not divisible into smaller units of furniture, clothing, etc. It is part of the *meaning* of such an NCN that like a typical CN, it ranges over discrete pieces, units or elements of what the NCN denotes; indeed the very *identity* of some furniture is not to be distinguished from that of some pieces of furniture. And group (b), though not thus atomic, are object-involving in that they may be said to be semantically 'particulate': it's part of their meaning that what these words denote consists of discrete grains, flakes, bits, etc. etc., the difference being that the identity of some sand (snow, rubble, etc.) is not dependent on that of certain particular grains (flakes, etc.).

In contrast with groups (a) and (b), no such object-involving concepts enter into the meanings of the group (c) terms. Thus whereas to say that there is furniture or clothing in some region is to say that there constituent pieces or units of furniture or clothing in that region, to say that there is wine or mashed potato in some region is not to say that there are objects characterisable as 'pieces' or 'units' of wine or mashed potato in that region. In the nature of the case, there is here no comparable notion of a constituent piece or unit.[16] Justice can and should be done to the ontic contrast between the idea of a range of discrete countables and that of an homogeneous medium, or what Michael Hallet calls an 'undifferentiated material'; but it must be done within the framework of an appropriately semantical conception of the CN / NCN contrast itself.

4. The essential non-singularity of quantified non-count sentences

CNs are either singular or plural, and as such, they or their occurrences are prone to vary in semantic value, even within the scope of a single (valid) argument.[17] But NCNs are neither singular nor plural, and are incapable of variations in semantic

[16] To emphasize, the point is *not* a point about the semantic status of the denotation of 'the wine in this bottle' as against, e.g., 'the snow in our garden' — these, I maintain, are equally non-singular. It concerns rather the contrast between the semantically non-particulate nature of 'wine' as opposed to 'snow'.

[17] The argument 'All cars pollute; Guzzler is a car; therefore, Guzzler pollutes' involves just such an obvious shift from plural into singular.

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value. Whether in the context of regular quantified sentences, or in referential contexts, they are non-singular in all of their occurrences; I review the former contexts first. And here, the non-singularity of NCNs is reflected in the twin facts that they do not combine with the singular quantifiers such as 'each' and 'every', 'a' and 'one'; and that the quantifiers with which they do combine, for instance 'all' and 'some', are themselves essentially non-singular, combining also with plural, but never with singular CNs.[18] And being also non-plural, there is a truistic sense in which the non-count form is never, unlike that of many plural sentences, reducible to singular form.

Consider then what are, from the standpoint of their quantifiers, a group of 'standardly' quantified non-count sentences:

[1a] All water is pure
[2a] Some water is pure
[3a] No water is pure
[4a] Some water is not pure.

The quantifiers here involved are more familiarly combined with plural nouns — 'men', 'Greeks', 'cars' etc. — and plural verbs; in fact group [a] may be compared with what I'll call a 'classical base set' of simple quantified CN sentences such as are represented in the standard predicate calculus —

[1b] All cars pollute
[2b] Some cars pollute
[3b] No cars pollute
[4b] Some cars do not pollute.

So far as the group [a] sentences are concerned, however, there is no standard recipe for the representation or understanding of their structure; non-count constructions do not enter into standard predicate calculus. There are simply, as I have noted, a variety of more or less undeveloped suggestions and proposals, with nothing like consensus as to the way forward.

Now the group [b] sentences are, in fact, uniformly plural; but the quantifiers they involve are not peculiarly plural; they are rather, and more generally, non-singular, governing both plural count and non-count sentences alike. That is, subject to certain qualifications, the *only* bare general terms or predicates with which 'all' and 'some' *cannot* be conjoined are those which are singular in form. 'All tree', 'all person' and 'all number' just make no sense; they are not grammatically well-formed.[19] The case of 'some' is more complex than that of 'all' since 'some' is ambiguous; it's necessary to distinguish its use with either singular or plural CNs to speak of unidentified individuals — 'Some turkey / turkeys spilled my wine' — from the sense I here intend, in which it calls for either plural CNs or NCNs, as in, e.g., 'I'll have some soup' or 'We'll boil some eggs', where it may be

[18] 'Any' however is an all-purpose quantifier which combines with singular, plural and non-count nouns alike.

[19] While 'all' may be conjoined with proper names (in constructions which are perhaps elliptical — 'all Rome', 'all Gaul', etc.), and 'all *of the*...' (along with 'some *of the*...') may be conjoined with both plural and singular occurrences of nouns, the meaning of 'all' is such as to preclude its combination with unvarnished singular occurrences of common nouns.

said to be the non-singular indefinite article, signifying indefinite or indeterminate amount or quantity. In this latter sense, 'some' no more combines with singular CNs than does 'all'; to thus speak of 'some tree' or 'some person' would be to enforce a non-count sense on 'tree' or 'person'.

A key difference between groups [a] and [b] is that the latter group, being plural sentences of a relatively simple type, can be paraphrased as singular, whereas non-count sentences can never be so paraphrased. And singular paraphases of quantified plural sentences are crucial to their representation in standard predicate calculus, since sentences in that calculus are always cast as singular.[20] Insofar as there is such a thing as a 'standard' modern logic, that logic is not

[20] The categorical universal plural of [1b], 'All cars...', becomes the hypothetical universal singular 'If something *is a car*, then *it* ...'. Likewise the plural 'Some cars...' of [2b] becomes the existential singular 'There *is an x* — or, *at least one x* — such that *x is* ...'; and so on. Non-singular quantifiers and their corresponding verbs and plural CNs, in natural-language sentences, are invariably replaced by singular quantifiers and CNs or predicates in the artificial-language sentences. Natural-language plural grammatical subjects, such as 'All cars', are deconstructed into quantifiers, predicates and variables — quantifiers and predicates which are singular in form, and variables which range over individual objects taken individually, or take semantically singular terms exclusively as their substituends. Because of this, the calculus may be said to provide an *analysis* of sentences like [1b] - [4b] — an analysis which complements the introduction of a category of atomic sentences, referring to specific individuals by name, and which accords with that basic theoretical role commonly assigned to the notion of singular reference. All this is obvious enough, and is more or less explicitly recognized in standard expositions of the calculus. We are told for example in the *Encyclopedia of Philosophy* that 'Predicate logic begins its analysis with the very simplest type of sentence, the singular sentence' — which itself, we are told, asserts 'that a certain property is possessed by an individual object'. Beyond this, the next step 'is to extend the analysis to certain classes of *nonsingular* simple sentences', such as, for instance,

Everything is material.

And the analysis of such sentences 'requires the introduction of a second sort of term, *individual variables*' — items which 'do not name or refer to a particular object but, like pronouns, serve as placeholders for terms that do'. P. Edwards, (ed.), *The Encyclopedia of Philosophy* (New York: Macmillan and The Free Press, 1967), entry under *Logic, Modern*, italics in original. It will I think be evident that 'singular' in this passage does double duty both for the quantitatively specific concept of *being numerically one* and for the quantitatively or numerically neutral concept of *being non-general*; and that it is the latter concept, but not the former, which is carried over into the meaning of 'nonsingular' as used above. (It is a nice question why the most typical natural-language examples of universal sentences with which we are confronted for analysis are ones beginning with 'all', when the irony is that it is *precisely* in the divergence between 'all', and 'any', 'each' and 'every', that some of the more serious limitations of this calculus come clearly into view). The passage illustrates the fact that there is a certain equivocation surrounding the use of the expression 'singular term' in much theoretical discourse, reflecting the conflicting pressures of both formal canon and natural language. There is a tendency, consonant with formalism, to take the expression as co-extensive with, and even perhaps as virtually synonymous with, 'referring expression'. Yet referring expressions may of course be plural; and the natural bizarreness of such constructions as 'plural singular terms', or — even more strikingly, 'non-singular singular terms' (which must plainly, if it is to be coherent, involve equivocation) — is self-evident. It will already be very plain however that by 'singular' in this work I always and only mean *numerically* singular — singular, that is, as it is contrasted with plural. In this manner it's clear that even among CN sentences themselves, the predicate calculus accords a massive privilege to those which are singular. And this too is unfortunate, since to understand quantification on NCNs, it is essential to understand its *overall* relationship to quantification on CNs — its relationship not only to quantification on CNs which are singular, but to quantification on CNs which are plural too. In turn, this requires that we identify the limitations imposed by the predicate calculus on accounts of quantification of this latter kind: we need to understand what it is about CN quantification which is excluded from the calculus account.

simply one of CNs — it is one of CNs which are either in, or are reducible to, the *singular form*, and correspondingly of predicates which are distributive in form, which are true of objects one by one. Group [a], however, cannot be paraphrased or reduced into singular form; they are *essentially* non-singular; and their kinship with the (*unreduced*) plural sentences of group [b] is absolutely vital to an understanding of their structure.

Nevertheless, it *is* the fact that these sentences are essentially non-singular, and not that they are *non-plural*, which precludes the possibility of singular paraphrase or equivalence; for there are plural sentences which are likewise irreducible. Much as grammar prohibits analysis of the quantified *non-count* group along the singular lines of the classical base set, so it prohibits representation of the *plural count* sentences

[1c] All cattle have tails

and

[2c] Some clothes are tailored

as the singular sentences

[1c'] *Each cattle has a tail

and

[2c'] *At least one clothe is tailored.

It's the merest truism that where a quantified subject expression involves a plural invariable noun, as in [1c] and [2c], no *non*-plural sentence, quantified or otherwise, involving that same noun can be constructed. These sentences are irreducible in an obviously weaker sense than that in which the group [a] sentences are irreducible; there are cognate CNs in terms of which they may be 'paraphrased' — [2c], for instance, may be paraphrased as 'At least one *item of clothing* is tailored'.[21] But *qua* plural invariable nouns, they cannot shift in semantic value from non-singular to singular; any string of words having a singular form which contains such a noun is bound to be ungrammatical. In this obvious if superficial sense they are *essentially* non-singular.[22]

Now it is precisely a feature of the essentially non-singular universal and existential quantifiers 'all' and 'some' that they are able to license inferences involving non-singular nouns in a way that 'each' and 'every' cannot. For instance, even when no move is possible to 'this F', 'all' permits a direct connection between 'all Fs' and 'these Fs'. An essentially plural quantified sentence such as

[1d] All clothes are made of polyester

has the power to *directly* license inferences between such non-quantified non-singular sentences as

These things are clothes

[21] There are a number of possible senses of 'irreducible'; one might for instance distinguish a 'merely' *syntactic* irreducibility such as that of [1c] from a *semantic* irreducibility such as that of Boolos' sentence 'The rocks rained down' where the irreducibility is enforced by the meaning of the (collective) predicate 'rained down', or again that of an *ontic* irreducibility, such as that of *certain* NCNs, e.g. 'water', to plural CNs, a point I will return to in due course.

[22] The same point applies, in a rather weaker sense, to such 'irregular' plural CNs as 'people', 'geese' and so forth.

and

These things are made of polyester.[23]

By the same token, [1d] might be said to 'distribute collectively' over whatever things *are clothes* — these clothes, the clothes on the first trans-Atlantic passenger flight, the clothes now in your bedroom, etc. Unsurprisingly, a similar point — and one which should seem almost equally trivial — may be made regarding NCNs. As with an essentially plural sentence like [1d], the essentially non-singular

[1e] All clothing is made of polyester

(or a compound hypothetical equivalent, e.g. 'If some stuff is clothing it's made of polyester') can directly license inferences between such non-quantified sentences as

This stuff is clothing

and

This stuff is made of polyester.[24]

Again, 'clothing' may be said to range over clothing much as 'clothes' ranges over clothes — that is, perforce, *en masse*. And since NCNs do not, like plural invariable nouns, vary in semantic value, being always non-singular, these non-quantified sentences in turn, like plural referential sentences, must be non-singular. In other words, the essential non-singularity of NCNs is reflected not only in the non-singularity of quantification involving NCNs; it is also reflected in the non-singularity of reference and definite denoting based upon such nouns. It is to this issue that I now turn.

5. The essential non-singularity of non-count reference

To say that non-count definite descriptions in particular are non-singular is to say precisely that they do not denote in accordance with Russell's Theory of Descriptions (RTD) — that their denoting mechanism is not the one identified by Russell. For RTD is explicitly a theory of *singular* descriptions — of '*the* in the singular' as Russell puts it — where a singular description is one commonly having the form of 'the F' and purporting to denote a single F, or denoting at most a single F. Briefly, Russell's theory involves the claim that it's a necessary condition of a definite description's counting as singular that if the description (or sentence containing it) is to denote, the term or concept 'F' itself should apply, contextually or otherwise, *uniquely*. This seems to me to be correct; and it seems, furthermore,

[23] To say that because an essentially non-singular universally quantified sentence is plural or collective, it is not *also* distributive, would then be misleading. Intuitively, such a sentence seems best characterised as *non-individually* distributive — distributing *en masse*, or several at a time, over all and any *things* which are so-and-so, and not over every and any *thing* which is so-and-so (there being no *such* things).

[24] 'Stuff' here is a dummy term standing in for any concrete NCN.

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that the nature of NCNs is such that they are simply *incapable* of having unique application.

To illustrate the Russellian point, consider a sentence whose semantic value is, on account of ambiguity, unclear. For example,

[5a] The sheep in Russell's meadow slept

may be read as either singular or plural, but such a sentence can be disambiguated in context by its truth-conditions. Thus if 'the sheep in Russell's meadow' is singular — if, that is, it purports to denote a *single* sheep — then the sentence must be construed as

[5b] The *one* (or *single*) sheep in Russell's meadow slept,

which in turn entails

[5c] *There is* exactly one sheep in Russell's meadow.

It follows that if the description 'the sheep in Russell's meadow' purports to designate a single sheep, then the contained predicate 'sheep in Russell's meadow' itself must be supposed to be true of just one thing — that is, to apply uniquely. If on the other hand 'the sheep in Russell's meadow' is non-singular, no such implication will obtain.

And given this bonding of the singularity of a definite description with the uniqueness of application of its contained predicate, it seems plain that non-count descriptions must indeed be semantically non-singular. For if, by parity of reasoning,

[5a'] The clothing in the warehouse is made of polyester

were semantically singular, thereby denoting a single 'clothing-object', individual or thing, then it could not but mean

[5b'] The *one* (or *single*) 'clothing-object' in the warehouse is made of polyester.

And this in turn could not but entail

[5c'] *There is* exactly one 'clothing-object' in the warehouse.

But since whatever stuff is *some of* the clothing in the warehouse is *also* clothing in the warehouse, [5c'], hence [5b'], could not generally be true. The fact that 'the clothing in the warehouse' can have denotation, consistently with the contained predicate 'clothing in the warehouse' having what may be called 'multiple applicability', demonstrates that 'the clothing in the warehouse' cannot possibly mean 'the *one* ___ of clothing in the warehouse'.[25] It cannot, in short, be

[25] Helen Cartwright is acutely aware of this difficulty and it motivates her theory of 'quantities', to be examined in the sequel. Cartwright considers the identity-statement

[1] The water Heraclitus bathed in yesterday = the water Heraclitus bathed in today,

and remarks that [1] might *seem* to entail

[2]There is exactly one x such that x is some water, and Heraclitus bathed in x yesterday.

But, as she goes on to note, [1] might be true even though [2] is bound to be false. 'Even if he took just one bath yesterday, Heraclitus bathed in most of what he bathed in; he bathed in all but a quart and all but a pint; and these things are surely distinct' [481]. What [1] therefore requires, it is suggested, is not [2] but rather

[3] There is exactly one x such that x is *all* of the water Heraclitus bathed in yesterday, and exactly one y such that y is *all* of the water Heraclitus bathed in today, and x = y.

Clearly, both [2] and its 'revision' or 'improvement' [3] give direct expression to the belief that [1] is semantically singular — that the denoting phrases here purport the designation of a single object each. It is

singular. There can be no such single thing or object as the clothing in the warehouse; that clothing is *so much* stuff (and, indeed, *so many* things) but it is no *unit*, hence no constituent element in the existence of clothing. There are of course objects which are made up of individual pieces of clothing, and which have their own 'higher' principles of individuation, such as individual wardrobes, outfits, and heaps of clothing. But the only *units* over which 'clothing' *as such* ranges are the individual pieces; there are no other objects into which clothing may be 'divided'.

Given a certain range of objects, the semantics of singularity determine that there is *exactly one* way in which that range can be exhaustively configured or divided into potential recipients of such reference — namely, *at the joints*. Where a predicate 'F' is singular, the semantics of 'F' determine that there is just one fixed set of one-one correlations between singular references involving 'F' and Fs. But the semantics of 'Fs' ('items of clothing', say) impose no constraints upon the number of ways in which a range of objects may be exhaustively configured or divided — any combinations of F-objects, without restrictions upon number, are the potential correlates of a plural reference. They determine neither how many items nor what combinations of items may be referred to as 'these Fs', 'those Fs' or 'the Fs'. There are potentially indefinitely many sets of one-many correlations between plural references and things. Contextual factors, not the semantics of plurality, determine the scope and content of any such reference — one such factor being the choices made by the speaker as to the scope and combination in a given reference or set of references. The correlations between plural reference and its objects are semantically *unconstrained*, and might be described as ontologically *arbitrary*. And as with plural reference there is — depending in part on one's interests, and in part on accidental or 'external' conjunctions of units — enormous leeway as to the scope and content of such non-count references, a leeway which also deserves to be characterised as ontological arbitraryness in the style of reference itself. And the *only* difference in these respects between a collective NCN like 'clothing' and a 'pure' NCN like 'water' is that there are *no* units over which the pure NCN might range.

The clothing in the warehouse may be said to be *some* clothing, and a certain *amount* of clothing, much as the sheep in Russell's meadow may be said to be *some* sheep, and a certain *number* of sheep, just in case the relevant occurrences of 'sheep' are non-singular. But there can be no such (single) object — hence no such *category* of object — as *some clothing*, and more generally, as *some stuff*. The *number* of items of clothing in the warehouse is not determined by, and has

precisely because she makes this assumption, that Cartwright naturally also supposes that the denoting phrases must somehow, a la Russell, involve uniqueness. But since this creates an obvious *prima facie* difficulty — the problem with [2] — she is faced with the task of circumventing it; hence [3]. In fact, however, it is not difficult to see that as it stands, the proposed analysis of [1] as [3] is wholly spurious. For the definite descriptions in [1] are not, as in 'On Denoting', *unpacked* in [3], but are baldly *reproduced* behind the quantifier. Cartwright might just as well have said 'There is exactly one x such that x is *the water*....' — were it not for the fact that it would then be patently obvious that no analysis of [1] had so far been produced. In an article in which she attempts to address these and related criticisms — criticisms which first appeared in my 'Theories of Matter', *Synthese* 31 (1975) — Cartwright has nevertheless acknowledged that her thesis here is mistaken. See her 'Parts and Partitives: Notes on What Things are Made of', in *Synthese* 58 (1984), 251-277, and especially pp.265-272.

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nothing to do with, the *kind* of thing they are. Nor, likewise does the *amount* of clothing in the warehouse have anything to do with the kind of stuff it is. While to be is, among other things, to be *an* individual, it is never to be *some* individuals or *some* stuff; there are simply no such categories of things. Non-singular referring expressions such as 'the (items of) clothing here' and 'the (items of) clothing there' do not underpin or 'ground' general or quantified statements concerning (items of) clothing — they do not indicate in what the existence of (items of) clothing *consists*.

6. Non-singularity as singular

Now I have urged that non-count reference is semantically non-singular much on a par with plural reference. The position is incompatible with that of those who, like Vere Chappell, suppose that non-count reference is straightforwardly singular — that 'the gold in this ring' denotes a unitary 'parcel' of gold much as 'the cat in this bag' might denote a unitary cat.[26] On the other hand, however, there are those who, on the basis of *acknowledging* the parallels of NCNs with those non-singular nouns which are plural, claim that non-count reference is nonetheless reference to units, to certain types of individual *aggregates*, so-called 'portions', 'quantities' or 'masses', and correlatively, that non-count quantification is quantification over such aggregates.

Unlike the 'naive' view of non-count reference, for which it is straightforwardly singular, this more sophisticated doctrine of non-count aggregates or 'quantities', which is due primarily to Helen Cartwright, is based upon the real parallels between non-count and plural reference, on the one hand, along with a singular (and what can only be reductive) construal of the plural, on the other. Thus Cartwright maintains that a plural identity-statement such as

> The cats we have in Boston are the same cats as the cats we had in Detroit

is equivalent to a singular set-theoretical identity-statement such as

> The set of cats we have in Boston = the set of cats we had in Detroit.

[26] 'It has been said that a mass noun... does not 'wholly determine criteria of distinctness and identity for individual instances' or 'provide a principle for distinguishing enumerating and re-identifying particulars of a sort' (Strawson); and that, whereas a cat 'is a particular thing, the concept "gold" does not determine an individual thing in this way' (Anscombe). Such statements are true enough so long as they are taken to mean just that there is not such a thing as "one gold" or that, as Geach puts it, 'the question "how many golds?" does not make sense'; for this much is guaranteed by the grammar of 'gold' as a mass noun. But it does not follow that what 'gold' is used for or applied to... as a general term, is not *one single thing*, as individual and capable of being counted as any cat... Suppose it is true that this lump is gold... This lump may be made into a ring, and the ring then cut up into a number of bits. *There is something that survives these changes, some one thing that we can pick out and follow through them*; and though this is always gold... for it is this gold that survives, and the same gold that is first a lump, then a ring, and then a collection of bits — it is not always a lump. *We need a count noun, therefore, that will be true of this thing and remain true of it so long as it keeps its identity as this same gold'*. V. C. Chappell, 'Stuff and things', 63-4, my italics.

'Identical cats', she writes, 'are one — one *cat* or one *set* of cats'. And on this basis, Cartwright maintains that identities such as

> The gold of which my ring is made is the same gold as the gold of which Aunt Suzie's ring was made

are 'equivalent to identities like

> The quantity of gold of which my ring is made = the quantity of gold of which Aunt Suzie's ring was made'.[27]

Correlatively, of the negative existential sentence

> There isn't any water left in the tub

Cartwright remarks that we 'may set out the (apparently) equivalent

> It is not the case that there is an x such that x is some water and x is left in the tub.'

Cartwright's view is grounded, then, in a conception of plural reference whereby such reference is supposed to denote 'classes', 'sets' or 'plural objects'. The conception is not unusual, and even traditional; thus for instance Russell writes in Chapter Seventeen of his *Introduction to Mathematical Philosophy*, that in

> the present chapter we shall be concerned with *the* in the plural: the inhabitants of London, the sons of rich men, and so on. In other words, we shall be concerned with classes.[28]

In an apparently similar vein, Max Black proposes to build

> the idealised set talk of mathematicians upon the rough but serviceable uses in ordinary language of plural referring expressions... to get the abstract notion of a set as... *several things referred to at once*.

And E. J. Lowe writes 'I treat a plural noun phrase like "the planets" as denoting a set... construed... as being, quite simply, *a number of things*... '. And, Lowe continues, 'sets so conceived qualify as *objects*... the principle of

[27] 'Quantities'. The need for the comparison between the non-count and plural sentences for Cartwright is grounded in the fact already noted that, as Cartwright clearly recognises in 'Heraclitus and the bath water', a straightforwardly singular construal of the definite non-count description violates the semantic conditions for such a description as identified by Russell.

[28] *Introduction to Mathematical Philosophy*, ch. 17. In a similar vein, Max Black proposes to build the idealised set talk of mathematicians upon the rough but serviceable uses in ordinary language of plural referring expressions... to get the abstract notion of a set as... *several things referred to at once*. ('The Elusiveness of Sets', *The Review of Metaphysics* 24 (1971), 614-636.)

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extensionality provides them with determinate identity conditions'.[29] In short, there is here a genre of views which maintain that non-singular reference — reference to either 'the many' or 'the much' — is reference to some aggregated or collective *one*.[30] In effect, according to such views, non-singular reference is really (sometimes or always) singular reference writ large. I consider plural reference first.[31]

It's not difficult to understand the motivation for collective object-oriented views. Plural reference has what it seems natural to characterize as a semantically *collective* form: it involves the use of a *single* grammatical subject-expression, simple or compound, to pick out *several* objects all at once, *tous ensemble*. And in referrring to several objects all at once, such reference circumscribes certain particular objects collectively, demarcating just these objects from the rest of what there is. And if we group or single out *these* objects and distinguish them collectively from *those*, do we not *thereby* single out two unique and distinct groupings or collections?[32]

The issue is one with which Russell's engagement is in my view outstanding, if at the same time agonisingly confused; I have in mind his struggle with the notion of the so-called 'class as many' in The Principles of Mathematics. But, insofar as what is here at issue is the *identity* of 'the many' with some 'one', then the answer to the question can hardly be 'yes' — many things cannot just *be* a single thing. It is a kind of truism that there is no such object — no such *single* object — as *the* object of a plural reference, that reference in the plural is to many

[29] 'The Metaphysics of Abstract Objects', *Journal of Philosophy* 1995, Vol XCII, no. 10, 522-23.

[30] In the case of *quantified* sentences, the venerable tendency to paraphrase the plural in terms of sentences ostensibly referring to *classes* — e.g. the paraphrase of a sentence with the form of 'All cars pollute' as 'The class of cars is included in the class of polluters' — is radically undercut by the Fregean-style formalisation. Such spurious talk of classes is a feature of the (*so-called*) Boolean algebra of classes — which need involve commitment to no objects beyond the individuals of the first-order predicate calculus. See, e.g., Quine's *Methods of Logic* on Boolean algebra. The inclination to suppose that a *grammatical* subject like that of [1b], 'all cars', is also a *logical* subject or *semantic* unit, supposedly denoting some one 'collective' entity, is dispelled in the singular recasting as [1d]. Such an inclination is also starkly manifest by Russell himself in the *Principles*, when he writes 'With regard to infinite classes, say, the class of numbers, it is to be observed that the concept *all numbers*, though not itself infinitely complex, yet denotes an infinitely complex object' (72). A more perspicuous relationship is arguably then established between syntactic form and semantic content — the paraphrase in predicate calculus may plausibly be said to render explicit the logical form of [1b]. Equally, this representation may be said to constitute a transparent rendition of the truth-conditional content of the sentence: [1b] is seen to count as true, just in case any value of x which satisfies the open sentence 'x is a car' also satisfies the open sentence 'x pollutes'.

[31] At the same time, it should be said that the apparent tendency to identify the objects of a plural reference with a single collective unit need not be seen as the doctrine I here call into question; it might also be viewed, in a particular context, as either [i] a mere *facon de parler* with no ontological significance or [ii] an ill-formulated assertion of a relationship not of identity but of constitution, or [iii] a reduction or replacement of multiplicity by talk of sets.

[32] Furthermore, such reference renders possible collective *predication*, as exemplified by Russell's 'Brown and Jones are two of Miss Smith's suitors', where the predicate is not applicable individually to each of the objects thus collectively referred to. I address this and related issues at considerable length in *Words without Objects*.

things and not to one.[33] Its semantically collective form notwithstanding, plural reference cannot bring a novel category of objects into being: the very objects which are designated thus 'collectively' in the plural might equally be designated 'individually' or 'distributively' in the singular. The *categories* of objects in the singular and plural cases are the same; it is exclusively the modes of *correlation* which are different.[34] The mode of correlation for the plural is one-many; for the singular it is one-one. And in designating several distinct units all at once, plural reference is collective *only* in semantic form. Whereas the semantic form of singular reference encodes the corresponding *ontic* category or kind — objects, individuals or things are each and every one of them *a* unit, and reference in the singular is reference *to* a unit, the semantic form of plural reference embodies nothing other than the *semantic* element of collectivity. Such reference may be described as reference to *a number of units*, or equally to *some* or *several* units; and while there are of course such things as units — while there is such a category of being as *a unit* (*the unit*, *units*) — there is no thing which is (merely) *some* or *several* units, and *a fortiori*, there is no such *category* of being as '*some* units', there are simply objects which are *units*.[35]

Max Black writes that set talk is a 'verbal pattern projected on the universe, and set-boundaries are as "real" or "imaginary" as territorial boundaries'; and given that for Black, sets are the objects of plural reference, his point is in reality a point concerning plural reference. The grouping work that such non-singular reference does is based on something other than a feature of *the objects* of that reference: the power to group things referentially is commonly *facilitated* by the fact that distinct objects can *appear* in groups; but the fact remains that the plural reference does not *designate* the group.[36] Indeed, something closely analogous

[33] It is difficult in this connection to improve on Alex Oliver's luminous remarks (directed, as it happens, against the 'mereological' view of classes advanced by David Lewis). To say that the many just *are* the one, as he observes, seems

necessarily false given our ordinary understanding of identity and counting. Everything is identical to itself and to nothing else, in particular, nothing is identical to many things, each of which is different from it. If we measure commitment by the number of objects in our ontology, then a commitment to a cat-fusion is a further commitment, over and above the commitment to the cats which are its parts. If we have ten cats, then the cat-fusion which has all the cats as its parts is an eleventh object. How else could we measure commitment? 'Are Subclasses Parts of Classes?', *Analysis* 54.4 (1994), 215-223.

[34] Of course, the inhabitants of London, whilst actually *in* the city, *constitute* a kind of *physical* mass or group; but *those individuals* may be so dispersed as to cease to constitute any such physical collection.

[35] There are, plainly, such things as units *of* units — groups of units, packs of units, bunches of units, and so forth; but for these, some further *principle* of unity is called for, beyond the *mere* phenomenon of multiplicity. The status of non-singular reference in general and of plural reference in particular is examined at considerable length in my *Words without objects* (Oxford: Clarendon Press, April 2006).

[36] It might seem that there are putative counterexamples here which suggest that this remark is straightforwardly false — for example, 'The Beatles' is a plural referring expression, it may be said, which nonetheless designates a group. The short answer to this line of criticism would involve distinguishing, once again, between syntactic and semantic matters, and noting that while 'The Beatles' is syntactically plural, nonetheless, insofar as it *does* designate a group, it is semantically singular ('The Beatles are one of his favourite groups'); and we may truly say that the Beatles, while nominally at any rate the same group, have not always been the same *individuals*. But again, the expression may sometimes also be used in a geninely plural manner, to designate the *members* of that group, as in such sentences as 'The Beatles are jogging'

might be said about the grouping work that is done by such straightforwardly material 'aggregators' or 'collectors' as sacks and bags and bottles. In any familiar sort of collection or collective object — a sack of potatoes, pile of pheasants, cup of sugar, etc. etc. — the principle of collection itself ('sack', 'pile', 'cup') enters into the concept of the corresponding object. The principle of collection, or intuitively, the 'form', has to be internal to the collection itself. On this basis, the objects of any plural or collective reference arguably *do* constitute a collection of sorts — precisely *in conjunction with* the act of plural reference, or with the grammatical subject which 'unites' or 'collects' them, such that by analogy, it too might be characterized as the 'form', and they as the 'matter' or the 'content' of the collection. We may say then that there is after all a unity involved in reference to 'the many', but it is nothing more (though also nothing less) than that — a unity involved in *reference*.

Russell himself writes that 'The collection is defined by the actual *mention* of the terms'.[37] A fact about grouping work — in the former case a fact about the power of referential symbolism — is not at all the same thing as a fact about the objects grouped (in this case, a fact about the non-symbolic world).[38] In just this sense, its form represents no first-order ontic class or category, but instead the imposition of a Black-style grid upon the world.

Elementary though it is, the contrast in the semantic forms of singular and plural — the contrast e.g. between the meaning of the singular and non-singular indefinite articles, as in '*a* car' versus '*some* cars' — is then of signal metaphysical importance.[39] Furthermore, and given the parallels of non-count and plural reference on which the theory of 'quantities' is based, a rejection of the singular construal of the plural undermines the theoretical basis for the suggested singular construal of the non-count case. Like plural reference, non-count reference is not just non-singular but is absolutely so. While some stuff may be supposed to *constitute* all sorts of things, 'quantity-theoretical' aggregates no less than regular physical aggregates, it is incorrect to think that it just *is* a single stuffy-object.

('fighting', etc. etc.). And it may be that a sentence such as 'We are the Beatles' involves both plural and non-plural elements.

[37] *The Principles of Mathematics*, 69, italics mine.

[38] This point is tantamount to the contrast between the concept of a *substituend* and that of a *value* for a variable.

[39] It would be possible to think of the objects of a plural reference, *as such*, as constituting a collection, only if the plural referring expression itself were thought of as the linguistic equivalent of a bag, box or other collecting device or container — in effect, as the *form* of the collection, serving as an 'external' unifying agent for its 'contents'. Yet here too, the 'bag' would have to be *distinguished* from its contents: what could be said to be *in* such a linguistic 'container' — the objective contents of the term — would be *simply* a number of individual objects (several objects, some objects, etc.); and what they would count as the 'contents' *of* would be a linguistic object of a certain kind. It is a matter of common sense that the plural 'principle of collectivity' does not exist *in rebus*, but is rather to be found in the semantics of the symbolism itself.

7. Non-singular variables

A kindred scepticism concerning 'plural objects' in particular is also voiced by George Boolos, for whom, though plural reference should be understood *as plural*, it has no distinctive ontological significance.[40] The world does not, he insists, contain both singular and plural things:

> It is not as though there were two sorts of things in the world, individuals and collections... There are, rather, two different ways of referring to the same things... neither the use of plurals nor the employment of second order logic commits us to the existence of extra items beyond those to which we are already committed.

Boolos disarmingly remarks that it is

> haywire to think that when you have some Cheerios, you are eating a *set* — what you're doing is eating THE CHEERIOS... it doesn't follow just from the fact that there are some Cheerios in the bowl that, as some who theorise about the semantics of plurals would have it, there is also a set of them all.[41]

Perhaps most notably, Boolos explores the issue of the (non-reductive) formal representation of plural sentences and inferences. His strategy is motivated in large part by cases such as that of the so-called Geach-Kaplan sentence 'Some critics admire only one another', proved by David Kaplan to be unformalisable in standard first-order predicate calculus (i.e. without the addition of the symbolism of set theory), and supposed on that account to require the introduction of such symbolism. However this sentence, Boolos suggests, may be represented without the use of such symbolism, but instead using plural variables, as

$$[\exists X][\exists x][Xx \,\&\, (x)(y)(Xx \,\&\, Axy \Rightarrow Xy \,\&\, x \neq y)].$$

The domain of discourse is here stipulated as consisting of the critics; the upper-case 'X' is a second-order plural variable ranging over individuals several at a time; and the expression 'Xx' is to be read as 'x is one of X'. This then gives as a reading 'There are some critics, each of whom admires someone, only if that person is one of them, and none of whom admires himself.'

[40] In this respect Boolos differs from e.g. Barry Schein, who rejects plural objects, but insists also upon singular reduction.

[41] 'To be is to be the value of a variable (or to be some values of some variables) *Journal of Philosophy* (1984), 448. It is indeed 'haywire' to think that when you have some Cheerios, you are eating a set, but this fact is entirely compatible with the supervenience of sets on the semantics of plurals. All that is required is that plural expressions *as such* should not denote such things.

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At the core of Boolos' work is the development of a formal representation for irreducibly plural sentences intended, among other things, to reflect his common sense 'ontology of Cheerios', a representation without recourse to the apparatus of set theory. Central to his approach, obviously, is the introduction of plural variables; and in addition to such relatively complex sentences involving cross reference, there are many relatively straightforward plural sentences which may be handled with the use of plural variables, being irreducible to singular form on account of their possession of collective predicates; one such example Boolos cites is 'The rocks rained down'.[42] Boolos' strategy is to develop a novel *symbolism* for the representation of plural sentences and inferences without a corresponding novel category of *objects* such as that of sets — a distinctive logic and semantics without a correspondingly distinct ontology. His plural variables are intended as a special *notational* device, which (he rightly insists) are to be construed as lacking special *ontological* significance. The collectivity of plural reference, hence of plural variables, is 'merely semantic' or non-ontological. Since, as Boolos in effect observes, there is no such thing as *the* (one, single) object of a plural reference, there is no such thing as *the* (one, single) value of a plural variable; such a variable has *some* values, *several* values, not just one.

But there is no need to confine a Boolos-type strategy to the hard or semantically irreducible cases exclusively. Thus the sentence

[1d] All clothes are made of polyester.

could be represented semi-formally and simply as

[1d'] For all / any objects ζ, if ζ are clothes, then ζ are made of polyester

— where ζ is a plural variable, and the substituends for the variable are themselves plural referring expressions ('these objects', 'those objects', 'the objects on the first trans-Atlantic passenger flight', and so on). Or again, purely symbolically, we may write

$$(\zeta)(C\ \zeta \Rightarrow P\zeta),$$

where (ζ) is the matching non-singular universal quantifier. And parallel with such a plural logic, a non-count logic is surely possible. The sentence

[1e] All clothing is made of polyester

might be recast, somewhat in the manner of [1d], as

[1e'] For all / any stuff μ, if μ is clothing, then μ is made of polyester,

or again, as

[1e''] $(\mu)(C\ \mu \Rightarrow P\ \mu)$.

[42] Of such a sentence, Boolos plausibly remarks that 'it would appear hopeless to try to say anything more about the meaning of a sentence of the form "The Ks M" other than that it means that there are some things such that they are the Ks and they M.'Reading the Begriffsschrift', repr. in *Logic, Logic and Logic*, 168.

Akin to the semantically plural ζ,μ is a non-count variable; the expression '(μ)' is to represent a non-count universal quantifier corresponding to 'For all / any stuff μ'. Expressions such as 'that clothing', 'the clothing in the warehouse', 'the clothing on the aircraft', etc., can then be treated as substituends for μ.

But given the non-singularity of non-count reference, there can be no such object as *the value* of a non-count variable. Much as a plural variable has *some values* (e.g. some clothes) and not one value, a non-count variable may be said to have **some value** (e.g. some clothing) and not one value. There is then a range of distinct substituends in cases of this kind; but there is no corresponding range of discrete values. The substituends range arbitarily over the scattered clothing-stuff. As with Boolosian variables, there is here a distinctive category of variable which is non-ontological — which corresponds to no distinctive category of *object*, and takes only a certain arbitrarily chosen amount of stuff (*some* stuff) as 'value'. The clothing in the warehouse will be **some value** of the variable; and *whatever* is clothing in the warehouse — the clothing in this corner of the warehouse, for instance — is *some of* the clothing in the warehouse, i.e. **some of the value** of the variable, and also **some value** of the variable. But this, given the nice neat role that variables have been traditionally assigned, strikes me as representing a complete breakdown in the role of the variable as construed 'objectually'. To say that a variable has not *a* value but (in the non-singular, non-count sense) **some value**, seems a travesty of the whole conception of an objectual, variable-based logic.

Quine asserts, famously, that to be is to be the value of a variable. And here 'the value of a variable' is a singular expression — Quinean variables are semantically singular; the value of a Quinean variable can only be a single object, of whatever sort. Insofar then as it is necessary or desirable to formally represent irreducibly non-singular sentences — a question which may itself occasion controvery — Quine's assertion cannot be accepted. It cannot be accepted in the context of irreducibly plural sentences, not because it gets the *ontic* categories wrong, but because it is *semantically* inadequate.[43] And it is semantically inadequate simply because there are possibly true statements about objects which cannot be represented with the use of singular variables exclusively.[44] However the semantic problem immediately becomes an ontological problem, when our focus shifts from irreducibly non-singular reference which is plural to irreducibly non-singular reference which is non-count. The *semantic* problem is that some reference is irreducibly *non-singular*; the *ontological* problem, on the other hand, is that some reference is also *non-plural*. And the only difference in this regard between collective NCNs like 'clothing' and 'pure' NCNs like 'water' is that the

[43] At any rate, it is not as if there is an ontic difference between singular and plural with regards to *substance*; at most, it is a difference over the importance of relations, over for instance, *pace* Boolos, what it is for rocks to be able to *rain down*, where this represents a 'collective action' of the rocks, or requires certain spatio-temporal relationships between a number of rocks.

[44] A Quinean manoeuver in such contexts might well be to invoke 'novel units' in the form of sets to play the role of values; but of course to exercise such an option is to embrace an anti-realist or pragmatic reductionism (which, indeed, was always part of Quine's well-tended 'desert landscape'); and this, for the realist, is to simply abandon the metaphysical quest.

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latter specify *no* units over which the pure NCNs range. In this sense, while there are lakes and clouds and rivers and so forth, there are *in* these lakes and clouds and rivers no *water-units*, no individuals or objects which are water. There is, quite simply, *water*.

8. The essential singularity of concept-script

But having acknowledged difficulties over irreducibly non-singular statements, there is a question as to whether these are really difficulties for a *Quinean* canonical notation, or whether they are difficulties for *any* well-constructed concept-script. We must consider in more detail what a concept-script requires. There are three chief factors in the notion of a concept-script; I sketch these out as follows: [i] First and foremost, the *semantics* of a sentence or a term in concept-script must be explicitly encoded in its *syntax* — syntax must directly encode meaning or semantic value. To take a very simple case, the syntactic form of 'The sheep slept' is conceptually non-ideal or defective — is sleep attributed to only one or to at least one sheep? [ii] Furthermore, to the extent that it has ontic significance, the *semantics* of the concept-script, in turn, must themselves directly codify the *ontic* categories involved. To understand the variables of the first-order predicate calculus, for instance, is to know that these take *individuals* exclusively as values. Whatever the metaphysical facts of the matter, the existence of a category of *predicables* is not acknowledged in this calculus; predicate letters do not here count as referential terms. Whether the calculus is indeed adequate, *qua* concept-script, depends of course on whether such a category is actually implicit in natural-language constructions, *independently* of the formal system, or not. [iii] Thus, a third constituent in the notion of concept-script is the requirement that the formal reconstruction must reproduce or replicate (and not reduce, replace or 'explicate') the ontic categories of the natural-language fragment at issue.[45] At any rate, given a realistic view of categories, such as I myself embrace, clarification is one thing and explication quite another; in this regard I must dissent from Quine.

To the extent that it has ontic significance, then, the semantics of concept-script must directly codify the ontic categories which it involves. And in this regard, a plural referential symbolism is ruled out: '*some* ___s' is *not* an ontic category, as Boolos himself makes crystal clear. The syntactico-semantic character and the ontology of plural reference are just incongruent; such reference is intrinsically non-ideal. There is a simple disconnect between semantics and ontology in one-many correlations. The semantic *form* of plural reference does not embody or reflect its ontic *content*; hence the symbolism for such reference *cannot* 'show at a glance the logical structure of the facts asserted or denied' and, to put it bluntly, invites the one who reflects on it to see its collective form as the form *of a collection*. There is an understandable and entirely sponteneous tendency to regard

[45] At the same time, there is in my view no guarantee that all natural-language sentences are *capable* of being represented in concept-script; a sentence might be intrinsically and irredeemably 'unclear'.

such nonsingular terms as designating distinct objects, a tendency for the reflective thinker to interpret such reference as designating a collection — a tendency the existence of which I take to 'empirically' confirm the ideal language thesis. It is for this reason that there is an atmosphere of uncertainty surrounding the significance of plural reference; it is hardly surprising that it is a matter of contention. Its collective form gets in the way, or intervenes. In not directly encoding the category of separate discrete units, but instead expressing collectivity, the semantic form of plural reference is non-ideal or defective. No such difficulties surround the understanding of reference which is singular; no parallel disagreement is widespread concerning the significance of singular referring expressions; hence, perhaps, their attraction to reflective thought..

Frege writes that clarity demands simply 'the closest possible agreement between the relations of the signs and the relations of the things themselves'.[46] According to J. Alberto Coffa, the Fregean project involves 'identifying a fragment of the German language' — that which constitutes the natural-linguistic basis for Frege's concept-script — such that 'the grammatical form of every sentence in this fragment *mirrors isomorphically* the constituents of the content it expresses, as well as their arrangement in that content'.[47] And, in a strikingly similar fashion, Russell writes that in

> a logically correct symbolism there will always be a certain fundamental *identity of structure* between a fact and the symbol for it... In a logically perfect language the words in a proposition would correspond *one by one* with the components of the corresponding fact, with the exception of such words as 'or', 'not', 'if', 'then', which have a different function.[48]

And I have in effect argued here that Frege and Russell are exactly right: *qua non*-ideal, the character of plural and more generally non-singular reference strikingly exemplifies and validates this principle. An ideal referential symbolism can countenance only *one-one correlations* between the signifier and the signified; only here is there a structural isomorphism of syntax and being — a single symbol for a single individual or thing. Singular reference, with its one-one correlation between term and object, is rightly taken to be relatively well understood. And Quine, in particular, seems right in insisting than a canonical notation can have room only for referential terms, variables included, which are singular. On the other hand, in what is tantamount to simply *reproducing* the natural-language plural form within his formalism, Boolos has reproduced just those features of natural-language plural reference on the basis of which one of the original conceptions of a class or set arose. In short, so far as Boolos' formalism is

[46] Quoted in Coffa, *op. cit*, p. 12.

[47] Coffa, *Op. Cit.*, 66, my italics. Fregean content is of course *thought*-content, a matter of sense and not of reference. But the point is much the same: a clear connection between word and thought is one in which, for instance, a single referring expression is correlated with the *thought* of a single object.

[48] *The Philosophy of Logical Atomism*, 198, my italics.

concerned, it remains open to one of the 'collective entity' persuasion to continue to insist that a number of objects — with which a plural variable is indisputably correlated — just *are* a 'single many' (and as a matter of fact, this is *exactly* the response to Boolos which Cartwright in particular has made). [49]

Now the formal quest to clarify the nature of the 'facts asserted or denied' involves, of course, a quest to clarify what there is *not* — no shady present kings of France, no golden mountains and so forth. A clear expression of the logical might then involve the isolation of what Gilbert Ryle once called 'systematically misleading expressions' — constructions which are alike in 'misleading in a certain direction' to the philosophical or reflective consciousness — expressions which may 'suggest the existence of new sorts of objects', expressions which 'are all temptations to ... "multiply entities"...'.[50] By recasting natural-language sentences containing such expressions into ontically explicit form, their baneful influence is overcome. The application of a logico-semantical analysis to sentences or terms which might appear to posit 'novel categories of objects' aims at the liberation of reflective thought from reifying tendencies to which it is, notoriously, prone. And just such reifying tendencies assert themselves, so I have urged, when we are confronted by reference which is semantically non-singular.

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[49] See especially her 'On Plural Reference and Elementary Set Theory', *Synthese* 96 (1993). 201-254.

[50] 'Systematically Misleading Expressions', *Proceedings of the Aristotelian Society* 32 (1932). Ryle writes that 'People... use expressions which disguise instead of exhibiting the forms of the facts recorded'. And in overlooking or failing to recognise this gap, serious philosophical confusions, mistakes, etc., can occur. Quine subsequently speaks of the (reflective) tendency to be 'carried away by the object-directed pattern of our thinking to the point of seeking the gist of every sentence in things it is about' (*Word and Object*, 239). Meinong is commonly cited as perhaps the most dramatic example of such a tendency; but less extravagant or more modest cases are not hard to find. Again, Wittgenstein observes that most philosophical 'questions and propositions' result from the fact that 'we' (that is, of course, *philosophers*, semanticists, etc.)

> do not understand the logic of our language... It is a merit of Russell's work to have shown that the apparent logical form of the proposition need not be its real form (*Tractatus*, 4.002-4.0031).

Following in Russell's footsteps, Quine speaks of logical theory as advancing our understanding of 'the referential work of language and clarifying our conceptual scheme' (158), a project that sometimes involves what, following Carnap, he calls 'explication'. 'We have', he writes,

> an expression or form of expression that is somehow troublesome. It behaves partly like a term but not enough so.... or encourages one or another confusion.... In the case of singular descriptions.... Russell dissolves [the problems] by showing how we can dispense with singular descriptions.... (*Word and Object*, 260-1).

A vital dimension of 'casting a statement into logical form', then, consists not *simply* in suiting it to formal manipulation, but also and precisely in undermining the all too frequent philosophical tendency towards what may be called the 'hypostatisation of non-existent entities', by way of rewriting its syntax so as to transparently reflect its true significance. The 'failure to give clear expression to the logical' may be thought of as one side of a coin — the underside, as it were — the upper side of which is precisely our proneness to misconceptions as to the significance of what we say; and it may seem tempting to think of this 'failure' as corresponding precisely to a gap between grammatical and logical forms.

There is not typically any suggestion, in this sort of outlook, that natural language are in *any* way misleading or defective for purposes which are *other than* philosophical. As Ryle himself stresses, this susceptibility to being misled is a purely reflective or philosophical hazard. The natural-language user, he writes, 'does not pretend to himself or anyone else that when he makes statements containing such expressions as "the meaning of x" he is referring to a queer new object; it does not cross his mind that his phrase might be misconstrued as a referentially used descriptive phrase'. Natural language, in short, creates no problems for its immediate user, and the grammar / logic gap is there for a variety of reasons.

Conceiving of the the realm of the concrete as isomorphic with the discrete character of reference is eminently suited to the case of Newtonian bodies — discrete, 'point-like' substances, things which are *essentially* Aristotelian 'this-somethings'. Simple reference involves talk of this and that, and Aristotle's basic category is just *a* 'this' or 'that': substance by its very nature lends itself to being pointed out, distinguished and identified. The conception is tailor-made for boulders, horses, rabbits, snowflakes, planets and the like — things which can be counted and identified (and counted, of course, *one by one*). But while what Aristotle's 'horse' or 'man' is true of is by nature a 'this-something' — remaining identifiable *qua this* so long as it endures — what 'air' or 'water' is true of is not. The application of demonstratives to stuff, if intended as picking out determinate and self-identical amounts of stuff, depends upon the stuff being *absolutely* 'fixed' or 'bottled' in discrete aggregates or bodies, or occupying physically demarcated volumes of space, a status it may and does lose without thereby ceasing to be. To the extent that it exists, the supposed isomorphism of reference with the realm of the concrete would seem to be a misconceived extension of the Aristotelian principle. And rather than attempting to contrive an account of this domain which fits into a pre-conceived and neat yet supposedly comprehensive notion of the mechanics of word-to-world relationships, our conception of those mechanics needs to be adapted to the shape of the particular domain of reality with which they are engaged.[51]

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[51] Three publications overlap the present article, all stemming from work in progress on a single major project; they are listed below, in order of appearance.
'Words without objects', Principia, vol. 2 no. 2 (1998), 147-182.
'Mass nouns, Count nouns and Non-count nouns: Philosophical aspects', in Encyclopedia of Language and Linguistics, ed. Keith Brown (Oxford: Elsevier, 2005).
Words without Objects (Oxford: Clarendon Press, April 2006).

Primary Relations and a Non-Standard Form of Identity

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Abstract. This paper is dedicated to a new concept of "primary relation" which permits the introduction of a dynamic, non-standard form of identity, with applications in mathematics. The idea to base the dynamical system only on one kind of primitive notion - relation, helps to bring about a shift of perspective: to start from unity and then from within unity to pass to distinction. To express some of our terms we apply the relation of the part to the whole (in the mereological sense) introduced by S. Lesniewski (1992, Nijhoff International Philosophy series 44). It is hoped that the concept of "primary relation" will find application in various fields where self-referential structures are implied.

1. Introduction

One can note that many natural phenomena exhibit a complex, nonlinear character. Nobody is surprised to hear about the fractal nature of different phenomena, the name introduced by the famous mathematician B.Mandelbrot, who claimed that everything was fractal (lat. fractus = broken) [12]. Seeing the progress in science, one can feel a growing need for elaborating new instruments to express its interrelatedness and self-referential dynamical nature. We can observe an increasing interest in interdisciplinary research in this sense. Cellucci [3], a philosopher, says that the main task of science today is to elaborate e new form of logic, a logic dealing with a body of knowledge in continual expansion. He adds that the new logic should take into account open systems, systems which can be represented by many systems with fixed rules and able to communicate with one another. But to achieve this aim it is necessary to develop a new kind of logic capable of meeting these requirements. The author goes so far as to affirm that such a step forward would imply a change of paradigm.

At the begining of the 20th century, when B. Russell [16] began the discussion on antinomies, Stanislaw Lesniewski [11] (in 1916) as an answer to it, proposed a universal language to meet the problem of describing self-referential phenomena. He found a way to omit antinomies. His system is based on the key idea of the relation of "being a part" and gives rise to a logical system which is open and in continual expansion and which does not lead to contradictions [19, 18].

Bearing this in mind, the work of E. De Giorgi, M. Forti and G. Lenzi on "Axiomatic Systems for 2000..." [9, 7] is also of great interest. The authors propose "a new, non reductive, self-descriptive, open-ended axiomatic framework" which it is hoped will be applicable not only to the foundations of mathematics, logic and computer sciences, but which can serve also as a common language for different fields of natural and human science [8]. But neither De Giorgi's nor Lesniewski's system provides a clear method for describing the pattern of "dynamic oneness" which seems to emerge from self-referential phenomena. In a paper dedicated to primary relations in a new foundational axiomatic framework [13] we proposed a system based on the axiomatic language of [9] and [7] and incorporating the mereological intuition of Lesniewski [11] but going beyond this in an attempt to add a kind of "dynamic identity".

In the present paper, in Section 2, we will give a brief presentation of Lesniewski's concept of a class and of being a member of a class.

In Section 3 we define a new type of relation ("primary relation") which permits us to shift our perspective from the two primitive notions of quality and relation to a single primitive notion: relation and this shift permits the introduction of a dynamic non-standard form of identity for whose definition Lesniewski's interpretation of inclusion is used.

Section 4 presents a practical application of this extended axiomatic framework and Section 5 - some conclusions.

2. Mereology

Mereology is an axiomatic logical system (one could say it is a form of set theory) first formulized by Stanislaw Lesniewski at the beginning of 1911 and then reformulized in the period 1916 -1927. This system is collective in the sense that a set is a whole (a collective aggregate or class) composed of "parts" and the fundamental relation is that of being a "part" of the whole, an element of a class. Thus being an element of a class is equivalent to being a subset (proper or improper) of a class. In this sense it is clear that every class is an element of itself [11].

Initially Lesniewski did not distinguish between "part" and "element" but in 1916 he specified his terminology in a more restrictive sense. Being a "part" implied being a proper subset whereas being an " element" - or equivalently an "ingredient" - included the meaning of proper or improper subset. Let us consider the definition of class proposed by Lesniewski:

Definition 2.1 *P is an ingredient of object Q if and only if P is the same object as Q or is a part of object Q.*

Definition 2.2 *P is the class of objects a, if and only if the following conditions are fulfilled:*

(a) P is an object;

(b) every a is an ingredient of object P;

(c) for any Q, if Q is an ingredient of object P, then some ingredient of object Q is an ingredient of some a (cf. [11]).

Examples:
Segment *AB* of Figure 1 is the class of parts of segment *AB*, because the conditions of Definition 2.2 are fulfilled.

Segment *AB* of Figure 1 is not the class of parts of segment *AD*, because although conditions (a) and (b) are fulfilled, condition (c) is not fulfilled (segment *EB* is an ingredient of segment *AB*, while not (some ingredient of segment *EB* is an ingredient of some part of segment *AD*)).

Segment *AC* of Figure 1 is not the class of ingredients of segment *AB*, because conditions (a) and (c) are fulfilled, but condition (b) is not fulfilled. (Segment *AB* is an ingredient of segment *AB*, but not an ingredient of segment *AC*).

One can notice the difference between mereology and Cantor's original set theory, which is said to be distributive, ie. the starting points are elements which are then considered together as a whole [2]. However Lesniewski's theory maintains the basic intuitive notion of Cantor of a set as "a many which can be considered as a one" [2] and represents a major shift of perspective in that the "part" is defined in terms of the whole and not the whole in terms of the "part".

Figure 1

Thus *x* is defined as being an element of the class *y* if there is a *z* such that *y* is the class of objects *z* and *x* is a *z*.

As a consequence, in Lesniewski's mereology all objects are classes and all classes are elements of themselves (as opposed to Russell's concept that a set which contains itself is "improper", ie. an exception). In this sense Lesniewski saw mereology as a way to avoid the antinomies arising from Cantor's theory.

Both in the collective sense as well as in the distributive sense an aggregate (or set) is formed of objects considered together as a whole. In a collective sense however, by definition, an aggregate has meaning only if its objects really exist, while in a distributive sense this is not the case. For example the set of all months

with 32 days has meaning for Cantor, but is meaningless for Lesniewski. As a logical consequence of collective set theory no class is empty.

3. Relations and Qualities

In the introduction we have said that we would like to base our system on a single primitive notion: relation, thus we will treat qualities as articulations of relations. To do this, first we need to clarify what we mean by quality and relation and then to express quality in terms of relation. Thus, we begin with some primitive notions of quality and relation based on the work of De Giorgi et al. [9] and [7].

1. We will say that *q* is a quality and *qx* means that the object *x* has the quality *q*.

2. The object *r* is a binary relation (*relb r*).

3. Given two objects *x,y* of any nature and a binary relation *r* we will write *r x,y* to say that "*x* and *y* are in the relation *r*". At times instead of saying the *x* and *y* are in the relation *r* we will say that *x* is in the relation *r* with *y*.

   Analogously:

4. The object *s* is a ternary relation (*relt s*).

5. The objects *x,y,z* are in a ternary relation *s* (*s x,y,z*), etc.

Following de Giorgi, we introduce fundamental relations: *Rqual, Rrelb, Rid* which describe the behavior of qualities, binary relations and the identity relation:

Axiom 3.1 *Rqual is a binary relation.*

1. if Rqual x,y then Qqual x

2. if Qqual q then Rqual q,x≡ q x

Axiom 3.2 *Rrelb is a ternary relation.*

1. if Rrelb x,y,z then Qrelb x

2. if Qrelb r then Rrelb r,x,y≡ r x,y

Axiom 3.3 *Rid is a binary relation such that:*
Rid x,y holds if and only if x and y are the same object.

Let us observe that *Rrelb* is defined as *ternary* relation which describes the behavior of *relb* (cf. Axioms 3.2 above). On the other hand both *Rqual* and *Rid* are defined as *binary relations* (Axioms 3.1, 3.3). Continuing in this scheme it would

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seem natural to consider *qual* and *id* as objects of a type we called *primary relations-relp* [13] which we will define further on (see Definition 3.8)

We can note that, in general, $R*$ is used for relations which qualify or describe the behavior of $*$ and that, if $R*$ is an *n-ary* relation, $*$ is an *(n-1)-ary* relation.

We will take as given the concept of one-many and one-one relations to be understood in the usual sense [9]:

Axiom 3.4 *Qrun is a quality describing one-many relations.*

1. *if Qrun x then Qrel x*
2. *if Qrelb r and Qrun r and r x,y and r x,z then y=z.*

Axiom 3.5 *Qrbiun is a quality describing one-one relations.*

1. *if Qrbiun x then Qrelb x and Qrun x*
2. *if Qrbiun r and r y,x and r z,x then y=z.*

We will also need a concept of operation (and operation of inversion). An operation is considered as an object which acts (operates) on objects and produces a result (cf. [7] Axiom 2.3 and 2.7).

Axiom 3.6 *Qop is a quality and Rop is a ternary relation such that:*

1. *if Rop x,y,z then Qop x*
2. *if Rop f,x,y and Rop f,x,z then Rid y,z*

(we will write if f(x)=y and f(x)=z then y=z).

Axiom 3.7 *Inv is an unary operation and f is an unary operation, too. If Qop f then inv f exists and is a simple unary operation such that:*

1. *if (inv f)(x)=y then f(y)=x*
2. *for every x there exists y such that:*
   (inv f)(x)=y if and only if there exists y such that: f(y)=x

Intuitively, we could say that *op* is a special kind of binary relation which "transforms" *x* into *y*.

Now we are ready to define the notion of primary relation - *relp*.

Definition 3.8 *A primary relation relp is any " relation" such that Rrelp is a binary relation.*

One can observe that primary relations underlie those binary relations in which at least one of the objects is defined in terms of the other. In view of Axiom 3.1, the primitive notion of quality can be considered, if we wish, as a primary relation. In fact *Rqual* is a binary relation. The nuance of shift of perspective lies in the fact that in the original perspective *Rqual x,y* evokes the idea of a quality existing independently from an object possessing it and of a relation "joining" the two, whereas in the perspective of primary relation, quality is viewed as a relational entity, in as far as it can be conceived of only "in" some object. It can be noted that this perspective is in accord with Aristotle's fundamental analysis of qualities as predicables.

Once having established this basic shift of perspective, we will continue using the notation used by [9, 7]: *qx* to signify that "*x* has the quality *q*". *Qqual x* to signify that *x* is a quality, ie. a primary relation. *Qrel x* to signify that *x* is a relation, etc.

In our new scheme the fundamental relations are: *Rrelp, Rrelb, Rrelt*, etc. *Rqual* and *Rid* are variations of *Rrelp*.

In the same way *id* can be considered as a primary relation. Here we would like to add a comment. In Axiom 3.3 it is not clear, if *Rid* itself is intended to be the identity relation or whether, as in the case of *Rrelb* etc., *Rid* describes the way in which the identity relation *id* operates. In the former case how can *Rid* be "binary", when for every expression *Rid x, y*; *x* and *y* are "the same object"? Or are *x* and *y* to be considered at the same time as "the same object" and "not the same object"?

The second case - with the introduction of the concept of primary relation; i.e. considering *id* as the identity relation and *Rid* the binary relation describing the way in which *id* operates - seems preferable. Perhaps from another perspective we could view a primary relation such as *id* as a relation which "mediates itself". Considering *id* as a primary relation, a more precise definition of *Rid* would become Axiom 3.9.

Axiom 3.9 *id is a primary relation. Rid is a one-one binary relation for which:*

1. *Rid x, x means that x has the quality of being x, and*

2. *Rid x,y iff x and y are the same object*

From (1) we can see that *Rid* is defined in terms of a quality (and indeed the concept of "identity" could also be defined equivalently as a kind of "singular quality", i.e. "the quality of being this and not that"). However, in mathematics, identity is normally defined in terms of relation. In (2) we can note the subtle shift of meaning, respect to Axiom 3.3, since there is now a nuance of difference between the *x* on the left side of the proposition (intended as a primary relation: the identity relation or the quality of being *x*) and the *x* on the right (the object *x*). This permits *Rid* to be considered as binary.

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4. Non-standard identity

Continuing the previous section, we will define now a new primary relation - *tr* - to represent a new kind of "non-static identity". It is our impression that in configurations of self-referential phenomena a pattern of "non-static" identity is suggested, which we attempt to describe in Axioms 4.1 - 4.2 with the introduction of the primary relation *tr*.

Axiom 4.1 *tr is a primary relation.*
Rtr is a one-one *binary relation such that:*
Rtr x,y implies that x is not y (x and y are NOT the same object).

This axiom is not incompatible with Axiom 3.9.

One can note that *Rqual* is *binary* and may be *multivalued*. Thus *Rqual x,y* and *Rqual x,z* does not imply that *y* and *z* are the same object; *Rid* is a *binary*, *one-one* relation in which *Rid x,y* iff *x* and *y* are the same object; *Rtr* is a *binary*, *one-one* relation in which - for *Rtr x,y* - *x* and *y* are *NOT* the same object.

We will now introduce three variations of *tr*: *tr*1, *tr*2, *tr*3 with the respective binary relations *Rtr*1, *Rtr*2, *Rtr*3 which describe a dynamic behavior which we call the "one-threeness pattern".

tr is an object - a primary relation - which is described in terms of *Rtr*, that is, in terms of a binary relation of which it is itself one of the terms related. The same can be said analogously for *tr*1, *tr*2, *tr*3.

Axiom 4.2 *Three subvariations of tr: tr*1, *tr*2, *tr*3 *are described by three binary relations Rtr*1, *Rtr*2, *Rtr*3 *or, alternatively, by Rtr*3 *(see Axiom 4.3) and by two unary operations (Optr*1, *Optr*2*) which act in the following way:*
*If Rtr*1 *x,y then there exists Optr1 such that:*

1. *Optr1(x)=y_x where y_x means "y with x in y", which is to be interpreted in the mereological sense of x being an ingredient of y (cf. Section 2), and*

2. *inv Optr1(y_x)=x*

*If Rtr*2 *y,x then there exists Optr*2 *such that:*

*Optr*2*(y_x)=x_{y_x}*

The operation *inv Optr*1 is similar to *Optr*2, in that both have the same domain y_x, but the image of *inv Optr*1 is *x* alone (*inv Optr*1 represents "being generated"), whereas the image of *Optr*2 is x_{y_x} (the "generator" with what has been generated by it in it; it represents that which has been generated "returning" to the generator). Clearly this is the start of an infinite reflexive relation.

The primary relation *tr*3 is defined as follows:

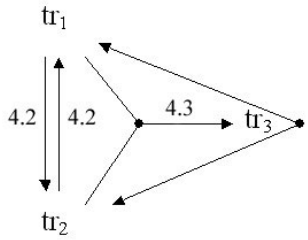

Figure 2: A static representation of primary relation *tr*. The two points indicate the simultaneous implication described in Axiom 4.3.

Axiom 4.3 *tr*3 *is the primary relation such that:*

*1. tr*3 *iff tr*1 *and tr*2

*2. Rtr*3 *z,tr*1 *iff Rtr*3 *z,tr*2

It can be seen that *tr*3 does not distinguish between *tr*1 and *tr*2 although, by Axiom 4.2 *tr*1 and *tr*2 are not the same object. Moreover *tr*3 is not the same object as *tr*1 or *tr*2. We can consider the relationship of *tr*3 with respect to *tr*1 and *tr*2 as reminiscent of the mathematical relation " between" ([13]).

In conclusion one can observe:

Proposition 4.4 *tr*1 *iff tr*2 *iff tr*3.

From this it can be seen why we have called this model the *one-threeness* model.

5. Application

In the introduction we said that B. Mandelbrot stated that everything was fractal. In his opinion figures like rectangles, circles and triangles were artificially invented by people to simplify the description of the surrounding world. He justified his opinion by the fact that these figures have no corresponding thing in the real world. If we look at trees, clouds, mountains, one can think that Mandelbrot was right [12]. Before Mandelbrot, no homogenous theory existed capable of describing these phenomena. With the concept of fractals, science made a step forward.

Fractals are objects which are the same in every piece and on every scale and there exists a very simple mathematical method to describe them. The most important thing is that one can construct fractals repeating the same *affine transformation* [5] on the object we take in the first step of its construction. This method is called – *iteration* [6]. One can say, in a very general way that, an *affine transformation* is one which does not conserve distances and angles.

Now, we will attempt to describe the formation of the Cantor set - the simplest fractal - in terms of the primary relations: *tr*1, *tr*2, *tr*3, and more precisely, in terms of this iterated application of *Optr*1 and *Optr*2.

Let the behavior of *tr*1 for the Cantor set be described by *Optr*11 and *Optr*12 (later we will indicate this double operation simply by *Optr*1=(*Optr*11,*Optr*12)) where the action of *Optr*11 and *Optr*12 is determined by the classical rule:

for every x which is a point element of [0,1], $Optr11(x) = 1/3\ x$
for every y which is a point element of [0,1/3], $Inv\ Optr11(y) = 3y$
for every x which is a point element of [0,1], $Optr12(x) = 1 - 1/3\ x$
for every y which is a point element of [2/3,1], $Inv\ Optr12(y) = 3 - 3y$.

We can consider *Optr*11 as a transformation which "shrinks" the interval [0,1] by a factor of 1/3 and shifts it to the left, which is just an example of an affine transformation. *Optr*12 is a transformation which shrinks the interval [0, 1] by a factor of 1/3 and shifts it to the right. (*Optr*11,*Optr*12) gives the first stage of the formation of the Cantor set depicted in Figure 3.

With reference to Axiom 4.2 we have (see Figure 3):

x:=[0,1]
y_x:= a shrunken [0,1] reflected twice, at the left and at the right.

*Optr*2 is the operator which transforms y_x into a singular (mereological) class, (as it were, a new x, but now, x with y_x in it, see Figure 4). Thus the result of *Optr*2 is not to be intended as a "return" to the original segment, but as a "shift of perspective" of the entire system, whereby several parts are now considered as an undivided unit.

We see that repeating the process $(Optr11,Optr12)(x_{y_x})$ or alternately $(Optr11,Optr12) \circ Optr2 \circ (Optr11,Optr12)(x)$ with the operation of composition understood in a usual way, gives the second stage of the Cantor set, and so on.

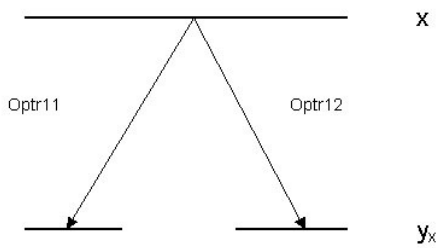

Figure 3: A sketch of Optr1 for the Cantor set.

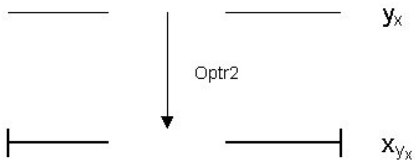

Figure 4: A sketch of Optr2 for the Cantor set.

The process itself of iteration can be seen as an expression of the primary relation *tr*3, which, according to Axiom 4.3 does not distinguish between *Optr*1 and *Optr*2.

The application of the dynamic identity in the example gives a new insight to the process involved in the formation of the Cantor set.

The second example we present is more complex and is two-dimensional. It is described by the following two equations:

$$\begin{cases} x_{n+1} = a x_n + b y_n + c \\ y_{n+1} = d x_n + e y_n + f \end{cases}$$

where (x,y) represents a point in R^2;

$$\begin{pmatrix} a & b \\ d & e \end{pmatrix}$$

represents a linear transformation in R^2;

$$\begin{pmatrix} c \\ f \end{pmatrix}$$

represents a linear translation in R^2;

The whole matrix

$$\begin{pmatrix} a & b & c \\ d & e & f \end{pmatrix}$$

represents an affine transformation in R^2.

At this point, one can note that in this way we can produce a variety of equations, which corresponds to different transformations, which have impact on the point-movement on the plane. To emphasize those transformations which are more important we could add another factor, which represents the probability of acting of each transformation.

Now we take into consideration three transformations instead of one, as in the case of the Cantor set, with the following parameters:

$$\begin{cases} a = 0.5 \\ b = c = 0 \\ d = 0.5 \\ e = f = 0 \end{cases}$$

$$\begin{cases} a = 0.5 \\ b = c = 0 \\ d = 0.5 \\ e = 1.3 \\ f = 0 \end{cases}$$

$$\begin{cases} a = 0.5 \\ b = c = 0 \\ d = 0.5 \\ e = 0.6 \\ f = 0 \end{cases}$$

In this way we obtain the system of equations for every group of parameters. We assume that the probability is equal to 1/3 for each transformation.

Coming back to primary relations, we can express *Optr*1 as:

*Optr*1=(*Optr*11,*Optr*12,*Optr*13), where: *Optr*11,*Optr*12,*Optr*13 each represent one of the transformation described above. For the first transformation one can see clearly that its two iterated functions are invertible:

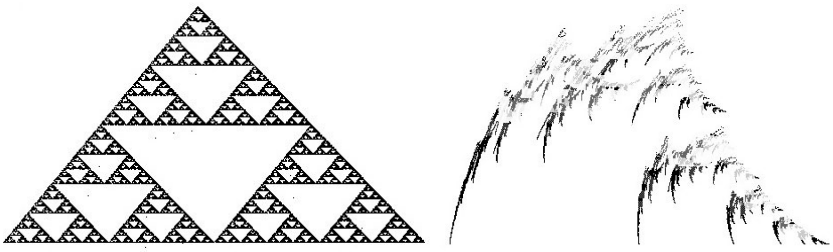

Figure 5: Sierpinski triangle and its modification.

$$\begin{cases} x_{n+1} = \frac{1}{2} x_n \\ y_{n+1} = \frac{1}{2} x_n \end{cases}$$

For the other two, it is not so obvious, thus we have to calculate the Jacobian matrix [10, 1]. The Jacobian matrices for them are non singular:

$$\begin{pmatrix} 1/2 & 0 \\ 1/2 & 1.3 \end{pmatrix}$$
$$\begin{pmatrix} 1/2 & 0 \\ 1/2 & 0.6 \end{pmatrix}$$

which means that these transformations are invertible, thus according to Axiom 4.2, $tr1$ can be represented by $Optr1$. $tr2$ as for the Cantor set, is a shift of perspective and $tr3$ the process of iteration. As a result we obtain the Sierpinski triangle (see Figure 5), [14]. Moreover changing parameters produces new transformations and as a consequence we can generate more interesting figures (see Figure 6). In conclusion admiring all these beautiful fractals that sometimes reflect nature very well, it would not seem unnatural to posit the kind of dynamical (unity) identity which we called *non-standard identity* as having to do with natural phenomena.

Figure 6: Barnsley fern

6. Conclusions

In this paper we have tried to present the new concept of primary relation as a useful tool in describing self-referential phenomena. Introducing the concept of primary relation has led to a fundamental shift of perspective in that the new system is based totally on relation. With the introduction of the special primary relation *tr*, where the unique value of Lesniewski's mereology is more than ever evident, it seems possible to express a new kind of "dynamic identity" suited to self-referential phenomena.

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Some Ontological Remarks on the Maxim of Identification of Indiscernibles

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Abstract. In Körner 1970, a pluralization of categorial frameworks is put forward. This project can be interpreted as a transcendentalism relativized in respect of several systems of reference. Here I attempt to apply this idea to what may be the most fundamental category, that of *identity* and *unitariness*, employing to this end the maxim discussed in Quine 1950. In §§ 1-2, I introduce the notion of "object" (or "entity") through the (intuitively more immediate) example of material objects, point to the problematical nature of this notion with reference to the principle of *identity*, and state the aim of my contribution, which is to make this problematical aspect (at least partially) meaningless by a pragmatic move, involving the *constitution* of identity (*identification*). In § 3, I refer explicitly to the Kantian perspective and introduce some notions that are useful for what I propose to do here. In §§ 4-5, I express some doubts with regard to essence and necessity and these doubts lead to the identification solution in discourse contexts (§ 6). In § 7, I draw on two examples used in Quine 1950: sociology and propositional logic. In § 8, I present a general consequence (ontological pluralism) and in §§ 9-11 more specific consequences in relation to token-type bifurcation (§ 9), the need for a sortal expression for statements of identity (§ 10), and nominalism (§ 11). The argument used in § 11, which is simply a collateral application of the consequence presented in § 8, draws on some remarks developed more fully in Valore 2006.

1. Initial problems

Objects can be of many different kinds. Among the least problematical we are perhaps prepared to admit first of all material objects (in the broad sense of not necessarily being identified with *physical objects*). Let us consider then, for the sake of simplicity, the case of material objects like books and bookshops. One of the criteria we usually rely on is the principle of unitariness, according to which a

(material) object is something essentially unitary (even if not necessarily coherent), distinct – as it were, conceptually carved out – from the background. It is sufficiently clear, at least intuitively, that a book is a unitary object distinct from the bookshop in which it is found and from the bookmark inside it. However, when we ask why we make this assumption, difficulties may arise. Although some of these later turn out to be only apparent (or apparent in their traditional formulation), the fact remains that they force us to account for our idea of *identity*: in what sense is an object identical to itself and different from the others?

The difficulties with material objects can be of (at least) two kinds, are widely reported in philosophical literature, and can easily be generalized to objects in general.

A first kind of difficulty lies at the *diachronic* level. What we have here is a set of problems that are well exemplified by the traditional argument regarding identity in change: how is it possible to identify the same object despite change in its properties? Do we have to postulate a substance as the substratum for identity? A second kind of difficulty lies at the *synchronic* level; the problems here are connected, for example, with the difficulties encountered in tracing the degree of unitariness of a material object. Is a book a unitary object? What about the cover or the glue that holds the pages together? And can we speak of a single object if we move from the descriptive level that recognizes a macroscopic object to the level that traces innumerable subatomic particles?

It is evident that the intuitive instruments to which we have recourse in common sense contexts are insufficient. Such instruments are, for example, the idea that a material object is something coherent and continuous or that the object is unitary because basically homogeneous. However, it is easy to come up with counterexamples to both these solutions (salt is found in part in my kitchen and in part in the Ocean, and a single table can be made out of many different materials).

Objects of a different kind from material objects, providing they are allowed into our "inventory of the world", suffer of course from additional problems, which it is not worth going into detail here. Suffice it to recall the attempts made throughout the history of philosophy to (in some way) reduce non-material objects (like, for example, abstract entities) to material objects or even *physical* objects in a narrow sense to the ordinary objects of common sense. This is true of, among others, Toulmin 1953, who as a criterion for the *existence* of problematic entities, like a neutrino or magnetic field, suggests «cloud-chamber pictures of a α-ray tracks, electron microscope photographs or, as a second-best, audible clicks from a Geiger counter», arguing that this would be «sufficiently like being shown a live dodo on a lawn» (p. 136). Such attempts reveal the misconception that material objects are easier to justify philosophically. But this is obviously not so; or, to be more exact, the identity principle for material objects themselves is waiting to be justified from a philosophical standpoint. And as long as that does not occur, "why the table is the object it is" is an unresolved question on a par with "what kind of object is √9".

2. Aim of this paper

The above-mentioned difficulties will not be gone into any further. What this paper proposes instead is to consider the principle of *identity* of an object and not only a material one. This principle is understood as the principle of *identification* of indiscernibles. It starts from the maxim put forward in Quine 1950, although I develop it along different lines from what he intended. In particular, I propose to reject the moderation in applying the maxim recommended in Quine 1960 (p. 230).

The intention behind choosing the term "identification" is to underline the logical priority of the act of constituting identity, whose outcome is a *unitary object* (and not the reverse). Clearly, "identification" is not here synonymous with "individuation", especially if, by individuation, one means an *exclusively* epistemic principle (one that would allow us to trace individuals, things and events on the basis of certain criteria). What is required is not an answer to the question: "How can we *know* if *a* is identical to *b*?" or "How can we *know* if the object considered at a moment t_1 is identical to the object considered at a moment t_2?". Identification shares with identity an ontological characterization. I hope to succeed below in elucidating in what sense the identification of indiscernibles within contexts is able to combine the ontological level of identity with the level of the epistemic criteria for identity (which at most can be informative) and with the level of individuation.

Naturally, there is no intention here to resolve all the difficulties of both a diachronic and synchronic kind referred to above. Rather, the aim is to point out that these difficulties are often tied to the nostalgia for a readymade object, in itself, of which we may have no need. A further consequence is the pluralization of the absolute principle of identity into a principle of identity in contexts of discourse. Furthermore, I also intend to deny that the privilege of material objects and the attempts to reduce other kinds of entities to material objects (or, at a more sophisticated level, of non-physical entities to physical objects) have any meaning or that they are of any use from a metaphysical standpoint.

3. Thing and representation

In section 1, I referred to the complications in respect of the identity of an object and its characterization as a unitary object. If we look closely, we see that the unitariness of the object seems to represent the correlate, on the synchronic level, of identity in change. The object is what it is on the basis of a "substantial" characterization, which allows us to trace it independently of further determinations; we can even think of the latter as "accidental". It is substance that gathers the multiple features into a unitary object, that represents the object's stable support. On the other hand, at the diachronic level, it is still substance that allows identity, that is the permanence of the same object, independently of the loss or acquisition of further determinations, which also can be thought of as "accidental". Once again, this is possible on the basis of the assumption of a "support" for the determinations.

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This conceptual connection between substance as unitariness and permanence of identity in change is very clear in Kant's "First Analogy of Experience" (Kant 1787), in which he draws attention to the necessity that something is permanent if there is to be a change. Without permanence through time, there would also be no «*identity* of the subtratum, wherein alone all change has thoroughgoing unity» [B 229, p. 216, my underlining]. Indeed, «the substratum of all that is real, that is, of all that belongs to the existence of things, is *substance*; and all that belongs to the existence can be thought only as a determination of substance. Consequently the permanent, in relation to which alone all time-relations of appearance can be determined, is substance in the [field of] appearance, that is, the real in appearance, and as the substrate of all the change remains ever the same» [B 225, p. 213]. The catch-phrase "in all the changes in the world, substance remains and only accidents change" is a commonplace in the history of philosophy; but Kant inverts it in his own revolutionary way and gives it an entirely new meaning: «this permanence is, however, simply the mode in which we represent to ourselves the existence of things» [B 229, p. 216]. Even if substance is "the real", this does not mean that it is not, *in toto*, introduced into the representation.

What I want to do here is to develop this observation in some respects independently of the Kantian formulation even though, within a still transcendental perspective, I believe that the observation emphasizes the *constitution* of the level of objectuality (the essential difference probably consists, as we will see, in the fact that I prefer to link substance not to time but to a *purely conceptual* determination). The basic idea is that pointing to substance as that which allows us to characterize an object as unitary, with respect to other objects, and as identical within the change it undergoes through time should not reflect a fact *of the thing* but, rather, indicate a way in which we represent it. More precisely, pointing to "substance" is only possible within a conceptual scheme, without which no object would be given; better still, this indication is a fundamental element in the conceptual scheme itself. Paradoxically, for this very reason it is a fact *of the thing*, but of the thing as object, that is as having meaning within a categorial framework.

4. Essence

A further characterization of substance, which allows it to play an essential role in the process of identification, is traditionally associated with essentiality. Essence is that which is relevant to the unity of the object, to recognizing the object for what it is and to finding the same object in the modification of its features. If, for example, we identify the individual substance Socrates as essentially a "rational animal" of a certain kind, Socrates may even turn bald without ceasing to be what he is, that is a rational animal. Being bald or not is irrelevant to the object.

But how do we establish what is and is not relevant? In his *Metaphysics* (*Met.* Z 17, pp. 29-30), Aristotle already pointed out this difficulty: if we ask ourselves why a man is artistic, we want to explain a connection between "a thing" and something else that is its predicate. However, when we ask ourselves for what reason a man is a man, «why a thing is itself, is to ask nothing at all» (*Met.* Z 17,

1041 b, p. 19). We cannot avoid acknowledging the presence, in the thing, of the essence of that object which it is (for example, the essence of man).

The problem with this treatment of individuation is that we do not in fact have any instrument for tracing (in the strong sense of "discovering") what the, already always existing, essence is of a given something that we recognize to be an exemplary case of a certain kind. Indeed, substance as essence is not contained, already given, in the thing itself but relates to our characterization of a determined object as a case of this or that thing. This aspect is clearly emphasized by Quine 1951, who says that it is meaningless to characterize a property as essential to the object itself. Take the above case of man: for a man to be rational is essential and to have two legs accidental. Now, we can even concede that rationality is implicit *in the meaning of the term* "man" and that to have two legs is not; but, without doubt, to have two legs is implicit *in the meaning of the term* "biped" whereas rationality in not. Faced with a real individual, who is both a man and a biped, it makes no sense to say that rationality is essential to him and having two legs accidental rather than the contrary (Quine 1951, p. 22). The decision is not based on an essence already given in the thing but on choosing to characterize a real individual conceptually as "man" rather than as "biped". It has to do with the meanings of the terms "man" and "biped" and not with real determinations. What we have is a common confusion in metaphysics: a semantic distinction crystallizes into an ontological bifurcation. But it would be too easy to conclude *sic et simpliciter* that we can do without this bifurcation and still maintain unitary objects identical with themselves. Although the characterization of substance has to do with a definition at the conceptual level, this is no reason why it cannot be transferred to the level of the real thing. If substance indicates "our mode of representation", this does not exclude (on the contrary, it implies) that «the substratum of all that is real, that is, of all that belongs to existence of things, is *substance*» (Kant 1787, B 225, p. 213).

5. *"De dicto"* necessity and *"de re"* necessity

Another way to present the question of essential characterization is to make a two-tier division: *de dicto* and *de re*. To hold that a definition is essential in that we have decided to characterize a real individual conceptually in this way means to derive *de re* necessity from a *de dicto* necessity. And it is not by chance that Quine's project, which denies essence in the Aristotelian sense, also rejects *de re* necessity: the necessity that Socrates is a man depends on the predicate "man" that we attribute to him and not on a real necessity. Likewise with "biped". It is obvious that if we characterize an individual as a biped it is necessarily true that this individual is a biped (this comes down to something like this: it is necessarily true that a biped is biped); but this does not mean that the individual is necessarily biped, that is that he could not lose a leg in an accident. This project rejects as senseless any *de re* modality: for example, any necessity referring to *res*. From this rejection derives the parallel rejection of the distinction between necessary and contingent properties of individuals, that is the denial of essence. Once more,

however, it would be too simple to conclude *sic et simpliciter* that we can happily do without *de re* necessity. To put forward this conclusion means to repropose, without justifying it, the metaphysical idea in a negative sense according to which essence concerns the representation of the object *but not* the thing in itself. It means merely moving the problem of substance a bit further away: we cannot attribute to the *res* (in itself) legitimate distinctions only at the level of conceptual characterization. Quine's rejection of Aristotelian essence is, then, employed here in certain respects against him so as to stress, instead, Kant's thesis that substantial characterization is legitimate, pertains to the object (and not only to its representation) and makes sense solely in a categorial framework of reference. As we will see below, this interpretation – only at first sight (I hope) paradoxical – derives moreover from the extension of another thesis in this same project: identification in contexts of discourse.

6. Identification in contexts of discourse

The strategy in which the maxim of the identification of indiscernibles consists (Quine 1950, p. 626) is to translate *in contexts* an economic principle analogous to Ockham's razor: objects that are indistinguishable from one another in the terms of a given discourse should be considered as identical for that discourse. In this way, the number of objects in our "local" ontology is reduced by rethinking the initial objects (defined in another context) as a single new object. At first sight, this principle may appear simply to repropose Leibniz's principle of identity of indiscernibles (so-called Leibniz's law, in one of its formulations): if x and y share exactly the same properties, we have two identical entities, that is *the same entity*:

$$\text{(ID)}\ \forall x\ \forall y\ (\forall F\ (Fy \leftrightarrow Fx) \rightarrow x = y)$$

where F is a predicative variable and $\forall F$ a second-order quantification. But this is not the case. In reality, the term "identification" indicates the activity of *constructing* identical objects by applying one from among all the conceptual schemes we need. (ID) is applied to each *Li* and, of course, extended to any number of variables x, y, z, ..., within each *Li*.

Objects are not identical *in themselves*; they are or are not the same objects, they are one or more objects depending on the context and the language we adopt for our cognitive ends (and, I would add, also for other ends). It is evident that, in absolute terms, what we have is a principle contrary to Ockham's razor considering that it entails a proliferation of objects in relation to diverse discourses. But, if we take each context in turn, what we have is certainly a principle of economizing and frugality.

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7. Exemplification: sociology and prepositional logic

It may be useful to present again the example of "income group" found in Quine 1950 (pp. 625-626). From the viewpoint of the privilege attached to physical objects or events and units of sensorial experience, income group is merely a *façon de parler*, which refers to a characteristic abstracted from the really existing physical individuals. "Income group" is then only a convenient abbreviation and from an ontological perspective does not commit us at all to assuming that there are corresponding entities. What the identification principle proposes, on the contrary, is to simplify our discourse and reduce as far as possible the number of objects we have by means of a *conceptual integration*. Let us suppose that our context is a branch of sociology and in this context everything that can be said about a person's life stage can be applied to all the life stages of those earning the same amount. We can consider our sociological discourse as a discourse on an innumerable series of such life stages (individual 1 or life stage of individual 1, individual 2 or life stage of individual 2, …, individual *n* or life stage of individual *n*). *Or else*, following the identification principle, we can simplify our discourse by replacing the *n* life stages of those earning the same with *a single object*: "income group". What we have done is to reconsider the initial objects, indistinguishable within the context of discourse in which we find ourselves, and rethink them as a single new object.

Another useful example is the interpretation of propositional calculus. We can consider the letters *p*, *q*, *r*, etc. as if we are referring to propositional concepts (initial objects). From the standpoint of propositional calculus, however, concepts that have the same truth value are indistinguishable and interchangeable. As long as our context is propositional calculus, we can reconstruct the initial objects by bringing in a smaller number of objects that are identical because indistinguishable. The referential objects of letters *p*, *q*, *r*, etc will then be the same truth values, which in this context is all that we are interested in (see Frege 1892, pp. 162-165).

8. Some general considerations

To tie the idea of a unitary object to referential language means to make the idea of an object depend on the "local" conceptual scheme, that is the categorial framework in which we operate. As has already been mentioned, this entails a plurality of object levels with reference to the plurality of languages, that is of the conceptual schemes which we employ. The notion of substance, understood as that which ensures the unitariness of the object and its permanence within change, thus acquires meaning only in far as it is *introduced* by the categorial system of reference, assembled – as it were – in the act of identification and not already traced in itself in the object.

We can switch from one theory to another, defining a class of objects of one theory in terms of another class of objects belonging to another theory. In order to do this, however, we need to establish "projective functions" that allow us to

project the universe of one theory into that of another theory. But this cannot be done *from nowhere*, that is from a position that is external and neutral with respect to the theories in question. We can say something about the objects only in relation to a theory and, what's more, we cannot engage in a discourse on the objects of a theory in total disregard of another theory in relation to which we interpret this first theory. We either define objects within the language of a theory or we speak of the objects of this theory, but in any event we have to define them within the language of another theory. There is no super theory or privileged super language to refer to. The objects are not already there: they are *assumed*. What precisely counts as an object can only find an answer in one of the world's various "systems". Beyond this general consequence, some particular consequences can be pointed out which, I believe, require us to revise the intuitive idea of identity.

9. Token Identity vs Type Identity

The systematic application of the maxim of identification of indiscernibles leads to negation of the usual distinction between token-identity and type-identity, that is the distinction between a kind of identity that regards concrete things and one that regards abstract entities such as species and types. Token-identity should allow the replacement of individuals as values for the variables in statements of identity such as "*x* is identical to *y*", and type-identity the replacement of entities different from individuals (given that our ontology permits this kind of replacement). The identification of indiscernibles, in the sense that I have attempted to repropose here, allows a token-identity also for species and types, trivially considering such entities as individuals within a certain discourse. There is no other identity, in the proper sense, than token-identity *in contexts*.

10. Narrow Identity vs Wide Identity

Strictly speaking, the above conclusion needs to be reformulated as follows: there is no other identity than narrow identity. Wide identity presupposes that for a statement of identity such as "*x* is identical to *y*" a sortal expression *F* is to be introduced, at least implicitly, for which

x is identical to *y* in as far as *F*.

It seems *prima facie* that explication of the sortal expression acts precisely as the restriction on the principle of identity that leads to a pluralization of identity according to the relevant characteristic in a determined context of discourse. This is not the case. The sortal expression allows us to specify the (qualitative) equality of two individuals which are (quantitatively) different in that, for example, they belong to the same class. From this point of view, two different models of car are identical in that they are cars. It would better in this case to speak of equality rather than identity (or wide identity in opposition to identity in the proper sense or

narrow identity; or again, qualitative as opposed to quantitative identity). In order to specify a sortal expression, there must be two numerically different individuals. The expression of a narrow identity through a sortal specification contains no informative content: each thing is identical to itself in that it is that kind of thing which it is! The identification of indiscernibles leads to a completely opposite conclusion: two (or more) entities that are identical in relation to the context of discourse must be considered as one and only one entity and not two entities of the same kind. A qualitative identity relative to the context of discourse becomes a quantitative identity relative to that context of discourse. There is no other identity in the proper sense than quantitative identity.

11. Nominalism

Empiricists are usually suspicious of abstract entities such as properties, relations, propositions, classes, numbers and so on. As a result, contemporary empiricists tend for the most part to avoid referring to entities with a dubious ontological consistency and often confine themselves to a nominalistic language. Giving up abstract entities, for example in mathematics and logic, can be justified as follows: sensorial qualities do not provide an adequate enough basis for the limitless universe of numbers, functions and classes as, let us say, values attributed to variables in mathematics. If, however, we take the identification of indiscernibles in its objectual outcomes seriously, this link between the idea of unitary object and sensorial qualities is no longer significant. An object is not that object which it is on the basis of experience but, rather, on the basis of the categorial structure of the context in which, on each occasion, we are interested. Nominalism, on the contrary, puts forward again the idea of a, so to speak, *zero level* of basic objects.

Giving up abstract entities means not only not adopting as the subject of predication expressions that refer to abstract entities, but also any predicate that is not a predicate of concrete individuals or which cannot be restated in terms of predicates of concrete individuals. It goes without saying that each definition that commits us, in the *definiens*, to adopting abstract entities must be rejected, even if it is brought in to elucidate predicates of concrete individuals in terms of other predicates of concrete individuals. The only legitimate instruments, therefore, are predicates of concrete individuals, individual variables, quantifiers for these variables and truth connectives. Note that the legitimacy of such a basic vocabulary is not grounded in logical or mathematical necessity but rather, as stated above, in a philosophical intuition that cannot be justified any further by appealing to something more fundamental.

Why this privilege of basic objects? The general application of the maxim also to entities which within a given context of discourse (say, fundamental physics) can be considered abstract provides, in my view, a good argument against this privilege. In § 6, it was stated that, for reasons of greater frugality, references to the original objects should be rethought as if they referred to a smaller number of objects so that the original indiscernibles give rise to a same new object. The essentially metaphysical question is how to understand the expression "original

objects". I believe that the best way to understand it is as "initial objects" and *not* as "zero level objects". That is to say: there is no privileged point from which to conduct this translation; we can only translate the objects of a context of discourse into the objects of another context, but what we obtain is a series of *new* objects. Every objectual context, constructed within a particular categorial structure, has the same ontological dignity as any other. The choice between contexts concerns what we intend to do with our theories and descriptions of the world.

In this sense I also interpret the *principle of indispensability*, sometimes referred in the literature as the Putnam-Quine principle. Not to acknowledge more than is necessary means allowing as legitimate entities those that a given discipline requires for its own ends. In other words, the arguments for admitting entities must be *a posteriori*, that is dependent on the growth of knowledge and on its needs. We are committed to acknowledging as entities what is indispensable for our scientific theories and if some of the latter need to adopt certain mathematical entities, then we have to acknowledge that such mathematical entities "exist". Where this is not required, on the basis of a principle of austerity and economizing, we have to conclude that non-necessary entities "do not exist" (see Putnam 1975). This appeal to the arguments of indispensability can be puzzling, above all because existence and reality are made to depend on the level of theories. We might find this astonishing, but only if we adopt the *standard view*, according to which reality is what is given independently of our theory and descriptions.

To sum up, the idea of establishing *a priori* what kind of entity to privilege seems to me wholly incompatible with the constructivist idea in general. In particular, nominalism and physicalism betray nostalgia for an object in itself, outside of our descriptions, for which we have no real need.

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Mereology in Approximate Reasoning about Concepts

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1. Introduction: Approximate Reasoning

Approximate Reasoning is also known under other names: Reasoning under Uncertainty, Soft Computing. The second name reveals the essence of this paradigm: uncertainty of knowledge.

Uncertainty of knowledge is due to such factors, among others, as: complexity of the context of reasoning that makes it impossible to account for all essential parameters, e.g., in control of large systems; lack of precise values of parameters, e.g., in sensing by means of infrared sensors, ultrasonic sensors, etc., etc. and in consequence knowledge of these values up to a degree; necessity of dealing with partially missing or restricted sets of data, e.g., in medical diagnosis.

Approximate reasoning differs from precise reasoning both in its formal shape as well as in methods.

Formal reasoning processes may be divided, according to I. M. Bochenski [5], cf. Łukasiewicz [20], Hintikka [10], into two main classes: deductive reasoning and reductive (encompassing so called inductive reasoning) reasoning. Deductive reasoning proceeds formally in systems endowed with implicative assertions, according to the schema:

$$\text{if } A \text{ implies } B \text{ and } A \text{ then } B \tag{1}$$

On the contrary, reductive reasoning proceeds according to the schema:

$$\text{if } A \text{ implies } B \text{ and } B \text{ then } A \tag{2}$$

In reasoning with certainty, it is assumed that all assertions are assigned values of truth states, from falsity (0) through intermediate states, to truth (1). Deduction

rules assign truth values to the assertion B on the basis of the truth values of the implication $A \Rightarrow B$ and the assertion A. This is especially manifest in axiomatic systems, see, e.g., Hilbert and Ackermann [12] in which one assumes a collection of assertions called axioms, assigned by default to be true, from which by deduction rules other true assertions are derived.

Reductive reasoning poses more problems: in order to infer that A on the basis of $A \Rightarrow B$ and B requires usually the evidence B of satisfactory strength. Problems related to this inference are expressed, e.g., by the well–known Hempel "paradoxes of confirmation" [11], [3].

By these problems, reductive reasoning borders on reasoning under uncertainty, as many approximate schemas, e.g., based on probability or Bayesian reasoning, were proposed for it, see, e.g., [1], [37].

Approximate reasoning in the strict sense seems to be a form of reasoning in which assertions are assigned degrees of uncertainty, i.e., one is aware that the actual value of truth of an assertion is known only to a degree; in this there seems to lie the main distinguishing feature of this form of reasoning. Clearly, it is not possible to draw a clear line between this form of reasoning and some forms of formal deductive reasoning; for instance, the state $1/2$ of truth in the 3–valued logic of Łukasiewicz [21] may be interpreted as the state "do not know" i.e., an expression of uncertainty to degree of $1/2$. However, in 3–valued logic, the state of truth of $1/2$ is known "exactly" and it is not subject to evaluation on the basis of evidence or knowledge.

We will in this exposition adhere to the point of view that approximate reasoning involves clear realization of uncertainty of knowledge which is expressed in its formal shape by some numerical factors that convey the degree of uncertainty and are evaluated on the basis of actual knowledge.

Among known forms of approximate reasoning, one may mention: rough set–based reasoning, see, e.g., [26], [27], in which knowledge is expressed by means of information systems and the underlying logic is the decision logic of descriptors (loc.cit., loc.cit.); fuzzy set–based reasoning, see, e.g., [42], [9], in which degrees of uncertainty are expressed by means of membership degrees of an element in a set; various forms of nonmonotonic reasoning, see e.g., [6].

2. Ontology

Reality that is the subject of our cognition, description, and analysis, can be approached by means of categories (concepts, names) that classify objects of which the reality is composed.

The subject of Ontology is a calculus of categories. We restrict ourselves to ontological scheme proposed by Leśniewski, see [15], cf. [38].

The primitive predicate in this theory is ε, read "is". Elementary formulas of Ontology are expressions of the form $x \varepsilon y$, that are read "x is y".

One may ask whether there is any difference between the ontological symbol ε and the set–theoretical "esti" symbol $\in$. As pointed to by, among others,

Słupecki and Łoś (see [36]), ε is closest to $\in$ when the latter is used as the predicate denoting the relation of membership between an individual and a set of individuals (see, [38], p. 71), albeit, even in this case, the semantic difference persists: $\in$ may be used to denote the relation effected between objects of various logical types whereas arguments of $\mathcal{E}$ should belong in the same semantic category (loc. cit.).

The meaning of ε is explained by means of Axiom of Ontology (AO):

$$\begin{gathered} x\varepsilon X \Leftrightarrow (\exists z.z\varepsilon x) \wedge \\ (\forall y,z.y\varepsilon x \wedge z\varepsilon x \Rightarrow y\varepsilon z) \wedge \\ (\forall y.y\varepsilon x \Rightarrow y\varepsilon X). \end{gathered} \tag{3}$$

The identity predicate $=$ id defined by means of,

$$x = y \Leftrightarrow x\varepsilon y \wedge y\varepsilon x. \tag{4}$$

In the light of (4), (AO) states that the statement $x\varepsilon X$ is true if and only if the three conjuncts in the right–hand side of (3) are true, viz., it is true that

$$\exists z.z\varepsilon x, \tag{5}$$

i.e, x is a non–empty concept, and it is true that

$$\forall y,z.y\varepsilon x \wedge z\varepsilon x \Rightarrow y\varepsilon z, \tag{6}$$

i.e, $y = z$, so x is an individual concept as any two objects falling under it are identical, and it is true that

$$\forall y.y\varepsilon x \Rightarrow y\varepsilon X, \tag{7}$$

i.e., any object falling under x falls as well under X (x is "contained" in X).

A consequence to (AO) is,

$$x\varepsilon x, \tag{8}$$

that, in turn, along with the left–hand side of (AO) indicates that x and X are in the same semantic category.

To sum up: $x\varepsilon X$ means that x is an individual being, that falls under X (responds to the name of X).

3. Knowledge and its representation: the information system

As already mentioned the notion of knowledge is central to reasoning in Computer Science as reasoning is effected on the basis of available knowledge.

In Computer Science and its applications, knowledge is expressed very often by means of a set of sentences in some logical language.

Here, we propose to represent knowledge by means of *information systems* that may be perceived as windows open into the scrutinized world, giving only partial information about it – the primary source of uncertainty. Hence, the need for approximate descriptions of concepts derived from the representation only.

3.1 Information systems

Objects in the world under investigation are described by means of attributes (features, qualities) the number of which is finite; the set of attributes A consists of attributes $a_1,...,a_n$. Each attribute a_j is regarded formally as a mapping from the set of objects $U = \{u_1,...,u_m\}$ into the *value set* V_j of admissible values of a_j.

The pair (U, A) is the information system. It encompasses available knowledge about the world under investigation. For each object u_i, the *information set* $inf(u_i) = \{(a_1, a_1(u_i)),...,(a_n, a_n(u_i))$ encodes knowledge about the object u_i.

3.2 Rough set approach

This approach based on classical ideas of Frege [8] that, informally, concepts are divided into two categories: exact (precise) ones and inexact ones. Inexact concepts should be characterized by the presence of a boundary consisting of objects that can be classified – upon the available knowledge – neither to the concept nor to its complement.

To render this idea in the context of an information system (U, A), the relation of indiscernibility is introduced, see Pawlak [26], as follows,

$$u_i INDu_j \Leftrightarrow inf(u_i) = inf(u_j). \tag{9}$$

Objects u_i, u_j satisfying (9) are said to be indiscernible.

By concepts we understand here subsets of the set U; a concept X is *exact* if and only if

$$x \in X \wedge yINDx \Rightarrow y \in X, \tag{10}$$

otherwise, X is inexact, or, rough. We will introduce predicates $E(x)$, satisfied with a subset $x \subseteq U$ if and only if x is exact, and $R(x)$ satisfied with x if and only if x is rough.

Thus, concepts are classified in the rough set approach into two semantic categories: exact concepts, that we denote with the symbol $\mathcal{E}$, and rough ones, denoted $\mathcal{R}$.

3.3 Ontology of information systems

The general scheme of Ontology will be applied in the context of information systems. We will render two semantic categories of E, R in a generic information system (U, A).

Our individual objects will be concepts in U, i.e., subsets of the universe U.

We let, for a concept $x \subseteq U$

$$x \varepsilon \mathcal{E} \Leftrightarrow E(x), \tag{11}$$

and,

$$x \varepsilon \mathcal{R} \Leftrightarrow R(x). \tag{12}$$

The two categories of concepts are related by means of approximations: given a concept $X \subseteq U$, one lets [26],

$$\underline{X} = \{x \in U : [x] \subseteq X\}; \overline{X} = \{x \in U : [x] \cap X \neq \varnothing\}, \tag{13}$$

where $[x]$ is the equivalence class of x with respect to the relation IND.

The concept $\underline{X} \varepsilon \mathcal{E}$ is called the lower approximation to X whereas $\overline{X} \varepsilon \mathcal{E}$ is said to be the upper approximation to X.

These concepts are related by duality,

$$\overline{X} = U \setminus \underline{U \setminus X}. \tag{14}$$

4. Mereology

The next ingredient of the complex scheme of notions originating with Ontology is Mereology. When Ontology may be called the theory of distributive concepts, Mereology is the theory of collective concepts, treated as a whole.

It is therefore not surprising that Mereology starts with relations on individual objects.

The scheme of Mereology we like to work with here is that proposed by S. Leśniewski [14], also [16], [18]. In his approach, the primitive predicate is that of *being a part*, denoted here with the symbol π.

The elementary formula form is $x\varepsilon\pi y$ read "the individual x is a part of the individual y, i.e.,

$$x\varepsilon\pi y \Rightarrow x\varepsilon x \wedge y\varepsilon y. \tag{15}$$

The predicate π is subjected in Mereology due to Leśniewski to the following requirements,

$$\begin{aligned} &1.\text{it is not true that } x\varepsilon\pi x; \\ &2.x\varepsilon\pi y \wedge y\varepsilon\pi z \Rightarrow x\varepsilon\pi z. \end{aligned} \tag{16}$$

Thus, the relation of a part is, in any model of mereology, a relation of pre–ordering.

In a standard way, as prescribed by the classical Schröder theorem, this pre–order is turned into an order encoded by the predicate of *being an ingredient*, $ingr$ in symbols,

$$x\varepsilon ingry \Leftrightarrow x\varepsilon\pi y \vee x = y. \tag{17}$$

Thus,

$$\begin{aligned} &1.x\varepsilon ingrx; \\ &2.x\varepsilon ingry \wedge y\varepsilon ingrx \Rightarrow x = y; \\ &3.x\varepsilon ingry \wedge y\varepsilon ingrz \Rightarrow x\varepsilon ingrz. \end{aligned} \tag{18}$$

A decision problem whether $y\varepsilon ingrx$ is true can be decided by means of the following proposition shown true by Leśniewski [14].

Proposition 1. [Leśniewski 1916]

If for each z such that $z\varepsilon ingry$ there exists w such that $w\varepsilon ingrz$ and $w\varepsilon ingrx$ then $y\varepsilon ingrx$.

4.1 Class operator

The notion of a set is introduced in Mereology by means of the predicate of ingredient.

Consider a property Ψ of individuals, such that there exists an individual x with $\Psi(x)$ true (we say that Ψ is non–vacuous). The idea of a set is that of an

individual that does represent a non–empty collection of individuals that satisfy Ψ.

An individual X is a set of objects Ψ, in symbols, $X\varepsilon set(\Psi)$ if and only if

$$\begin{gathered} 1.\exists x.x\varepsilon ingrX; \\ 2.x\varepsilon ingrX \Rightarrow \exists y,z.y\varepsilon ingrx \wedge y\varepsilon ingrz \wedge \Psi(z). \end{gathered} \tag{19}$$

Thus: $X\varepsilon set(\Psi)$ is true when there is some ingredient of X and each ingredient of X has an ingredient in common with an object satisfying Ψ.

When a set X of objects Ψ has the property that

$$\Psi(x) \Rightarrow x\varepsilon ingrX, \tag{20}$$

then X is called the *class of objects* Ψ. We denote this fact symbolically with $X\varepsilon Cls(\Psi)$. It is assumed [14] that $Cls(\Psi)$ be unique for each non–vacuous Ψ.

5. Approximate Mereology

In the setting of information systems, the phenomenon of inexactness, calling for approximate description of concepts, has been taken into account by means of notions of exact, respectively, rough concepts (sets). Inexactness of concepts bears also on part relation, as for inexact (rough) concepts, the part relation cannot be ascertained exactly.

An idea poses itself here, to consider relations (predicates) of *being a part to a degree*.

Predicates of this kind were introduced in Polkowski and Skowron [32] as *rough inclusions*, i.e., predicates of the form $\mu(x,y,r)$, where x,y are individuals, $r \in [0,1]$, that satisfy the requirements,

$$\begin{gathered} 1.\mu(x,y,1) \Leftrightarrow x\varepsilon ingry; \\ 2.\mu(x,y,1) \Rightarrow [\mu(z,x,r) \Rightarrow \mu(z,y,r)]; \\ 3.\mu(x,y,r) \wedge s < r \Rightarrow \mu(x,y,s). \end{gathered} \tag{21}$$

In (21), Property 1. assures that rough inclusions are properly related to underlying mereological structures as expressed by their ingredient relations. Property 2. is a monotonicity requirement, and Property 3. does express the semantics of $\mu(.,.,r)$ as *a part to degree at least r*.

5.1 Rough inclusions in information systems

Following our knowledge representation, we will define rough inclusions in information systems.

Consider an information system (U, A). We assume that,

$$x, y \in U \wedge x \neq y \Rightarrow inf(x) \neq inf(y). \tag{22}$$

The condition (22) may be achieved in any information system (U, A) by selecting the representative in each class of the indiscernibility relation IND and declaring the universe U to be the set of representatives, i.e., by passing from U to the quotient set U/IND and defining new attributes as the quotients relative to IND of the original attributes.

For $x, y \in U$, we define the set

$$DIS(x, y) = \{a \in A : a(x) \neq a(y)\}. \tag{23}$$

Our construction of rough inclusions will proceed in two steps: for objects in U and for their sets, and it will depend on certain mappings called triangular norms (t–norms).

A mapping $t : [0,1] \times [0,1] \rightarrow [0,1]$ is called a triangular norm, when the following requirements are satisfied,

$$\begin{gathered} 1. t(x, y) = t(y, x); \\ 2. t(x, t(y, z)) = t(t(x, y), z)); \\ 3. t(x, 0) = 0; \\ 4. t(x, 1) = x; \\ 5. x \geq x' \Rightarrow t(x, y) \geq t(x', y). \end{gathered} \tag{24}$$

A t-norm t is archimedean when in addition,

$$\begin{gathered} 6. t \text{ is continuous;} \\ 7. t(x, x) < x. \end{gathered} \tag{25}$$

Examples of archimedan t–norms are, **the Łukasiewicz t–norm**,

$$t(x, y) = max\{0, x + y - 1\}, \tag{26}$$

denoted by us in the sequel with the symbol $\otimes$ (the Łukasiewicz tensor product), or,the Menger t–norm,

$$t(x, y) = x \cdot y. \tag{27}$$

On the other hand, the t–norm,

$$t(x,y) = min\{x,y\}, \tag{28}$$

is not archimedean.

It was shown, [17] that any archimedean t–norm t admits a representation on the lines of the Hilbert problem XIII, see [13],

$$t(x,y) = g(f(x) + f(y)), \tag{29}$$

where $f:[0,1] \to [0,1]$ is continuous decreasing and g is the pseudo–inverse to f, see[17].

In case of the Łukasiewicz t–norm, (57), one has,

$$f(x) = 1 - x = g(x), \tag{30}$$

whereas for the Menger t–norm (27),

$$f(x) = lnx; g(y) = e^{y}. \tag{31}$$

It is known that in case of the t–norm min, the function f that would satisfy (29) can be neither continuous nor decreasing [2].

Representation of the form (29) will be applied by us now in defining some rough inclusions, viz., for an archimedean t–norm t, with the representation (29), we let,

$$\mu_t(x,y,r) \Leftrightarrow g(\frac{|DIS(x,y)|}{|A|}) \geq r. \tag{32}$$

Then,

Proposition 2. *μ_t is a rough inclusion when $g^{-1}(0) = 1$ in which case the underlying ingr relation is the identity $=$.*

In terms of Proposition 2, the monotonicity condition (21).2 and the condition (21).3 are satisfied.

For instance, in case of the rough inclusion induced by the Łukasiewicz t–norm $\otimes$ (57), one has,

$$\mu_{\otimes}(x,y,r) \Leftrightarrow 1 - \frac{|DIS(x,y)|}{|A|} \geq r, \tag{33}$$

which may be paraphrased with the help of the set

$$INDIS(x,y) = \{a \in A : a(x) = a(y)\} = A \setminus DIS(x,y),$$

as,

$$\mu_{\otimes}(x,y,r) \Leftrightarrow \frac{|INDIS(x,y)|}{|A|} \geq r. \tag{34}$$

The Reader will notice that (34) does express the probabilistic mode of reasoning and that this formula is prevalent in many applications in Machine Learning.

An important consequence of this choice of rough inclusions is the transitivity rule of deduction, of the form,

$$\frac{\mu_t(x,y,r), \mu_t(y,z,s)}{\mu_t(x,z,t(r,s))}. \tag{35}$$

An argument will be useful here; let us observe that,

$$DIS(x,z) \subseteq DIS(x,y) \cup DIS(y,z),$$

so $\frac{|DIS(x,z)|}{|A|} \leq \frac{|DIS(x,y)|}{|A|} + \frac{|DIS(y,z)|}{|A|}$;

let

$g(\frac{|DIS(x,y)|}{|A|}) = r_1$, $g(\frac{|DIS(y,z)|}{|A|}) = s_1$, $g(\frac{|DIS(x,z)|}{|A|}) = u$.

Hence,

$\frac{|DIS(x,y)|}{|A|} = f(r_1)$, $\frac{|DIS(y,z)|}{|A|} = f(s_1)$, $\frac{|DIS(x,z)|}{|A|} = f(u)$,

and,

$f(u) \leq f(r_1) + f(s_1)$, hence, $u = g(f(u)) \geq g(f(r_1) + f(s_1)) = t(r_1, s_1) \geq t(r,s)$, witness to $\mu_t(x,z,t(r,s))$. The argument is concluded.

6. Ontology of granular concepts

Assume that a rough inclusion μ_t induced by an archimedean t–norm t according to (32) is given in a universe U of objects endowed with a relation *ingr* of mereological ingredient, consistent with μ_t in the sense of (21).1.

The class operator (19), (20) of mereology will be now applied as a tool in creating individual objects, i.e., subsets of U.

6.1 . Granules

The notion of a granule of knowledge is now very essential when paradigms for approximate reasoning are involved. The main idea is to collect objects "similar" with respect to some criteria into one individual that represents all of them; traditionally, such objects were, e.g., those called "clusters". Granular computing is an idea of computing on granules of objects [35] and rough mereology offers here a unifying framework [33].

For an object $x \in U$, and a real number $r \in [0,1]$, we define a predicate $\psi_{r,x}$,

$$\psi_{r,x}(y) \Leftrightarrow \mu_t(y,x,r). \tag{36}$$

We denote by $g_r^{\mu_t}(x)$ the class of property $\psi_{r,x}$,

$$g_r^{\mu_t}(x) \varepsilon Cls(\psi_{r,x}). \tag{37}$$

The individual object $g_r^{\mu_t}(x)$ is called the *granule of radius r about x*; it does represent all individual objects in U that are close to x with respect to the similarity μ_t .

A characterization of granules may be derived from properties of μ_t , in particular, from (35).

Proposition 3. *For a granule* $g_r^{\mu_t}(x)$ *, where* μ_t *is induced by an archimedean t–norm* t *, and* $y \in U$ *, the following is true:* $yingrg_r^{\mu_t}(x)$ *if and only if* $\mu_t(y,x,r)$.

We provide an argument in favor of Proposition 3.

Clearly, the implication $yingrg_r^{\mu_t}(x)$ if $\mu_t(y,x,r)$ follows immediately from (20). Assume now that $yingrg_r^{\mu_t}(x)$. Consider $zingry$, i.e., $\mu_t(z,y,1)$. As $zingrg_r^{\mu_t}(x)$, there exist in virtue of (19) objects u,w with properties,

$$(i) uingrz, (ii) uingrw, (iii) \mu_t(w,x,r). \tag{38}$$

Hence, $\mu_t(u,w,1), \mu_t(w,x,r)$ imply by (35) that $\mu_t(u,x,t(1,r))$, i.e., $\mu_t(u,x,r)$; it follows by (21).2, (38).(i), that $\mu_t(z,x,r)$ and, again by (21).2, $\mu_t(y,x,r)$. The argument is concluded by Proposition 1.

6.2 Granular topologies

It is natural to regard granule system $\{g_r^{\mu_t}(x) : x \in U; r \in (0,1)\}$ as a neighborhood system for a topology on U that may be called the *granular topology*. In order to make this idea explicit, we define classes of the form $N(x,r) = Cls(\iota\psi_{r,x})$, where

$$\iota\psi_{r,x}(y) \Leftrightarrow \exists s > r.\mu_t(y,x,s). \tag{39}$$

We declare the system $\{N(x,r) : x \in U; r \in (0,1)\}$ to be a neighborhood basis for a topology τ_μ. This is justified by the following

Proposition 4.

1. $y\varepsilon ingrN(x,r) \Rightarrow \exists\delta > 0.N(y,\delta)\varepsilon ingrN(x,r)$;
2. $s > r \Rightarrow N(x,s)\varepsilon ingrN(x,r)$;
3. $z\varepsilon ingrN(x,r) \wedge z\varepsilon ingrN(y,s) \Rightarrow \exists\delta > 0.N(z,\delta)\varepsilon ingrN(x,r) \wedge N(z,\delta)\varepsilon ingrN(y,s)$.

An argument is like follows. For Property 1. $y\varepsilon ingrN(x,r)$ implies by (35) and (21).2, that there exists an $s < r$ such that $\mu_t(y,x,s)$. Let $\delta < 1$ be such that $t(u,s) > r$ whenever $u > \delta$; δ exists by continuity of t and the identity $t(1,s) = s$. Thus, if $z\varepsilon ingrN(y,\delta)$, then $\mu_t(z,y,\eta)$ with $\eta > \delta$ and, by (35), $\mu_t(z,x,t(\eta,s))$ hence $z\varepsilon ingrN(x,r)$. Property 2. follows by Proposition 1 and Property 3. is a consequence to Properties 1. and 2. This concludes the argument for Proposition 4.

7. Issues of similarity

Similarity relations are of great importance in various considerations and applications. Among them, tolerance relations [43] are of importance due to the fact that they are the closest similarity relations to equivalence relations. A tolerance relation on a universe U of objects is a relation $\tau \subseteq U \times U$ that is reflexive and symmetric. Let us observe that any rough inclusion μ_t on the universe U of an information system (U,A), defined as in (32), induces a family of tolerance relations of the form $x\tau_r y \Leftrightarrow \mu_t(x,y,r)$, where $r \in [0,1]$.

A global view on μ_t as a tolerance relation is possible as well at the cost of regarding it as a fuzzy similarity relation [41].

The following properties hold.

$$x\tau_1 x. \tag{40}$$

$$x\tau_r y \Leftrightarrow y\tau_r x. \tag{41}$$

$$x\tau_r y \wedge y\tau_s z \Rightarrow x\tau_{t(r,s)} z. \tag{42}$$

We will write $\tau(x,y)$ instead of $\chi_{x,\tau}(y)$ in cases when we treat τ as a fuzzy set, where $\chi_{x,\tau}$ denotes the fuzzy membership function [42].

Thus, τ is by (40), (41), and (42), a t*-fuzzy similarity* [41].

Following [41], we may define *similarity classes* $[x]_\tau$ as fuzzy sets satisfying

$$\chi_{[x]_\tau}(y) = r \Leftrightarrow \tau_r(x,y). \tag{43}$$

The following are true

$$\chi_{[x]_\tau}(x) = 1. \tag{44}$$

$$t(\chi_{[x]_\tau}(y), \tau(y,z)) \leq \chi_{[x]_\tau}(z). \tag{45}$$

$$t(\chi_{[x]_\tau}(y), \chi_{[x]_\tau}(z)) \leq \tau(y,z). \tag{46}$$

Equations (44, 45, 46) follow by corresponding properties of τ given by (40), (41), and (42).

Finally, the family $\{[x]_\tau : x \subseteq U\}$ does satisfy the requirements to be a *fuzzy partition* [41], viz.,

$$\forall x \exists y . \chi_{[x]_\tau}(y) = 1, \tag{47}$$

$$\begin{gathered} [x]_\tau \neq [z]_\tau \Rightarrow \\ max_y \{min\{\chi_{[x]_\tau}(y), \chi_{[z]_\tau}(y)\}\} < 1, \end{gathered} \tag{48}$$

$$\bigcup_x [x]_\tau \times_t [x]_\tau = \tau, \tag{49}$$

where $A \times_t B$ denotes the fuzzy set defined via,

$$\chi_{A \times_t B}(u,v) = t(\chi_A(u), \chi_B(v)). \tag{50}$$

Indeed, (47), (48), (49) follow directly from properties of τ. For instance, (49) is justified as follows: if there was y with $\tau(x,y)=1=\tau(z,y)$, we would have $\tau(x,z),\tau(z,x)\geq t(1,1)=1$ hence $x=z$.

8. Logical issues

Rough inclusions may be adopted to the task of defining logics reflecting the reasoning mode of rough mereology. We define an intensional logic, cf. [4], [23], whose intension is the mapping $I:\mathcal{E}\times\mathcal{F}\longrightarrow[0,1]$, where $\mathcal{F}$ is the set of meaningful formulas over a set $Pred$ of unary predicates interpreted in the set U. We denote with $[[p(x)]]$ the meaning of a unary predicate $p(x)\in Pred$, i.e., $[[p(x)]]=\{u\in U: p(u)\}$. [29],[30].

Given $\Lambda\varepsilon\mathcal{E}$, we denote with $I_\Lambda(\phi)$ the extension of I at the set Λ, valued at a formula ϕ, i.e., $I_\Lambda(\phi)=I(\Lambda,\phi)$.

We adopt the following interpretation of logical connectives N of negation and C of implication,

$$[[Np]]=U\setminus[[p]], \tag{51}$$

and,

$$[[Cpq]]=(U\setminus[[p]])\cup[[q]]. \tag{52}$$

For a regular rough inclusion μ on $(2^U)^2\times[0,1]$, where 2^U is the powerset of U, we define the value $(I_\Lambda^\mu)(\phi)$ of the extension of I relative to μ at Λ,ϕ as follows,

$$(I_\Lambda^\mu)(\phi)\geq r\Leftrightarrow\mu(\Lambda,[[\phi]],r). \tag{53}$$

We denote the rough mereological logic based on a rough inclusion μ with the symbol RML^μ.

We call a meaningful formula ϕ of RML^μ a *theorem with respect to* μ if and only if $(I_\Lambda^\mu)(\phi)=1$ for each exact set $\Lambda\in\mathcal{E}$ and for each instance of RZF.

In particular, adopting (34) to the case of a rough inclusion on concepts, we have,

$$\mu_\otimes(x,y,r)\Leftrightarrow\frac{|x\cap y|}{|x|}\geq r. \tag{54}$$

We give some facts concerning the rough inclusion $\mu_{\otimes}$. In what follows, by $(I_{\Lambda}^{\mu_{\otimes}})(p(x))$ we understand the maximal value equal to $\frac{|\Lambda \cap [[p(x)]]|}{|\Lambda|}$.

The extension $(I_{\Lambda}^{\mu_{\otimes}})(\phi)$ does satisfy the following with respect to negation and implication,

$$(I_{\Lambda}^{\mu_{\otimes}})(Np(x)) = 1 - (I_{\Lambda}^{\mu_{\otimes}})(p(x)), \tag{55}$$

and,

$$\begin{aligned}(I_{\Lambda}^{\mu_{\otimes}})(Cp(x)q(x)) &= \frac{|\Lambda \cap (U \setminus [[p(x)]] \cup [[q(x)]])|}{|\Lambda|} \\ &\leq 1 - (I_{\Lambda}^{\mu_{\otimes}})(p(x)) + (I_{\Lambda}^{\mu_{\otimes}})(q(x)),\end{aligned} \tag{56}$$

so finally,

$$(I_{\Lambda}^{\mu_{\otimes}})(Cp(x)q(x)) \leq 1 - (I_{\Lambda}^{\mu_{\otimes}})(p(x)) + (I_{\Lambda}^{\mu_{\otimes}})(q(x)). \tag{57}$$

The formula on the right hand side of inequality (57) is of course the Łukasiewicz implication of many–valued logic [19], [34]. We may say that in this case the logic $RML^{\otimes}$ is a sub–Łukasiewicz many–valued logic, meaning in particular, that if a sentential form of the formula $\phi(x)$ is a theorem of $[0,1]$–valued Łukasiewicz logic then $\phi(x)$ is a theorem of the logic RML.

We explain in more detail the last statement; for a formula $p(x)$, we denote with the symbol $p^{\dagger}$ the formula p regarded as a formula of sentential logic, subject to $(\neg p)^{\dagger}$ is $\neg(p^{\dagger})$ and $Cp(x)q(x))^{\dagger}$ is $C(p^{\dagger})(q^{\dagger})$. Then, as one may check easily,

Proposition 5. *If a formula $\phi^{\dagger}$ is a theorem of the Łukasiewicz sentential [0,1]–valued logic then the formula $\phi(x)$ is a theorem of RML^{μ}, for each regular rough inclusion μ.*

One verifies directly that derivation rules:

(MP) $\frac{p(x), Cp(x)q(x)}{q(x)}$ (modus ponens)

and

(MT) $\frac{\neg q(x), Cp(x)q(x)}{\neg p(x)}$ (modus tollens)

are valid in the logic RML^{μ} for each regular rough inclusion μ. In the context of intensional logic RML, we may discuss modalities L (of necessity) and M (of possibility). To introduce these modalities into RML, we make use of rough

approximations $\underline{X}$, respectively $\overline{X}$, the lower, respectively, the upper, approximation to X cf. (13).

8.1 Mereological: Necessity, possibility

We define, with a regular rough inclusion μ, functors L of necessity and M of possibility (the formula $L\phi$ is read "it is necessary that ϕ" and the formula $M\phi$ is read: "it is possible that ϕ") with partial states of truth as follows,

$$(I_{\Lambda}^{\mu})(Lp(x)) = r \Leftrightarrow \mu(\Lambda, \underline{[[p(x)]]}, r), \tag{58}$$

and, similarly,

$$(I_{\Lambda}^{\mu})(Mp(x)) = r \Leftrightarrow \mu(\Lambda, \overline{[[p(x)]]}, r). \tag{59}$$

It seems especially interesting, to look at operators L, M with respect to the rough inclusion $\mu_{\otimes}$ of Łukasiewicz. Then,

$$(I_{\Lambda}^{\mu_{\otimes}})(Lp(x)) = \frac{|\Lambda \cap \underline{[[p(x)]]}|}{|\Lambda|}, \tag{60}$$

and,

$$(I_{\Lambda}^{\mu_{\otimes}})(Mp(x)) = \frac{|\Lambda \cap \overline{[[p(x)]]}|}{|\Lambda|}. \tag{61}$$

It follows that,

Proposition 6. *In the logic $RML^{\otimes}$, a meaningful formula $\phi(x)$ is satisfied necessarily (i.e., it is necessary in degree 1) with respect to the exact set Λ if and only if $\Lambda \subseteq \underline{[[\phi(x)]]}$; similarly, $\phi(x)$ is possible (i.e., possible in degree 1) with respect to the exact set Λ if and only if $\Lambda \subseteq \overline{[[\phi(x)]]}$.*

Clearly, by the property (14) of rough set approximations, the crucial relation,

$$(I_{\Lambda}^{\mu})(Lp(x)) = 1 - (I_{\Lambda}^{\mu})(MNp(x)), \tag{62}$$

holds between the two modalities with respect to each rough inclusion μ.

A brief examination shows that the rough set interpretation of necessity presented here (and of possibility as well) differs from the interpretation proposed by fuzzy set theory [40], [7], i.e.,

$$(I_{\Lambda}^{\mu})(L(p(x) \wedge q(x))) \leq min\{(I_{\Lambda}^{\mu})(Lp(x)), (I_{\Lambda}^{\mu})(Lq(x))\}, \tag{63}$$

and the equality required by the fuzzy set theory [40], [7] does not hold in general.

Using the properties of rough set approximations (13), we may present within our intensional logic $RML^{\otimes}$ the otherwise well–known fact, obtained within different frameworks , e.g., in [24], [39], that rough sets support modal logic S5.

Proposition 7. *The following formulas of modal logic are theorems of RML with respect to every regular rough inclusion* μ *:*

1. *(K)* $CL(Cp(x)q(x))CLp(x)Lq(x)$.
2. *(T)* $CLp(x)p(x)$.
3. *(S4)* $CLp(x)LLp(x)$.
4. *(S5)* $CMp(x)LMp(x)$.

Proof Indeed, let us verify that the formula (K) is a theorem of $RML^{\otimes}$. Other formulas are theorems by virtue of (14). We have,

$$\begin{gathered}[[CL(Cpq)CLpLq]] = \\ U \setminus \underline{U \setminus [[p]] \cup [[q]]} \cup (U \setminus \underline{[[p]]}) \cup \underline{[[q]]}.\end{gathered} \tag{64}$$

Assuming that $u \in U$ is such that $u \notin U \setminus \underline{[[p]]}) \cup \underline{[[q]]}$, we have that

$$(i)[u] \subseteq [[p]]; (ii)[u] \cap [[q]] = \emptyset, \tag{65}$$

where $[u]$ denotes the equivalence class of u with respect to the indiscernibility relation $IND(A)$.

It follows by (65) that $x \notin \underline{(U \setminus [[p]]) \cup [[q]]}$ (as $x \in \underline{(U \setminus [[p]]) \cup [[q]]}$ would mean that $[x] \subseteq (U \setminus [[p]]) \cup [[q]]$ hence by (65)(ii) one would have $[x] \subseteq U \setminus [[p]]$, contradicting (65)(i)), i.e., the meaning of (K) is U. That concludes the proof.

9. Applications

Rough inclusions discussed above, play important roles in approximate reasoning in various applications within computer science. Among their applications, let us mention

1. As a framework for approximate reasoning in multiagent and distributed systems [33].
2. As a framework for rough–neural computing [25], [28].
3. As a framework for granular computing and granular decision algorithms [31].

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Resemblance Relations and Higher-Order Information Systems

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1. Introduction

Reasoning about reality is possible through our ability to grasp individual objects in their relations to others and not only as isolated entities. This is not just because human mind is not capable of coping with a *complete* diversity of individuals: without thinking of objects in relation to others, one could not form any concepts, the building blocks of knowledge. Thus, *our* universe cannot be a mere collection of objects but a structured whole.

Moreover, our universe may not be *the* universe. For, the alleged structure of "the universe" may not be anything other than our imposition. The Universe is what it is independently of our epistemological needs. In a sense, we are just a part of the universe. Let us assume that we know what the individual objects in the universe are. Then, our first questions will include: what features these individuals really have and what relations really hold among them.

My main concern here is the relation of *resemblance* which is an important notion to explain the structure of the universe. My aim is twofold: (1) developing a feasible philosophical notion of resemblance and (2) describing a rich class of abstract structures by means of which resemblance and other identity-like relations can be explicated in a philosophically significant manner. I will try to accomplish these by evaluating and developing former attempts at both (1) and (2). For (1), I will mainlyfocus on Armstrong's definition of resemblance (cf. [3], p.96). For (2), I will consider two special information systems; property systems and attribute systems as Vakarelov described in [9] and develop them so that more aspects of resemblance can be represented. While dealing with these formalisms, I will mainly concentrate on their explanatory power concerning the representation of

basic ontological relations and try to avoid unnecessary technicalities about the formal structures considered.

I will say a few things about the neighboring relations viz., identity, indiscernibility and interchangeability, to put resemblance in a proper frame. Let me start with a general remark:

Let $\mathcal{U}$ be a class of objects, $\mathcal{P}$ be any class of properties of objects and $\mathcal{R}$ be any class of relations among objects. In the study of these properties and relations, three aspects should be carefully distinguished.

(i) The *epistemic* aspect: Formation, revision and justification of our beliefs about

(a) Whether a given object a (a tuple of objects $(a_1, a_2, ..., a_n)$) lies in the extension of a property P in $\mathcal{P}$ (an n-ary relation R in $\mathcal{R}$),
(b) The nature of these properties and relations. That is, meta-properties of properties and relations.

(ii) The *linguistic* aspect: Symbolic representation of these properties and relations by means of a language. Comparison of expressive strength of competing languages...

(iii) The *ontic* aspect: All that is the case concerning these properties and relations independent of any mental or symbolic representation and any system of beliefs. One should respect the distinction between *ontic* and *ontological* and keep the word "ontological" for the study of the ontic (of what exists and nature of existents). One example may help clarifying the point: In the sense that I am using the words "ontic" and "ontological", the expression "ontic commitment" is improper: to say that it is proper is to say that the world may be committed to the existence of something while in fact only people can be properly said to be committed to existential beliefs on the face of their other beliefs.

An example of a "disagreement" arising from the failure to distinguish the epistemic and the ontic will be considered in the next section.

2. Identity, Indiscernibility, Interchangeability

Every object is identical with itself and with nothing else; the problem arises only if we question whether two descriptions in a language are the descriptions of the same thing or two different things. This is the common view of identity and I have no intention to question it. Formal properties of identity, or the axioms of identity, are the following:

(i) (Reflexivity) $x \equiv x$
(ii) (Substituvity in terms)

$$x \equiv y \rightarrow t(v_0 \ldots, v_{i-1}, x, v_{i+1}, \ldots, v_n) \equiv t(v_0 \ldots, v_{i-1}, y, v_{i+1}, \ldots, v_n)$$

(iii) (Substituvity in formulas)

$$x \equiv y \rightarrow \varphi(v_0 \ldots, v_{i-1}, x, v_{i+1}, \ldots, v_n) \Leftrightarrow \varphi(v_0 \ldots, v_{i-1}, y, v_{i+1}, \ldots, v_n)$$

Here, x, y are indeterminate nominal variables, $v_1, v_2, \ldots$ is the list of nominal variables, $t(v_1, v_2, \ldots, v_n)$ is any term whose variables are included among those in the parentheses. It is easy to see that symmetry and transitivity of identity follow from these axioms.

Identification of an individual object is something else and this notion does not lead to a diversity of identities as some philosophers claim: Geach, for example, clearly says that absolute identity is nonsense and we should further specify identity claims by referring to "being the same F" relations where F is a schematic expression for a count-noun. However, saying that x and y are the same F (where F is a count-noun) is nothing else but saying that x and y are identical and both are F. Thus there are no two objects which are the same F but not the same G. If there is a property F that x has while y fails to have, then x and y are not identical by the indiscernibility of identicals. Ontologically the interestingquestion is whether there are *weakenings of identity*. Or, if you wish, whether some neighboring relations should be considered as such. By labeling them as weakenings, I wish to emphasize that they are *not* relations competing with identity.

Indiscernibility is the coincidence of all properties. It is the closest relation to identity among the identity-like relations. Moreover, in the presence of the Leibniz' law, these are the very same relation.

Formally, indiscernibility is an equivalence relation. Confusing the epistemic with the ontic, one may claim that indiscernibility relation can be non-reflexive or non-symmetric or non-transitive. Such claims explicitly mention a method of justifying our discernibility claims about objects in a given domain. Consider the following:

> Reflexivity. It may seem reasonable to assume that every object is indiscernible form itself. But in some occasions this is not true, since it is possible that our information is so imprecise. For example, we may discern persons by comparing photographs taken of them. But it may happen that we are unable to recognize that a same person appears in two different photographs.
>
> Symmetry. Usually it is supposed that indiscernibility relations are symmetric, which means that if we cannot discern x from y, than we cannot discern y from x either. But indiscernibility relations may be directional. For example, if a person x speaks English, Finnish, and a person y speaks English, Finnish and German, then x cannot discern y form himself by the property "knowledge of languages" since y can

communicate with x in any languages that x speaks. On the other hand, y can discern x from himself by asking a simple question in German, for example.

Transitivity. Transitivity is the least obvious of the three properties usually associated with indiscernibility relations. For example, if we define an indiscernibility relation on a set of human beings in such a way that two person are indiscernible with respect to the property "age" if their time of birth differs by less than two hours. Then, there may exist three persons x, y and z, such that x is born an hour before y and y is born $1\frac{1}{2}$ hours before z. Hence, x is indiscernible form y and y is indiscernible from z but x and z are not discernible. [5]

Although it is possible to define and work with method-relative indiscernibility notions which may lack some (or all) of the formal features of indiscernibility, a general claim against reflexivity-symmetry-transitivity of indiscernibility cannot be supported that way. Indiscernibility or discernibility of objects depends only on the properties they have, not on any particular method of discerning them. *Indistinguishability* seems to be the proper epistemological counterpart of indiscernibility. Indistinguishability means inability to give an epistemic justification for the belief that *two or more* given objects have any distinguishing non-relational properties. Indiscernibility, on the other hand, refers to all properties, relational or non-relational, that the objects really have. In these terms, Black's famous counter-example to the identity of indiscernibles is an example of confusing the two. It is possible to imagine "two" indistinguishable spheres but they may not be indiscernible.

Confusing properties (the ontic) and predicates (the linguistic) may also mislead: the particular language we are using may not enable us to express the property discerning two objects. From this we cannot infer that they are indiscernible. For indiscernibility refers to the collection of all properties, not to a subcollection of properties. In particular those expressible in a language.

Let $\mathcal{U}$, the *universe*, be the class of all *entities* (abstract or concrete), whose elements are called *objects, particulars* or *individuals*, $\mathcal{P}$ be the class of all properties and $\mathcal{R}$ be the class of all relations with a special subclass $\mathcal{R}_{id}$ of *identity-like* relations: indiscernibility, interchangeability and resemblance. In the course of the following, I will present a unified account of relations in $\mathcal{R}_{id}$. Identity-like relations can be defined also on tuples of objects. Thus, we could allow $\mathcal{R}_{id}$ to include binary relations on $\mathcal{U}^n$ for any non-zero natural number n. The extension is straightforward.

Two objects o_1 and o_2 are indiscernible, $o_1 \cong o_2$ if they have the same properties, symbolically;

$$\forall P \in \mathcal{P}(P(o_1) \leftrightarrow P(o_2)) \tag{1}$$

Interchangeability is indiscernibility relativized to a subclass of $\mathcal{P}$. Thus, given a subclass $\mathcal{P}_0$ of $\mathcal{P}$, two objects o_1 and o_2 are interchangeable relative to $\mathcal{P}_0$, or $\mathcal{P}_0$-interchangeable, $o_1 \rightleftharpoons_{\mathcal{P}_0} o_2$, if

$$\forall P \in \mathcal{P}_0 (P(o_1) \leftrightarrow P(o_2)) \tag{2}$$

Just like indiscernibility, the relation of interchangeability is reflexive, symmetric and transitive on the universe. Explicating interchangeability this way has the following immediate consequences conforming to the commonsensical conception of interchangeability:

(i) Any two objects having all of the $\mathcal{P}_0$-properties are $\mathcal{P}_0$-interchangeable,

(ii) Any two objects having none of the $\mathcal{P}_0$-properties are $\mathcal{P}_0$-interchangeable,

(iii) No object lacking all the $\mathcal{P}_0$-properties is $\mathcal{P}_0$-interchangeable with an object having a $\mathcal{P}_0$-property,

(iv) No object having all $\mathcal{P}_0$-properties is $\mathcal{P}_0$-interchangeable with an object lacking a $\mathcal{P}_0$-property.

Plausibility of these consequences show that (2) is superior to another possible definition of interchangeability according to which we would have

$$\forall (P \in \mathcal{P}_0)(P(o_1) \wedge P(o_2)) \tag{3}$$

or, equivalently

$$\forall (P \in \mathcal{P}_0) P(o_1) \wedge \forall (P \in \mathcal{P}_0) P(o_2) \tag{4}$$

We may say that o_1 and o_2 are *strongly interchangeable for-* Δ, or strongly Δ-interchangeable if o_1 and o_2 satisfy (3) (or (4)). In fact (3) is covered by (2) while (ii) will no longer be true under (3). By means of the clause (ii), we label some entities as "useless" in the given context. We may say that two entities both failing to satisfy any $\mathcal{P}_0$-property, two $\mathcal{P}_0$-uselessobjects, are *negatively interchangeable with respect to* $\mathcal{P}_0$.

Intuitively $\mathcal{P}_0$ is the class of significant properties. What significant means depends on the context. In daily life, significant properties are often functional properties: if you want to peel an apple, any two fruit knives in the drawer are interchangeable although as objects they may have many distinguishing properties such as color or brand.

Later I will defend a notion of resemblance defined in the spirit of (2). My main contention is that particular interchangeability and resemblance notions can be accounted for by choosing suitable sets of significant properties.

Some overall stipulations can be made about sets of significant properties: to mention the most obvious, they should contain ontological properties and relations in so far as we regard interchangeability and resemblance as ontological relations (just likeindiscernibility). One may protest and say that the above notions of interchangeability and resemblance cannot correspond to ontological relations since there is no objective (ontic) ground for choosing sets of significant properties. My reply would be: Yes but for any set of properties $\mathcal{P}_0$, $\mathcal{P}_0$-interchangebility (and, later, $\mathcal{P}_0$-resemblance) has its proper ontological sense.

Resemblance, or similarity, is much weaker than indiscernibility and interchangeability. While comparing rival theories of resemblance, the following two points seem to be useful:

(i) The theory should *explain* (not just accept) the fact that a particular may resemble some particulars more closely than others. The theory should not be so robust to the extent that all resemblances are just yes or no cases.

(ii) The coverage of theory should be as broad as possible. Other things being equal, the theory which explains among more entities should be preferred.

The big deal about the resemblance debate is whether it is a primitive relation or a derivative of properties of objects. For a resemblance nominalist, resemblance is a primitive relation in terms of which properties can be eliminated. His aim is to explain away properties in terms of particular resemblances among objects. According to resemblance nominalism, a has the property F if a resembles a set of F*-paradigms*, F*-exemplars*, as closely as they resemble one another. Thus, the class of objects which are F has a structure: it is the set of exemplars which keeps the class together.

3. Criticisms of Resemblance Nominalism

There are many criticisms of resemblance nominalism. The strongest motivations of resemblance nominalists to regard resemblance as basic seem to be epistemological. Indeed, some of our experiences with resembling particulars suggest that it is too simple to be analyzed in terms of properties of objects. Moreover, there are cases where we cannot identify the property common in both individuals while we immediately observe that they resemble each other. The argument against realism about resemblance out of simple, or, better, immediate resemblances seems to rest on the confusion of the epistemic with the ontic. Although epistemic and linguistic facts (e.g., the way we come to believe that objects o_1 and o_2 resemble each other and the way we state our beliefs in a language) may be illuminating while doing ontology, we cannot use them

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uncritically. Even the overall epistemic simplicity of a relation cannot be taken as a direct justification of ontic simplicity. The alleged existence of immediate perceptions of resemblance may well be an evolutionary matter of fact rather than ontological necessity. Moreover, it is not obvious that we have immediate perceptions of any entity: How can we have immediate perceptions concerning say closed regions in 8-dimensions?

The source of one objection against resemblance nominalism is the relation of resemblance itself. If the resemblance nominalist cannot give a successful eliminativist account of it, this relation should be a universal too. Obviously, he should try a similar strategy and refer to second order resemblances among resemblance situations. A resemblance situation is a concrete case employing a tuple of individuals $(a_1, a_2, ..., a_n)$ standing in a particular resemblance R_1. Any two cases of resemblance $(a_1, a_2, ..., a_n), R_1$ and $(b_1, b_2, ..., b_m), R_2$ would resemble each other by both of them being resemblances. Thus this second order resemblance should be explained away by introducing a third order resemblance and so on. I should note here that, infinite regress arguments like this should be examined closely before deciding about whether they really undermine the theory.

One may legitimately question F-ness of F-exemplars as well. Since an account of their F-ness by means of their resembling each other gives no information, resemblance nominalists suggested that instead of a fixed set of F-exemplars, $a_1, a_2, ..., a_n$, we may use alternative sets of exemplars. The explanation for the F-ness of objects in the group of exemplars $a_1, a_2, ..., a_n$ is then given by their resemblance with objects in another group of exemplars $b_1, b_2, ..., b_m$. This account has the following deficiency: How can we choose this second group of exemplars? These must be somehow F (to help the explanation of a_i's F-ness) but how can we claim that any of these b_js are F? Referring to the F-ness of a_is would be illegitimate since their F-ness is under question. Then, we must pick another set of exemplars $c_1, c_2, ..., c_k$ whose F-ness again raises the same problem... Thus we are led to an infinite regress. It seems to me that the use of alternative sets of paradigm objects is of no use although it seems to be a natural explanation. In many cases existence of an infinite regress do not present a decisive problem at all. In this case, however, I think that the infinite regress argument leads to the conclusion that resemblances cannot explain objects' having properties. Even if we accept this conclusion, it does not by itself show that properties, or at least some of them, are irreducible. For there may be other attempts to evaluate. And indeed there are. Nevertheless, I believe that resemblance nominalism is the strongest rival against realism. Since, for above reasons, I rejected primitive resemblances of resemblance nominalism, I will assume that at least some properties are irreducible. I can gladly admit that once the class of objects which have the property F has been formed, use of (derived) resemblances on this class with well-chosen paradigm objects seems to be a good and natural way, e.g., to teach someone who does not just know F-ness or to justify claims about F-ness.

So what is problematic is not the relation of resemblance itself but relying heavily upon it to define F-ness.

4. Some Remarks on Possible Rival Formalisms of Resemblance

Before the issue of realist [1] conception of resemblance and its formal explications, let me add a few words about some possible formal explications of primitive resemblances. If one accepts that resemblance is a primitive relation, the obvious first move towards formal explication of resemblance would be to stipulate that resemblance is a binary, reflexive and symmetric relation. Corresponding first order structures consists of a non-empty set T together with such a relation on it. Reflexive and symmetric relations are called as *tolerance relations* by Schreider (cf. [8]) and the relational structures $\langle T, \tau \rangle$ consisting of a non-empty set T and a tolerance relation τ on it, are called *tolerance spaces*.

Even if it is accepted that resemblance relations are indeed reflexive and symmetric (and not necessarily transitive), not every such relation would correspond to a resemblance relation. It will be a consequence of the notion of resemblance defended here that resemblance relations need not be reflexive but *weakly reflexive* [2].

Another possible formal strategy corresponding to a nominalist conception of resemblance is to work with relative resemblances. Here one determines degrees of resemblances between pairs of objects relative to other pairs. More specifically, such a strategy requires a binary relation, say δ, on the set of pairs of objects. If S is a non-empty set and $\delta \subseteq (S \times S) \times (S \times S)$, then $(s_1, s_2), (s_3, s_4) \in \delta$ means that the resemblance between s_1 and s_2 is closer than the resemblance between s_3 and s_4.

5. Real Resemblances

The roughest realist notion of resemblance is that, it is a coincidence of at least one property. That is, objects o_1 and o_2 resemble each other, $o_1 \oplus o_2$ if they satisfy a common property:

$$\exists P \in \mathcal{P}(P(o_1) \wedge P(o_2)). \tag{5}$$

[1]The label "realist" here is used in the sense that it is used in the context of universals. In the sense used here, realist means "a philosopher who believes that universals exist". Note that primitive resemblances may well be "real" that is, they may form a part of independent reality.

[2]A relation R is weakly reflexive if every object that is R-related to an object is R-related to itself

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The same feature presents itself in several individuals or the same pattern is observed among several groups of individuals. When a feature, a property, say the property of F-ness is present in objects $o_1, o_2, ..., o_n$ we say that these objects resemble with respect to F-ness.

Generalizing resemblances with respect to properties, suppose that we propose the following as the definition of resemblance: Two or more objects resemble each other if there is a feature, F, present in all of them. Is this an appropriate definition? No: in this case the notion of resemblance would be too broad to be useful. For observe that under this definition, any two individual objects would resemble each other in so far as both are, say, self identical.

Fixing a respect of resemblance seems to be vital. It seems to be confusing though: should we say two or more objects resemble if they are both F? (F is the fixed respect of resemblance). No: I would rather say that there are many resemblances and the seemingly significant notion of unrestricted resemblances is the outcome of the careless talk. In cases where we assert simply e.g., that Fred resemble his father, we in fact intend to say that Fred resemble his father in some significant respects and presuppose that our listener knows which properties are significant in the context of utterance.

Universals theory of resemblance (UTR) explains resemblances of objects as mutual possession of the *same or resembling* properties of these objects. I will concentrate on structures in which resemblances, besides other identity-like relations, can be adequately represented in terms of property satisfaction and hope that the naturality of these structures will contribute the plausibility of the UTR.

Although daily use of resemblance is that it is a binary (2-ary) relation between particulars, it turns out that substantial part of the UTR must turn around the notion of resemblance between universals. This may appear to conflict with our intuition since we may think that what we perceive are not resembling properties but objects and this relation is too basic to admit any further analysis. It is true that, we do not perceive the universal red but always some red object and even *if* we should admit that universals resemble each other, this is not a matter of perception but conception.

However, I believe that, resemblances among universals are epistemologically as fundamental as resemblances among individuals. My knowledge that some properties of a resemble some properties of b is a *part of my knowledge* that and not just a *part of the explanation of my knowledge* that "a resembles b". I perceive the color of a, the color of b and quite possibly some other properties of a and b. Even "the sameness of the colors of a and b" is not something perceived. It is a conception. Perhaps trivial but a conception.

Thus, on this view, not only particulars resemble each other; universals may resemble each other too. Resemblance of universals is extremely important to develop a theory of resemblance inside a theory of universals. The resembling pairs of particulars may have a common identical property. However, one has to admit, this is just a theoretical *possibility*. In most (almost all) of the cases particulars resemble not in virtue of instantiating the same universal but again similar universals. For Armstrong, exact resemblance of universals is identity while

inexact resemblance of universals is *partial identity*. Rather than giving a general argument in support of this claim, he gives the example of "being five kilograms in mass". Using the language of states of affairs, being five kilograms in mass consists of "something's being four kilos plus something else's being (non-overlapping something else) being one kilo state of affairs". Thus, since being five kilos in mass involves having a (proper) part having two kilos in mass and being four kilos in mass [1].

Armstrong's view of resembling universals can be outlined as follows: Universals may resemble each other with degrees. Only complex universals can resemble each other. Some, like Hume, may think that simple universals may also resemble. This is out of false beliefs about the nature of resembling properties under consideration. It is true that colors, for example, may resemble each other: We say that red is more like orange than it is like yellow. It is also true that our perception of colors is somewhat simple. However, this does not mean that colors are simple properties and science gives us the reason why they are not so.

Although degrees of resemblance strongly suggest a quantitative analysis, there is nothing wrong with a qualitative analysis of this notion. The problem, to restate, is to give a viable analysis of the relation "the resemblance between a and b is closer than c and d".

Suppose that we are given a class of figures. Some of them may resemble each other with respect to planarity. Among these planar figures some would have a closer resemblance by virtue of their all being triangles. Among these we may further choose a smaller group upon noticing that they are all isosceles. Again, we may say that these figures have a closer resemblance. Armstrong, e.g., would accept that being an isosceles triangle is a legitimate conjunctive property. The first question to ask is that is it a first order property or not? It seems that it is. Being an isosceles triangle is a conjunction of two properties of first order although "being isosceles" cannot be meaningfully asserted for all objects. We cannot say that isosceles triangles are those objects lying in the intersection of "triangularity" and the extension of "isosceleshood". One may say that isosceleshood is a first order property even if it is weird to ask for example this piece of music is isosceles. After all there are objects (triangles) which are isosceles and the extension of the universal consists of these objects.

The apparent asymmetry arising in language due to awkwardness of the predicate "triangle isosceles" may suggest that the analysis of "isosceles triangularity" as a conjunctive property is not correct. For conjunction is commutative and thus there should be no asymmetry between "isosceles triangularity" and "triangle isosceleshood". However, in some languages, for example in French, the adjective "isosceles" comes after "triangle". Thus, the asymmetry is only apparent.

Moreover, there seems to be nothing special with the property "isosceleshood". The situation is not different than with the predicate "red car" in this respect. Although the possible extension of redness admits much more diversity, "car red" could be rejected as a natural predicate.

Resemblances between collections of objects can also be explicated in the spirit of UTR. As to the issue of resemblance between sets, two senses of resemblance are relevant. Two sets, to simplify the point, may resemble each other with respect to a property of sets. For example, two sets may resemble each other by being equinumerous, i.e., having the same number of elements. Or, they may resemble each other by carrying a similar structure. For example, two sets may resemble each other by both being partially ordered sets, or even by being order-isomorphic. Satisfying the same set-properties seems not to be the only way for sets to resemble each other. Sets, by being composite entities formed by particulars of lower levels, may resemble each other by virtue of properties of their members. Two sets may resemble each other by both being sets of ordinals or by being sets of finite sets...

Any theory of resemblance should explain the *degrees of resemblance*: we believe that there are pairs of particulars (o_1, o_2), (o_3, o_4) such that the resemblance between o_1 and o_2 will be closer than the resemblance between o_3 and o_4. However, according to (5) particulars seem to either resemble or not. Possible solutions have been suggested. Armstrong suggested, without much emphasis, that degrees of resemblance can be explained by the existence of scale-like orders on some classes of properties. Indeed, some properties can be thought to form a scale, e.g., shades of a color provide a good example. Another possible solution, based on the determinate-determinable relation on properties, has been suggested by Price [7]. Determinate-determinable relation is hard to define but, fortunately, easy to exemplify: Red is a determinable with respect to carmen-red and pink which are two determinates of it. There will still be determinates of carmen red and pink. Thus, two objects one is carmen-red the other pink resemble each other but this resemblance is weaker than the resemblance between any two pink objects. Thirdly, again by Armstrong, the structural-universals solution has been suggested. All these are still open pragrammes in the theory of universals. The last one, which I will follow and modify, by Armstrong, refers to resemblances between properties. Armstrong's definition of resemblance is that

> ...a particular a resembles a particular b if and only if: There exists a property, P, such that a has P, and that there exists a property, Q, such that b has Q, and *either* $P = Q$ *or* P resembles Q. [3], p. 96.

As in the case of interchangeability, resemblance claims are context dependent. As a first approximation towards the definition of contextual-resemblances, given a set of significant properties $\mathcal{P}_0$ and two objects o_1 and o_2, let us define: o_1 and o_2 resemble each other with respect to $\mathcal{P}_0$, $o_1 \mathbin{\textcircled{H}}_{\mathcal{P}_0} o_2$, if

$$\exists P \in \mathcal{P}_0 (P(o_1 \wedge P(o_2)) \tag{6}$$

In case $\mathcal{P}_0 = \mathcal{P}_{physical} \subseteq \mathcal{P}$ we obtain physical resemblance. One may extend (6) to cover *negative resemblance* so that o_1 resembles o_2 if

$$\exists P \in \mathcal{P}_0[(P(o_1 \wedge P(o_2)) \vee (\neg P(o_1 \wedge \neg P(o_2))] \tag{7}$$

A similar strategy would be to demand $\mathcal{P}_0$ to be closed under the so called negative properties. I will not follow this. Although negative resemblance is a genuine resemblance and should be accounted for in the theory, the extension of the followingto cover negative resemblances is straight forward and I will leave it for the purpose of brevity. Thus, "$\neg$" in (7) is not a property constructor applying to P but a sentential operator applying to $P(o_1), P(o_2)$. Similarly, I will not assume that $\mathcal{P}$ is closed under disjunctive or conjunctive properties.

An admissible but strange feature of relative resemblance is that an object having no significant property does not self-resemble. Note that (7) immediately solves the problem. The strongest point in favor of (6) is its generality. While resemblance nominalism tries to gain to strength from simple or immediate resemblances between common objects, it is weak if asked to explain resemblances between unobservable physical entities (and abstract entities too, if you accept them in your ontology). Ontologically, there is no point in denying resemblance for these entities just because we cannot causally interact with them properly to the degree that allows testing our beliefs about them (we may have no interaction at all, as in the case of abstract entities). We assume that they somehow exist either by definition (cf. mathematical entities) or as a result of our metaphysical beliefs and that they have properties (being prime, being odd...). Thus, (6) cover them easily.

Although not impossible, it is unreasonable to expect to observe resembling particulars in the sense of (6). In almost all cases, after closer examination, colors of two particulars, alleged to be the same at first, can be seen to be different. The same holds for other properties. Thus, one may claim that (6) has a serious explanatory weakness. Resembling properties of (*) motivate a further revision of (6). Granting that resemblance of properties belongs to a different type than resemblance of particulars, for any two properties $P_1, P_2 \in \mathcal{P}$, let $P_1 \Cap P_2$ denote "P_1 resembles P_2". Thus (*) becomes:

$$\exists P, Q \in \mathcal{P}[P(o_1) \wedge Q(o_2) \wedge (P = Q \vee P \Cap Q)] \tag{8}$$

Introducing contextuality into we get

$$\exists P, Q \in \mathcal{P}_0[P(o_1) \wedge Q(o_2) \wedge (P = Q \vee P \Cap Q)] \tag{9}$$

It remains to investigate the relation of resemblance $\Cap$ between non-identical properties. We need the notion of *attribute*. Before going any further about

attributes and their explication in information systems, I will consider property and relation systems.

6. Property and Relation Systems

In the usual semantics, properties and relations are identified with their extensions in the first order structures and the property instantiation is represented with set membership. However, "two" properties may have the same extension. In property systems, properties and property instantiation are taken as primitives.

Property systems are defined in [9] as follows:

Definition 6.1 A *property system* is a triple

$$\mathbf{S} = \langle Ob_S, Pr_S, f_S \rangle$$

where

(i) $Ob_S \neq \varnothing$ is the set of *objects*,

(ii) Pr_S is the, possibly empty, set of *properties*,

(iii) $f_S : Ob \longrightarrow \mathcal{P}(Pr)$ is the *information function*. For an object x, $f_S(x)$ is called the *information about* x in $\mathbf{S}$.

Thus, if x is an object then $f_S(x)$ determines which properties *from* Pr_S are instantiated by x. If the possibility that $f_S(x) = \varnothing$ obtains, this does not mean that x has absolutely no properties, just that it has no properties of the system. For applications, it is vital to restrict the properties and objects to manageable sizes.

For notational convenience, the subscript S may be dropped when the system we work in is understood. We write $\overline{f}(x)$ for $Pr \setminus f(x)$. If $Pr_S = \varnothing$, then S is called *trivial*, otherwise it is *non-trivial*. S is *total* if for every $x \in OB_S$, $f_S(x) \neq \varnothing$.

The assertion that "the object x has the property A in the system S" corresponds to the set-theoretical statement $A \in f_S(x)$. It is important to remark here that properties are not to be identified with sets. Since, note that, the definition of property systems does not forbid there being "two" properties A and B in Pr_S such that for every $x \in Ob_S$, $A \in f_S(x)$ if and only if $B \in f_S(x)$. While, if we identify a property P with the set $P' = \{x \in Ob_S : P \in f_S(x)\}$ then obviously $A' = B'$ by the extensionality criterion of set identity, thus the identification together with the transitivity of identity entails $A = B$.

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Definition 6.2 Let W be a non-empty set and let $V \subseteq W$ be any family of subsets of W. The *set-theoretical* P *system over the pair* (W,V) is the P-system $\mathbf{S}$ with $Ob_{\mathbf{S}} = W$, $Pr_{\mathbf{S}} = V$ and for any $x \in W$, $f_{\mathbf{S}}(x) = \{A \in V : x \in A\}$

The set theoretic model corresponding to a given property system $\mathbf{S}$ can be constructed as follows:

Definition 6.3 Let $\mathbf{S} = \langle Ob_{\mathbf{S}}, Pr_{\mathbf{S}}, f_{\mathbf{S}} \rangle$ be a property system. Let $W = Ob_{\mathbf{S}}$. For any $A \in Pr_{\mathbf{S}}$, let $|A| = \{x \in Ob_{\mathbf{S}} : A \in f_{\mathbf{S}}(x)\}$ and let $V = \{|A| : A \in Pr_{\mathbf{S}}\}$. Then the set-theoretical P-system over the pair (W,V), $P(W,V)$, is called the *set-theoretical* P*-system associated with* S. We write $|S|$ for $P(W,V)$.

Remark 6.4 Note that the set theoretic P system $|\mathbf{S}|$ corresponding to the property system $\mathbf{S}$ does not represent $\mathbf{S}$ truthfully. For, if A and B are two properties with the same extension, then in the set theoretic system these two properties will be counted as one. One may code the set theoretic system so that the same set appears two or more times perhaps with different indices.

Property systems may easily be extended to allow relations. Note that this extension is not only a formal possibility but it also has an ontological significance. Most of the natural binary relations cannot be defined on the basis of non-relational properties of objects. Similarly, many ternary relations of objects cannot be reduced to combinations of binary relations and properties. In some cases, reductions are possible: the relation of "betweenness" which may hold or does not hold for any triples of points on a line can easily be expressed by means of the relations "being on the left hand side of" and "being on the right hand side of". However, it is at least not clear, if not impossible, whether all natural ternary relations can be reduced to binary relations.

Definition 6.5 A *relational information system*, or relation system for short, is a structure

$$\mathbf{R} = \langle Ob; Rel^{(1)}, Rel^{(2)}, \ldots; f_1, f_2, \ldots \rangle$$

where

(i) $Ob \neq \emptyset$ is the set of *objects* of $\mathbf{R}$,

(ii) $Rel = \{Rel^{(1)}, Rel^{(2)}, \ldots\}$ is the set of *relations* of $\mathbf{R}$ so that for $n \geq 1$, $R^{(n)} = \{R_1^{(n)}, R_2^{(n)}, \ldots\}$ is a set of relations called n-ary relations of $\mathbf{R}$

(iii) For each $i \geq 1$, f_i is the i-th *information function* so that $f_i : Ob^i \longrightarrow \mathcal{P}(Rel^{(i)})$

Ob^i denotes the set of all i-tuples of objects. For any $\overline{x} \in Ob^i$, $f_i(\overline{x})$ tells which relations hold among the objects $x_1, x_2, ..., x_n$, in that order.

Remark 6.6 It should be clear that any relation system includes a property system and given a relation system $\mathbf{R} = \langle Ob; Rel^{(1)}, Rel^{(2)}, ...; f_1, f_2, ..\rangle$, the system $\langle Ob_R; R^{(1)}; f_1\rangle$ is the property system included in $\mathbf{R}$.

If one accepts the genuine possibility of *multi-grade* relations, then some relations may not have a fixed arity. Consider the relation "robbing a bank together". Any number of people may rob a bank together. Furthermore, it seems that, that relation cannot be reduced to relations of a fixed arity. For multi-grade relation symbols, we could drop the use of superscripts and allow tuples of objects of different lengths to be sent onto them by the information function f.

As in the case of property systems, some systems of sets give rise to relational information systems.

Definition 6.7 Let $W \neq \varnothing$ and $V = \{V^{(1)}, V^{(2)}, ...\}$ be a family of possibly empty subsets $V^{(n)} \subseteq (\mathcal{P}(W^n))$. Then the set theoretical relational system S over the pair $\langle W, V\rangle$ is the relational system with $Ob_S = W$, $R^{(n)} = V^{(n)}$ and for each $x \in W$ $f_{(n)}(x) = \{V_m^{(n)} \in V^{(n)} : (x_1, x_2, ..., x_n) \in V_m^{(n)}\}$

7. Identity-like Relations in Property and Relation Systems

Now we are ready to define identity-like relations in property and relation systems:

Definition 7.1 Let $\mathbf{S} = \langle Ob; Pr; f\rangle$ be a property system and x, y be two objects. Then

(i) x and y are *indiscernible*, $x \cong y$, if $f(x) = f(y)$

(ii) x and y are *interchangable with respect to* $Pr_0 \subseteq Pr$, $x \leftrightharpoons_{Pr_0} y$, if

$$f(x) \cap Pr_0 = f(y) \cap Pr_0$$

(iii) x and y *positively resemble*, $x \Sigma_S y$, if $f(x) \cap f(y) \neq \varnothing$,

(iv) x and y *positively resemble with respect to* $Pr_0 \subseteq Pr$, $x \Sigma_{Pr_0} y$, if

$$f(x) \cap f(y) \cap Pr_0 \neq \varnothing ,$$

(v) x and y *negatively resemble*, $xN_S y$, if $\overline{f}(x) \cap \overline{f}(y) \neq \varnothing$

(vi) x and y *negatively resemble with respect to* Pr_0, $xN_{Pr_0} y$, if

$$\overline{f}(x) \cap \overline{f}(y) \cap (Pr \setminus Pr_0) \neq \varnothing$$

Thus negative resemblance with respect to a class of properties is positive resemblance with respect to the complement of this class of properties: $xN_{Pr_0} y \Leftrightarrow x\Sigma_{Pr \setminus Pr_0} y$.

Generalization to the case of relation systems does not present any difficulty:

Definition 7.2 Let $\mathbf{R} = \langle Ob; Rel; f_1, f_2, ... \rangle$ be a relational information system and $\overline{x}$, $\overline{y}$ be two n-tuples of objects where n is any natural number. Then

(i) $\overline{x}$ and $\overline{y}$ are indiscernible $\overline{x} \cong \overline{y}$ if $f_n(\overline{x}) = f_n(\overline{y})$,

(ii) $\overline{x}$ and $\overline{y}$ are *interchangeable with respect to* $Rel_0^{(n)} \subseteq Rel^{(n)}$, $\overline{x} \rightleftharpoons_{Rel_0} \overline{y}$, if $f_n(\overline{x}) \cap Rel_0 = f_n(\overline{y}) \cap Rel_0$

(iii) $\overline{x}$ and $\overline{y}$ are *positively resemble*, $\overline{x}\Sigma\overline{y}$, if $f_n(\overline{x}) \cap f_n(\overline{y}) \neq \varnothing$

(iv) $\overline{x}$ and $\overline{y}$ are *positively resemble with respect to* $Rel_0^{(n)} \subseteq Rel^{(n)}$, $\overline{x} \rightleftharpoons_{Rel_0} \overline{y}$, if $f_n(\overline{x}) \cap f_n(\overline{y}) \cap Rel_0 \neq \varnothing$

(v) $\overline{x}$ and $\overline{y}$ are *negatively resemble*, $\overline{x}N\overline{y}$, if $f_n(\overline{x}) \cap f_n(\overline{y}) \neq \varnothing$

(vi) $\overline{x}$ and $\overline{y}$ are *negatively resemble with respect to* $Rel_0^{(n)} \subseteq Rel^{(n)}$, $\overline{x}N_{Rel_0}\overline{y}$, if $f_n(\overline{x}) \cap f_n(\overline{y}) \cap (Rel \setminus Rel_0) \neq \varnothing$

Considering again property systems, note that we did not put any restriction on classes of significant properties; they may be any subclasses of properties. This may cause a sense of arbitrariness. The notion of attribute systems enables us to eliminate this problem.

8. Attribute Systems

Definition 1. An attribute system, A-system for short, is a system

$$\mathbf{S} = \langle Ob, At, Val, f \rangle$$

where

(i) $Ob \neq \emptyset$ is called the set of objects,

(ii) At is a set whose elements are called the attributes of the system,

(iii) For each $a \in At$, $Val(a)$ is a set called the values of the attribute a,

(iv) f is a binary function, called the information function, such that for any $x \in Ob$, and any attribute $a \in At$, $f(x,a) \subseteq Val(a)$ called the *information about* x *with respect to the attribute* a.

Definition 8.2 An attribute system $\mathbf{S}$ such that $|f(x,A)| \leq 1$ for every object x and attribute is called a *single valued* information system. If $f(x;A) \neq \emptyset$ for every $x \in Ob_S$ and every $A \in At$, $\mathbf{S}$ is called normal.

Remark 3. As in the case of property systems, attributes should not be identified with sets of properties. The same set of properties may form the set of values for "two" different attributes.

Attributes can be thought to be properties of properties. For example "color" is an attribute and specific colors fall under the attribute "color", or, using the terminology of information systems, red is a value of color; red $\in$ $val(color)$. As I remarked above, it is possible that two properties may exist with the same individuals satisfying them and property systems allow us to represent this possibility. However, in property systems, we can not explain why these two properties are two instead of one. Since, no information besides information about objects can be coded in property systems. Attribute systems allow us to explain property differences (though to some extent).

The distinction between properties and attributes in attribute systems is justified by the fact that only attributes have properties as values. Attributes may be thought to be forming parts of the information about objects only indirectly since, note that, the information function in fact relates an object with a set of properties and as I remarked above, the same set of properties may form the set of values of two different attributes. This problem provides one of the main motivations for introducing higher order attribute systems. I will do this shortly.

There is a hierarchy on the class of attributes. For example, particular shades of red lie in the lowest level, red lies in the second level and color is in the third

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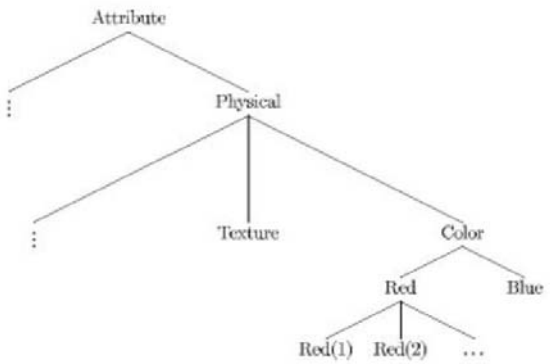
one... In the highest level we have the attribute "Attribute". A possible description of the hierarchy can be given by the following diagram:



Introducing higher order attributes to information systems is not only convenient but ontologically necessary. The principal use of first order attributes is that they enable us to differentiate two extensionally equivalent properties. The necessary equivalence of extensions may suggest that these two properties are in fact one. However, one may easily eliminate the possibility of this identification if he can find a feature of one of these properties such that the other one fails to have. It is hard to find good examples for that. However, the idea of such a possibility should be clear.

Concerning the representation of resemblance relations, the main role of higher order attributes is that by means of them resemblances of properties can be explained. In that respect, higher order attributes will function as determinables while their values will be the determinates.

If what I said before is indeed the case, we still have the possibility that the attribute immediately above the property may not give us the reason to differentiate one property from another and we may need to refer to a higher level attribute to do the work. In addition to the need for an explanation for non-identity of properties with the same set of extensions, higher order attributes are needed to explain non-identity of attributes with the same set of values too. First order attribute systems leave room to "represent" the contention that there may be "two" attributes with the same set of values. Thus an attribute is not identical with its set of values. However, in first order attribute systems there is no information as to what makes these two attributes non-identical. The representation is reduced to a mere notational convenience. Higher order attributes allows the "explanation" about non-identical extensionally equivalent properties and attributes be given.

From the above it should be clear that we need the higher order attributes to take full advantage of attribute systems over property systems. One should not think that the introduction of higher order information systems is a philosopher's trick. The hierarchy of attributes is a picture inherent in our conceptualization.

Consider a particular automobile. It is a car, it has the property of "being a car". This property falls under the attribute "being a vehicle" and "being a vehicle" falls under the attribute "being an artifact" and this in turn under the attribute "being a material being" and we may end all this by saying that "being a material being" falls under the highest level attribute "being an attribute" or, simply, "Attribute". Of course, this chain of attributes depends on the particular taxonomy of entities that one may not wish to commit himself to and he does not have to. For, the focus of the present study is not the categorization itself and we may just work modulo a categorization.

We must allow *relational attributes* or relation attributes into the system too. To give an example, both "lying under" and "being to the left of" are binary relations and they could fall under the attribute "spatial" or "being a spatial relation". In fact, allowing relations of different arities to fall under an attribute is a natural choice: both "betweenness" and "being to the left of" are spatial relations and thus the attribute "spatial" should include them both.

It may seem plausible that every property or relation falls under an attribute. However, in order to make the presentation more complete, the following generalization will not depend upon this assumption. I will immediately characterize systems based uponthis upon this assumption and thereafter I will work within systems in which every property and relation fall under an attribute. The reason for this choice is both philosophical and technical.

Definition 8. 4 A generalized attribute system is a system

$$\mathbf{S} = \langle Ob, Rel, At, Val, f \rangle$$

where

(i) $Ob \neq \varnothing$ is the set of objects,

(ii) $Rel = \{Rel^{(1)}, Rel^{(2)}, ...\}$ each set $Rel^{(n)}$ is called the set of n-ary relations of $\mathbf{S}$

(iii) $At = \{At^{(n)}_{(m)} : n, m \in \mathbb{N}^+\}$ is the set of attributes where each $At^{(n)}_{(m)}$ is the set of n-ary attributes of level m,

(iv) $Val : At \longrightarrow At \cup Rel$ is the valuation function so that

$Val(a) \subseteq At^{(n)}_{(m-1)}$ if $a \in At^{(n)}_{(m)}$ where $n \geq 2$

$Val(a) \subseteq Rel^{(n)}$ if $a \in At^{(n)}_{(1)}$

(v) f_n is the binary function such that for any $\overline{x} \in (Ob)^n$ and any $a \in At^{(n)}_{(1)}$ $f_n(\overline{x}, a) \subseteq Val(a)$. f_n is called the n-th information function and for any n-tuple of objects $\overline{x}$ and $a \in At^{(n)}_{(1)}$ $f_n(\overline{x}), a$ is called the *information about* $\overline{x}$ *according to* a.

Note that a relation $R \in Rel$ may not lie in the scope of any attribute. Thus we should explicitly mention our set of relations. As hinted above, I will now simplify generalized attribute systems by exploiting the assumption that every property or a relation falls under an attribute. This assumption justifies dropping the explicit mention of the sets of properties in an attribute system and relations since they will be "there" as values of attributes:

Definition 8. 5 A simplified generalized attribute system is a system

$$\mathbf{S} = \langle Ob; At; Val; f \rangle$$

where

(i) $Ob \neq \varnothing$ is the set of objects,

(ii) $At = \{At_{(m)}^{(n)} : n, m \in \mathbb{N}^+\}$ is the set of attributes and each $At_{(m)}^{(n)}$ is the set of n-ary attributes of level m,

(iii) Val is as in 4-(iv) except in the case of 1-th level attributes of arity n, $Val(a)$ is any set of n-ary relations among objects (while any two attributes of level 1 can still have the same set of values)

(iv) f_n is the same as the function f_n of 4-(v).

Definition 8.6 Given a generalized attribute system $\mathbf{S}'$, we may construct a corresponding relational information system $\mathbf{S}$ as follows:

(i) $Ob_S = Ob'_S$

(ii) $Rel_S^{(n)} = \bigcup\{Val(a) : a \in At_{(1)}^{(n)}\}$

(iii) for any $\overline{x} \in (Ob_S)^n$ $f_S(x) = \bigcup\{f'_S(\overline{x}, a) : a \in At_{(1)}^{(n)}\}$

9. Identity-like Relations in Attribute Systems

Now I will consider identity-like relations in attribute systems. For simplicity, I will give definitions for objects rather than tuples. The case of tuples is not difficult.

Definition 9.1 (Indiscernibility and interchangeability in attribute systems). Let $\mathbf{S} = \langle Ob; At; Val; f \rangle$ be an attribute system and let x and y be two objects. Then

(i) x and y are indiscernible in $\mathbf{S}$, $x \cong y$ if $\forall a \in At(f(x,a) = f(y,a))$,

(ii) x and y are interchangeable in $\mathbf{S}$ with respect to the attributes $A \subseteq At$, $x \leftrightharpoons_A y$, if $\forall a \in A f(x,a) = f(y,a)$

Attribute systems enable us to represent a wider variety of similarity relations:

Definition 9.2 (Similarity relations in information systems). Let $\mathbf{S} = \langle Ob; At; Val; f \rangle$ be an attribute system and let x and y be two objects. Then we may define the following notions of resemblance

(i) x and y are *weakly positively similar*, $x\Sigma_S y$, if $\exists a \in At : f(x,a) \cap f(y,a) \neq \varnothing$

(ii) x and y are *weakly negatively similar*, $xN_S y$, if

$$\exists a \in At : \overline{f}(x,a) \cap \overline{f}(y,a) \neq \varnothing$$

(iii) x and y are *strongly positively similar*, $x\sigma_S y$, if

$$\forall a \in At : f(x,a) \cap f(y,a) \neq \varnothing$$

(iv) x and y are *strongly negatively similar* $x\upsilon_S y$, if

$$\exists a \in At : \overline{f}(x,a) \cap \overline{f}(y,a) \neq \varnothing$$

We should consider restricting the set of attributes to smaller significant sets of attributes. In this way we obtain the following restricted similarities:

Definition 9.3 (Restricted similarity relations in attribute systems).
Let $\mathbf{S} = \langle Ob, At, Val, f \rangle$ be an attribute system, $A \subseteq At$ and x, y be two objects. We may define the following notions of resemblance

(i) x and y are *weakly positively similar w.r.t.* A, $x\Sigma_A y$, if

$$\exists a \in A : f(x,a) \cap f(y,a) \neq \varnothing$$

(ii) x and y are *weakly negatively similar w.r.t.* A, $xN_A y$, if

$$\exists a \in A : \overline{f}(x,a) \cap \overline{f}(y,a) \neq \varnothing$$

(iii) x and y are *strongly positively similar w.r.t.* A, $x\sigma_A y$, if

$$\forall a \in A : f(x,a) \cap f(y,a) \neq \varnothing$$

(iv) x and y are *strongly negatively similar w.r.t.* A $x\upsilon_A y$, if

$$\exists a \in A : \overline{f}(x,a) \cap \overline{f}(y,a) \neq \varnothing$$

If $A = \{a\}$, then we may write $x *_a y$ instead of $x *_{\{a\}} y$ where $*$ is any of the relations defined above.

Remark 4. Different assumptions about information systems yield different properties of these identity-like relations and different relations between these relations. For example, note that if S is a normal information system ($|f(x,a)| \leq 1$ for any $x \in Ob_S$ $a \in At_S$), then if $x\sigma_S y$ then $x \cong_S y$.

Let **S** be a generalized attribute system and let $A \in At_S$. Thus, $A \in At_{(m)}^{(n)}$ and each attribute belongs to a unique level m. A has a set of values forming a subset of attributes of level $m-1$. Similarly, each attribute of level $m-1$ has a set of values whose members are from the attributes of level $m-2$ and so on... Attributes of level 1 have values from the set of properties. Thus, for an attribute A we define an operation similar to transitive closure operation on sets. Namely

$$A^* = val(A) \cup \bigcup \{val(B) : B \in val(A)\} \cup \ldots$$

To give an example, Physical* would naturally include all physical attributes color, texture... and all specific colors, textures... and all specific shades of all colors, and if any specifications of specific textures...

We will use the following related notion as well:

$$A^{**} = A^* \cup \bigcup \{ext(P) : P \in A^*\}$$

where $ext(P) = \{a \in Ob : P(o)\}$. Note that since only properties have extensions this definition presents no difficulty.

The notion of resemblance of properties is defined by means of significant classes of attributes $\mathcal{A}_0$. Recall that I used the notation $P \Cap Q$ to denote "The property "P" resembles the property "Q"".

Definition 9.5 $P \Cap Q$ if and only if they are contained in A^* of some attribute A.

$$P \Cap Q \Leftrightarrow \{P,Q\} \subseteq A^* \tag{10}$$

Thus 9 becomes

$$\exists P,Q \in \mathcal{P}_0[P(o_1) \wedge Q(o_2) \wedge (P = Q) \vee \exists A \in \mathcal{A}_0 \{P;Q\} \subseteq A^*)]$$

It is reasonable to assume that $\mathcal{P}_0$ and $\mathcal{A}_0$ are related. One possibility could be to demand that

$$\mathcal{A}_0 \subseteq \{A \in \mathcal{A} : \exists P \in \mathcal{P}_0 \text{ such that } P \in A^*\} \tag{11}$$

Moreover, one can easily see by examining 9 and 11 does not lead to any restriction on the applicability of 9. In fact, the relation depends on the notion of attribute. The relation of "residing in" holding between one entity of level n and one of level $n-1$ (explicated by our Val function) is not transitive. Assume that an object o is red. Red is a color. But o is not (a) color. On the other hand, we ca formulate the relationship between, o and the attribute Color in the obvious manner: the property red resides in the object o and since red is a value of the attribute color, o is colorful. I will assume for present purposes that 11 is a reasonable constraint on the significant sets of properties and significant sets of attributes.

Let $\Theta(o_1,o_2)$ denote *the degree of resemblance between* o_1 *and* o_2. Thus $\Theta(o_1,o_2) \leq \Theta(o_3,o_4)$ means "the resemblance between o_3 and o_4 is closer than the resemblance between o_1 and o_2". I promised before that higher order attribute systems enable us to represent several degrees of resemblance. Among these we may state the following:

(i) The notions of degree of resemblance present in property systems is obviously present in attribute systems too. $\Theta^1_{P_0}(o_1,o_2) \leq \Theta^1_{P_0}(o_3,o_4)$ if o_3 and o_4 have more significant properties in common than o_1 and o_2 have:

$$|\{P \in \mathcal{P}_0 : P(o_1) \wedge P(o_2)\}| \leq |\{P \in \mathcal{P}_0 : P(o_3) \wedge P(o_4)\}|$$

(ii) Motivation for the following notion of degree should be clear from the preceding material: $\Theta^{1'}_{P_0}(o_1,o_2) \leq \Theta^{1'}_{P_0}(o_3,o_4)$ if o_3 and o_4 have more common or resembling properties than o_1 and o_2 have. That is

$$|\{(P,Q) \in (\mathcal{P}_0)^2 : P(o_1) \wedge P(o_2) \wedge (P = Q \vee P \Cap Q)\}| \leq$$

$$|\{(P,Q) \in (\mathcal{P}_0)^2 : P(o_3) \wedge P(o_4) \wedge (P = Q \vee P \Cap Q)\}|$$

(iii) I interpret what Armstrong pointed about scale-like ordering of some properties in terms of attributes as saying that there is a metric (a binary non-negative real valued function with some simple constraints) on the set of values of some attributes. Assume further that $A \in At$ be such an attribute and $P, Q \in A$. Let δ_A denote the metric on $val(A)$. Thus, $\delta_A(P,Q)$ denote the distance between P and Q with respect to the metric δ_A. Now define $\Theta_A^2(o_1,o_2) \leq \Theta_A^2(o_3,o_4)$ if

$$\exists(P,Q,R,S \in A)P(o_1) \wedge Q(o_2) \wedge R(o_3) \wedge S(o_4) \wedge \delta_A(R,S) \leq \delta_A(P,Q)$$

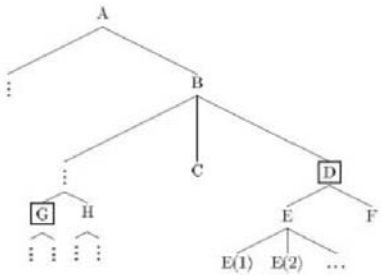
The hierarchy of attributes leads to several notions of degree of resemblance as well. To my knowledge, in the theory of information systems, the idea of hierarch of attributes has not been worked sufficiently yet. The attribute systems introduced so far work with one level of attributes and introducing the hierarch requires a lot of work. However, I believe that the introduction of higher order attribute systems will enable us to represent several important ontological ideas in the information systems.

Working with classes of attributes without mentioning properties we get a coarse but significant notion of degree. Let $A_0 \subseteq At$

(iv) $\Theta_{A_0}^3(o_1,o_2) \leq \Theta_{A_0}^3(o_3,o_4)$

if $min\{lev(A) : A \in A_0, \{o_3,o_4\} \subseteq A^{**} \leq min\{lev(A) : A \in A_0, \{o_1,o_2\} \subseteq A^{**}\}$

As an illustration of the notion consider the following diagram



This notion of degree corresponds to the determinable-determinate relation between universals. o_3 is closer to o_4 than o_1 is to o_2 if the lowest common determinable of o_1 and o_2 is lower than the lowest common determinable of o_3 and o_4.

These notions of degree are far from exhausting all philosophically significant possibilities. There are others including the combinations of those given. Moreover, even a complete list of degree notions will not bring a complete

order on the class of resembling objects. This is not a deficiency of the theory of the theory but a fact about resemblance. Even if we accept, for a moment, that resemblance is an undefinable primitive notion, this would remain as a fact about resemblance. Thus we should not except a definite answer for any two pairs $(o_1, o_2), (o_3, o_4)$ as to which pair has a closer resemblance. The reasonable constraint would be to cover unproblematic cases: if we think that $(o_1, o_2), (o_3, o_4)$ are comparable with respect to resemblance then the theory should explain why.

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Mereological and Causal Decompositions of Action

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1. Introduction

The postulation of actions as the individual entities quantified over in action sentences is the basis of the so-called event analysis, and the core tenet in Davidson's early and late philosophy of action.[1] Such an enduring concern with the unity of action survived even the discovery that sets of action descriptions once believed to co-refer are really satisfied by bundles of distinct complex events. The suggestion that such events *must* have a common origin, and that this *can* be a unified action (for example, a bodily movement) put new appeal on the mereology of actions.

The idea of understanding agency through one or another kind of decomposition of action must have arisen early. We know that according to Hobbes voluntary actions could be construed as the result of three phases: the first is passions as motivators for action; the second opinion-making through deliberation; the third is the will regarded as "the trigger at the end of the volitional sequence".[2] But there were also those, like Spinoza, who insisted on the distinction between the

[1] Cfr. D. Davidson, 'The Logical Form of Action Sentences', reprinted in *Essays on Actions and Events*, New York: Oxford University Press, 1980; 'Adverbs of Action', in B. Vermazen e M.B. Hintikka (ed.), *Essays on Davidson. Actions and Events*, New York: Oxford University Press, 1985; 'Aristotle's Action', reprinted in *Problems of Rationality*, New York: Oxford University Press, 2004.

[2] Robert Orr, 'Hobbes on the Regulation of Voluntary Motion', in G. Feaver (ed.), *Lives, Liberties and the Public Good*, London: MacMillan, Basingstoke, 1987, p. 49. Such an analysis helped Hobbes to solve "the determinism/free will conundrum. All the elements of voluntary motion are determined; but when assembled in order they compose a voluntary action, acquiring thereby properties not held by the separate components" (Ibid.).

active and passive behaviour but made no attempt to draw that distinction by decomposing behaviours into phases or temporal parts.[3]

In the last decades those who find it helpful to decompose the action to understand the nature of agency are majority. But the event analysis is often made appear untenable by the appeal to an unprincipled mereology of complex events. Untutored intuitions concerning the "springs" of bodily movements led some to believe that real actions ought to be located far before the overt movements you can observe when I raise my arm. The issue has then become that of deciding whether actions should be located at the peripheral level *or* by the inner events supposed to cause those moves. On the former, actions are identical with bodily movements, and it is an open question whether the physiological events supposed to shape the movements are part of the action or not. On the latter, the items which operate the analysis are *tryings*, where a trying is an event identified with the initial part of the action or, by some, with the action itself. Both the notion of an action as the peripheral movement and the notion of a trying as the initial (common) phase of the complex events we make happen face a number of problems, and this paper is an attempt to define more sharply their nature and source.

2. The peripheral view

Davidson's notion of actions as overt movements is among the most instructive positions on the matter. According to the peripheral view, as I will call it, my moving of my arm is identical with my arm's moving. Take the familiar distinction between transitive and intransitive forms of 'move' as applied to our bodies: to endorse the peripheral view is to say that bodily movings$_T$ are simply identical with bodily movings$_I$.

Recall Wittgenstein's question: "what is left over if I subtract the fact that my arm goes up from the fact that I raise my arm?".[4] A traditional answer to this question was that what is left over is the fact that I will my arm to go up, but according to Robert Jaeger Wittgenstein was not bent on solving the problem but on understanding why it arises and why we are free to reject it.[5] For Hugh McCann, "We would have at least part of the answer… if we could explain what it is, when a person raises his arm, that makes the motion of the arm the result of an action of arm-raising".[6] Also for Davidson the question made full sense, and after minor recasting it can receive a clear answer.[7] The recasting involves changing addition

[3] To the annoyance of scholars, Spinoza often speaks as if the distinction between action and desire (its cause, according to some) could be set aside as unimportant when it comes to understanding the nature of agency. See for example, Jonathan Bennett, *A Study of Spinoza's* Ethics, Cambridge: Cambridge University Press, 1984, pp. 259-261.

[4] Wittgenstein, *Philosophical Investigations*, § 621.

[5] Robert A. Jaeger, 'Action and Subtraction', *Philosophical Review*, 82, 1973, pp. 320-329.

[6] H. McCann, 'Volition and Basic Action', *Philosophical Review*, vol. 83, 1974, p. 453.

[7] What was Wittgenstein's answer is another question. As Fodor put it, "It was illuminating of Wittgenstein to ask what makes a mere event… an action. But he gave the wrong answer; at least he did as I read the text. (It's notorious that with Wittgenstein you can hardly ever tell how to read the text…)

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for subtraction, and replacing the ontology of facts with an ontology of events as the entities quantified over in action sentences. When we ask the new question, what must be added to my arm's going up to make it my raising of my arm,[8] Davidson's answer is "nothing", because when I raise my arm and my arm thereby goes up "nothing is added to the event itself that makes it into an action".[9]

The first thing to notice is that in this case, like in Wittgenstein's, the issue is not what happens when someone acts (Davidson is not making the weaker claim, "when I raise my arm, my arm goes up"[10]) but what it *is* to be a raising of one's arm.[11] As we may also see, there is no mention here of the physiological events supposed to shape bodily movements, but Davidson is not denying their causal role; his point is that they are no part of the action.

> When I say nothing has to be added to my arm going up to make it a case of my raising my arm, I don't mean no further conditions have to be satisfied to ensure that the rising of my arm is a particular case of my raising my arm; this much is obvious, since it can easily happen that my arm goes up without my raising it. But this is an addition to the description we give of the event, not to the event itself.[12]

Arm raisings differ from arm risings in that the former but not the latter are necessarily caused by some appropriate mental state (an intention, a reason, a desire, etc.) and according to Davidson the difference between active and passive behaviors (he dubs them "doings" and "sufferings") is not "endangered if we allow that some arm risings are arm raisings". The possibility "that my arm goes up without my raising it" shows that that difference *can* be expressed by the words, and Davidson seems to conclude that it is exhausted by those words. Still, that there must be more than descriptions to the difference between arm raisings and arm risings is at least suggested by one difficulty I would like to dwell upon.

Apparently, Wittgenstein's view is that to understand a movement as an action, rather than a mere happening, is to understand it as *contextualized* in a certain way… To understand an arm's rising as someone raising his arm is to set in a context that may include, but needn't be exhausted by, the agent's past and future actions and intentions. No doubt this account would be question-begging if a reductive analysis were the goal; but surely, in Wittgenstein, it's not". Jerry A. Fodor, '*Déjà vu* All Over Again: How Danto's Aesthetics Recapitulates the Philosophy of Mind', in M. Rollins (ed.), *Danto and His Critics*, Cambridge (Mass.): Blackwell, 1993, pp. 43, 44.

[8] D. Davidson, 'Problems in the Explanation of Actions', reprinted in *Problems of Rationality*, New York: Oxford University Press, 2004, p. 101.

[9] D. Davidson, 'Problems in the Explanation of Action', p. 102.

[10] The question what is an action is clearly distinct from the question what happens when someone acts. See Geert Keil, 'How Do We Ever Get Up?', *Grazer Philosophische Studien*, vol. 61, 2001, for an account that brackets the former question to concentrate on the latter.

[11] "[O]f the many individual events that are risings of my arm, some are cases of my raising my arm; and none of the cases of my raising my arm are events that include more than my arm going up. Nothing is added to the event itself that makes it into an action" (D. Davidson, 'Problems in the Explanation of Action', p. 102).

[12] D. Davidson, 'Problems in the Explanation of Action', p. 102.

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On the peripheral view, my raising of my arm is identical with my arm's going up. Physiology tells us that bodily movements are brought about by muscle contractions (not the other way round). So when I raise my arm, my arm's going up is caused by some appropriate muscle contractions. It would be no objection to contend that such physiological events may not be there when my arm goes up without my raising it, for what is at issue is the case where my arm goes up because I raise it. The difficulty I want to point out is that the physiological events (whatever they are) that are part of what causes my arm to go up when I raise it are *not* allowed to cause my raising of my arm, notwithstanding the fact that by hypothesis "my raising of my arm" names the same as "my arm's rising".

Davidson's version of Wittgenstein's question was designed to urge that the difference between arm risings and arm raisings is explained by placing the necessary conditions of action within the causal antecedents that rationalize the action (intentions and belief-desire pairs). The problem is that the cerebral and other physiological events that shape the overt movement are left no role between the peripheral action and the propositional causes. What is puzzling is that the difficulty cannot depend on the causal relation, which holds regardless of how we describe the relata: "being an action is a trait which particular events have independently of how they are described".[13]

One reaction is to ascribe the asymmetry to the intensionality of action descriptions: when my arm's going up is described as "my raising of my arm", it 'refuses' some of the proximal causes that make it occur. It is not clear, however, that action descriptions are intensional or "quasi-intensional" in causal statements.[14] A more plausible solution is to allow for the conclusion that my raising of my arm, just as my arm's going up, was caused by certain muscle contractions. Otherwise, a simple application of Davidson's criterion for the identity of events would yield that my raising of my arm *cannot* be my arm's going up, since they have different causes: while we can say that some cerebral events caused my arm's going up, we cannot say that they caused my raising of my arm.

It should be noted that Davidson's early strategy with the physiological events that shape our movements was to identify the movements with whatever we do that causes our body to move. This position resists the view of actions as processes while making a first step towards a causal decomposition of bodily movements. As Davidson put it: "It may be true that I cause my finger to move by contracting certain muscles, and possibly I cause the muscles to contract by making an event occur in my brain. But this does not show that pointing my finger is not a primitive action, for it does not show that I must do something else that causes it. Doing something that causes my finger to move does not cause me to move my finger; it *is* moving my finger".[15] This argument, though, goes too close to counting the bodily movement among the early effects of the action. Unless we read

[13] D. Davidson, 'Agency', reprinted in *Essays on Actions and Events*, p. 47.

[14] Cfr. D. Davidson, 'Actions, Reasons, and Causes', reprinted in *Essays on Actions and Events*, p. 5. Here I read Davidson as saying that action descriptions are quasi intensional *only* in rationalizations or other explanatory devices, i.e., that the quasi intensional character is due to explanation, not description.

[15] D. Davidson, 'Agency', pp. 49-50.

Davidson as saying that the action includes the movement *plus* whatever we do that causes it, a criticism can be leveled against his leaving out of the action the overt movement itself. The only way to prevent this objection is to read Davidson as saying that the action is identical with the referent of "[my] doing something that causes my finger to move" as a whole, where this is viewed as a complex event.[16] But in this case the sum from which we are asked to subtract my arm's going up has become my doing *something* that causes my arm to go up.

This slim addition should make us receptive to the answer which is nowadays predominant, the view that paradigmatic actions are (originated by) tryings.

According to Adams and Mele, "What Wittgenstein subtracts is the success of the trying. A successful trying to raise one's arm does involve the bodily actions that partially constitute the action of arm raising. Hence, when the bodily motions are subtracted the successful trying vanishes with them. But… even when no such motions occur, an unsuccessful trying may remain".[17] As an answer to Wittgenstein's question, tryings fare better than intentions in action, wishes and efforts, for the notion of trying emphasizes the temporal relation over the causal one. More than what causes the accomplishment, a trying is what precedes it. While it is not clear that my raising of my arm comes before my arm's rising, it is assumed that my trying to raise my arm comes before my arm's going up.

3. Tryings

How do we reach the notion of a trying as the common, initial part of the complex events we bring about when we act? Semantics provides both the need and the clue. One believed that "Booth pulled the trigger" and "Booth shot Lincoln" could be about the same event; but an elaboration of Davidson's analysis in terms of *thematic roles* proves that Booth's pulling of the trigger cannot be identical with Booth's shooting of Lincoln, for these events have different Patients.

1. ∃e[Pulling(e) & Agent(e, Booth) & Patient(e, the trigger)]

2. ∃e[Shooting(e) & Agent(e, Booth) & Patient(e, Lincoln)]

The friend of unified actions can insist that the action is the common, initial part of the various events in the bundle. Thus according to some the 'action' ought to be introduced as a way to make sense of the notion of an Agent of various complex events. This yields the notion of an action as a coarse-grained event

[16] In a recent article Benjamin Mossel urged that Davidson here "subscribes to inconsistent theses", since the above identity *plus* the so-called identity thesis seem to imply that we cause our actions, and in the same paper Davidson explicitly denied that we cause our actions. Cfr. B. Mossel, 'The Individuation of Actions', *Australasian Journal of Philosophy*, vol. 79, 2001, p. 272. This charge may misfire for in the quoted passage we are forced to read Davidson as endorsing no identity theses besides the claim that "doing something that causes my finger to move… *is* moving my finger".

[17] F. Adams and A.R. Mele, 'The Intention/Volition Debate', *Canadian Journal of Philosophy*, vol. 22 (3), 1992, p. 337.

constituting the initial part of a bundle of finer-grained events. The harmless assumption is that there is no agency over complex events unless there is agency over less complex ones. To defend the unified view while acknowledging the multiplicity of complex events, a number of philosophers invoked the ambiguity of action descriptions and put forward dual-reference accounts of them. A version of this was proposed by Paul Pietroski.[18]

Pietroski acknowledges that the referents of apparently co-referring action descriptions are distinct complex events. His program is to find a way to make room for Davidsonian unified actions as well. As Pietroski put it: "the important point is that 'Booth's shooting of Lincoln' is ambiguous".[19] This description could be used in referring to the complex event (which has Lincoln as its Patient) or the action that *initiates* the event. To avoid ambiguities we need subscripts. Thus even if the$_E$ shooting is not the$_E$ pulling, the$_A$ shooting *can* be the$_A$ pulling. Notice that the non-identity in this formula has nothing to do with universals, classes or predicates. No appeal is made here to type-token distinctions as explicative of, or relevant to, the contrast between identity and non-identity statements. It is the complex events satisfying the relevant sortals (pulling, shooting) *and* their adjunct phrases (with-a-finger, with–a-pistol) that differ from one another. But what *is* an action? What is it that the$_A$ pulling and the$_A$ shooting could have in common? Pietroski urges, following Hornsby,[20] that paradigmatic actions (e.g., bodily movements) can be thought of as *tryings*. Instead of discussing Pietroski's arguments for this claim, which are also borrowed from Hornsby, I would like to explain why I take tryings to be most unfitted to Pietroski's program.

According to Pietroski, trying to V "is doing *something*".[21] And on what I see a natural view, trying to V is doing something *with the goal* of performing the action (type) expressed by 'V'. In my opinion, the goal plays an essential ontological role, by providing the trying with its constitutive end. The words to clarify are "end" and "constitutive".

END. Both the verb 'try' and the complex predicate 'try to V' pass the standard tests for Activity predicates: they can be modified by *for*-adverbials but not by *in*-adverbials, and they can appear in the progressive. Moreover, while Accomplishments are characterized as telic, for they have a natural completion point, Activities are atelic, for they lack a natural completion point.[22] Although *tryings* are activities, they have nonetheless an inherent end. Thus trying to find one's passport is doing something with the goal of finding the passport, whether or not the agent really intends to find it. According to some philosophers,[23] it is not clear that the intention to try implies an intention to do the thing; so it may well be

[18] Paul M. Pietroski, 'Actions, Adjuncts, and Agency', *Mind* 107 (425), 1998 (reprinted in *Causing Actions*, Oxford: Oxford University Press, 2000. Page numbers in this paper apply to the *Mind* edition.)

[19] Paul M. Pietroski, 'Actions, Adjuncts, and Agency', p. 82.

[20] Jennifer Hornsby, *Actions*, London: Routledge and Kegan Paul, 1980.

[21] Paul M. Pietroski, 'Actions, Adjuncts, and Agency', p. 97.

[22] Zeno Vendler, *Linguistics in Philosophy*, Ithaca (N.Y.): Cornell University Press, 1967.

[23] Cfr. Frederick Adams, 'Trying, Desire, and Desiring to Try', *Canadian Journal of Philosophy*, vol. 24 (4), 1994.

the case that in order to try to find one's passport is not necessary to really intend to find it. Whatever one's view on this, my point is that the trying *has* the finding of the passport as its constitutive end, and this end must be fulfilled for the trying to succeed.

Generally speaking, the end of a trying to 'V' is the performance of an action belonging in the action type expressed by 'V'. So depending on the aspectual class of the embedded verb 'V', 'performing' that action will mean different things, and these differences have bearings on the fulfillment conditions of the trying's end.

If the embedded verb is an Achievement predicate (like *find a book*), the action-result (the finding) must obtain for the trying to succeed. If it is a State (like *remain calm*), the state must last unchanged for some given period of time.[24] When the embedded verb is an Accomplishment predicate (like *climb a mountain*), the action must be *completed* for the trying to succeed. The action to perform has a natural *completing point*, and this fixes the fulfillment conditions of the trying's constitutive end. (This end can be further specified by means of *in*-adverbials and adjunct phrases.) When the embedded verb is an Activity predicate (like *walk*), the action must simply *take off* for the trying to succeed. Since the action to perform has no natural *completing point*, the trying's end is fulfilled as soon as the action takes off convincingly enough to say that the act is being performed. Thus the constitutive end of a trying to walk is fulfilled (and the trying successful) as soon as the agent performs the first (two or three) steps.[25] Adding *for*-adverbials (and adjuncts) can serve to further specify the end of the trying, and this may radically change its fulfillment conditions. To give but an example, take an Activity predicate like *dance*, and suppose to make it the end of a trying. If the end of "Mary's trying to dance" is fulfilled as soon as Mary performs some adequate moves, the end of "Mary's trying to dance for 10 hours" requires completion of a task.

CONSTITUTIVE. A trying does not require the fulfillment of its end to occur, but to succeed. If the trying is to be clearly distinguished from other (co-occurring) tryings, the end must be fully specified by the content phrase of the try sentence (and nominalization). When it comes to the content property of a trying's content we ought not to allow for the partial connotation like that affecting other non-try nominals.[26] According to the general conception of the names of tryings I have been assuming so far, the content phrase of a proper trying's name expresses its whole content, and therefore it fully expresses its constitutive end. This is why I said that the end of a trying (and its fulfillment conditions) are "constitutive" of the trying's being the very trying it is.

We must assume that the end is fully expressed (by the trying's content phrase) because unless the fulfilment conditions of a trying are not clearly distinguishable from those of other tryings, there is no way to tell succeeded

[24] This could account also for negative tryings, like John's trying not to go, not to fall, not to die.

[25] The end of a trying to walk may be fulfilled from the very first steps, but the end of a trying to take a walk may require more.

[26] Jonathan Bennett, *Events and Their Names*, Oxford: Clarendon Press, 1988, Ch. 5.

tryings from failed ones. Perhaps we should make room for the notion of 'impossible tryings' (like Paul's trying to square the circle) and of tryings whose outcomes we will never be in a position to ascertain (like Sara's trying to make a bottle float to Sidney), but setting aside aberrant cases, there would be no point in holding a notion of *a* trying (that is, of an individual trying) such that it is *constitutively* impossible to tell success from failure.

4. Results

The discussion so far suggests that the following principle of identity for (co-occurring) tryings could be adequate: T_1 is identical with T_2 *if only if* the fulfilment conditions of T_1's end are the same as the fulfilment conditions of T_2's end.

I know of no way to avoid the conclusion that the referents of try descriptions are more finely cut than the complex events satisfying the parent action descriptions. Thus the referent of "John's trying to drink a beer" is not identical with the referent of "John's trying to drink a beer in 10 seconds"; consequently and more importantly, it doesn't follow from the fact that John tried the latter only once that he also tried the former, even if it is true that a success in doing the latter would entail the success at the former, in case the former were to be explicitly undertaken.

We saw that dual-reference accounts of action descriptions were designed to maintain that the$_A$ shooting *can* be the$_A$ pulling while conceding that the$_E$ shooting is not (and could not be) the$_E$ pulling. So where logical forms yield a multiplicity of fine-grained complex events, a philosophical concern with the unity of agency fuels the search for coarser-grained actions that could be assigned the role of unifying the various complex events. Now, the first lesson that the analysis of trying teaches us is that there is no reason to believe that tryings can play this wanted *unifying* role, for in no way Booth's trying to shoot Lincoln can be identical with Booth's trying to pull the trigger, as Pietroski and others seem to hold. This seems (to me) to defeat the hope to reconcile the unity of coarse-grained actions with the old attempt to reduce (paradigmatic) actions to tryings. In other words, it doesn't follow from the fact that a trying can be identified with the initial part, or phase,[27] of a complex event (say, an action) that every complex event we bring about is such that its initial phase could (or should) be identified with a trying.

One last point on the success of a trying. An agent's trying to V is said to succeed when the trying evolves into a V-event, that is, an action that meets the fulfillment conditions of the trying's constitutive end. A different matter is the event which culminates the action, for example, my arm's going up. This event is an effect of the trying and the result of the action.[28] What is then the *success* of a

[27] See Yael Sharvit, 'Trying to be Progressive: the Extensionality of *Try*', *Journal of Semantics*, vol. 20 (4), 2003, for the distinction between temporal parts and phases.

[28] According to Hugh McCann "Results are events which are necessary for those actions whose results they are". Cfr. H. McCann, 'Volition and Basic Action', *Philosophical Review*, vol. 83, 1974, p. 452.

trying? One proposal is to say that T is successful when T causes a result which meets the fulfillment conditions of the trying's end. The first thing to note is that "T caused R" could introduce no event over and above the mereological sum of T and R.[29] If this is true, then the occurrence of R (perhaps from T) is not sufficient for the success of T. To give necessary and sufficient conditions would require us to tell *how* T must cause R in order to succeed, and this is known to be a messy issue. One way to eschew it is to deny that the success of a trying is the culminating event of the action (as Mele and Adams seem to hold), and to view the success of T as the fact that T developed into a V-event. In other words, "that T developed into a V-event" is the name of a fact, and there may be no event of a trying developing into an action, for such an event would have another event as participant.

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[29] As Davidson would put it, there is no such thing as T's *causing* of R. Cfr. D. Davidson, 'Problems in the Explanation of Action', p. 103.

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Four-Dimensionalism and Modal Perdurants*

Jiri Benovsky

Abstract. This paper is about persistence of material objects through time and across possible worlds. It starts with the well-known argument from undetached parts, that is put as an objection to endurantism raised by four-dimensionalists who claim to have a nice treatment of it themselves. While it will be acknowledged that, indeed, four-dimensionalism has a good explanatory power here, and has an advantage over endurantism, we will then see a modified (modalized) version of the argument that will not be so easily dismissed by the four-dimensionalist. To provide a solution to this second puzzle, a proposal will be made to use the four-dimensionalist's strategy in the case of modality and use this notion of perdurance across possible worlds to answer the modalized version of the objection. Finally, I examine some objections to this theory of modal perdurants, and try to answer them.

1. Introduction

Four-dimensionalism in its most standard form is a doctrine consisting of the conjunction of two thesis : the perdurantist claim that material objects persist through time by being temporally extended, that is, by having different temporal parts at different times, and the eternalist claim that there is no ontological difference between past, present, and future times (and the objects that exist at those times) – all are equally real. It has often been argued[1] that a good reason to adopt four-dimensionalism is that it nicely solves a family of puzzles involving problems with coincident entities that yield difficulties for the competing views, especially *endurantism*. Endurantism is the doctrine that denies the first of the two

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[1] See for instance Sider (2001).

four-dimensionalist thesis : endurantists claim that material objects persist through time by being wholly located at different times.

In this paper, I wish to concentrate on one puzzle case raised against endurantism by four-dimensionalists, and on the treatment they themselves provide to it. It will appear that, while four-dimensionalism posses a genuine advantage over endurantism here, its own solution does not give complete satisfaction neither. A proposal will then be made to push the four-dimensionalist's strategy even further to yield a view that will deal with the puzzle case at best.

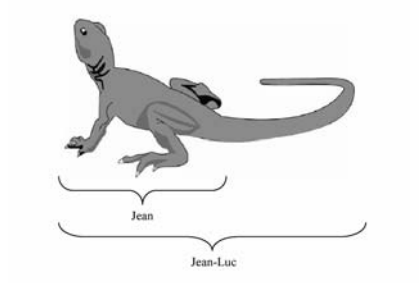
2. The undetached parts argument

So this is where the story begins : with the famous argument from 'undetached parts' against endurantism[2]. The argument goes as follows.

It is well known that some lizards can release a part or all of their tail when they are grabbed by a predator. Once the tail is broken off, the lizard quickly runs for shelter and is safe for the moment, leaving a squirming tail to confuse or distract the predator. Let us imagine that, at a certain time t, such an adventure happens to my pet lizard Jean-Luc. Certainly, Jean-Luc can undergo such an accident and survive it, so, favouring trans-temporal numerical identity, the endurantist would claim that

(1) Jean-Luc-before-t is numerically identical to Jean-Luc-after-t

But, the argument goes, there is, before t, an other object that is *the whole of Jean-Luc except its tail* – it is what can be called an *undetached part of Jean-Luc*. Let us call this thing, this part of Jean-Luc, simply, "Jean" (and we could call "Luc" its undetached part that consists in its tail), as pictured here :



Certainly, Jean, as well as Jean-Luc, survives the predator's attack, but then, according to endurantism it is to be accepted that

[2] A very clear exposition of the argument that I follow here at the beginning of my discussion of it is to be found in Loux (1998, p. 222-231). Discussions can be found in Heller (1990, p. 2-4; 19-20), Sider (2001, p. 142; 152-153), and Van Inwagen (1981, p. 79-84) who really introduced the argument (but for a different purpose, see below).

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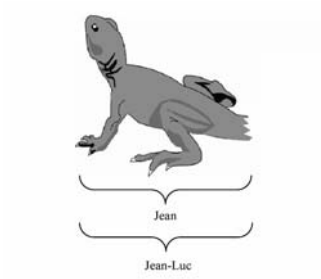
(2) Jean-before-t is numerically identical to Jean-after-t

If we don't want to endorse coincident entities, we'll have to say that after t

(3) Jean-Luc-after-t is numerically identical to Jean-after-t

for they occupy exactly the same space-time location and seem to be, in short, just one and the same thing :



But, by transitivity of identity, we get from (1), (2) and (3) that

(4) Jean-Luc-before-t is numerically identical to Jean-before-t

which is obviously false.

Here is now the complete argument in a more structured form :

(i) Before a time t, Jean-Luc had a tail. At t, he released his tail to escape a predator's attack.

(ii) Before t, there was un 'undetached' spatial part of Jean-Luc that is the whole of Jean-Luc except its tail. Let us call this thing "Jean".

(iii) Jean-Luc survives the accident, but doesn't have a tail any more after it.

(iv) Jean survives the accident, because nothing really happened to him.

(v) After t, Jean-Luc and Jean occupy the same spatio-temporal region, are made up of the same particles, have the same mass, shape, colour, …

(vi) So, after t, Jean-Luc and Jean are numerically identical, if we don't want coincident entities. (Jean-Luc-after-t is numerically identical to Jean-after-t)

(vii) Endurantist treatment of (iii): Jean-Luc-before-t is numerically identical to Jean-Luc-after-t.

(viii) Endurantist treatment of (iv): Jean-before-t is numerically identical to Jean-after-t.

(ix) But then, by transitivity of identity, it turns out that Jean-Luc-before-t is numerically identical to Jean-before-t – which is false.

(x) So, the endurantist assumption from (vii) and (viii) should be dropped, and thus, endurantism fails.

As mentioned, a way to avoid this conclusion would be to claim that (3) from above and (vi) from the argument are false because, after t, there are really *two* coincident non-identical things : Jean-Luc and Jean, but this line of response is, according to the objector, unpalatable – what we want is to provide a solution that does not force us to embrace coincident entities. So, the endurantist who is a friend of non-coincidence is in trouble, it seems.

The four-dimensionalist, on the other hand, has a nice treatment of this case – she will, of course, reject (vii) and (viii) while accepting (iii) and (iv). There are, on her account, two *space-time worms* – one of them ($Worm_{BIG}$, on the figure below) is the aggregate composed of Jean-Luc-before-t and the (one) thing that is there after t (call it "Jean-Luc-after-t" or "Jean-after-t" – it does not matter since those two names have the same reference), and the other ($Worm_{SMALL}$, on the figure below) is the aggregate composed of Jean-before-t and, again, the (one) thing that is there after t. The aggregates (the space-time worms) are not identical (because, for instance, they do not occupy the same space-time region) but they share a post-t temporal part. So, coincidence is avoided *via* the explanation that uses the idea that different material objects can share a part, be it a spatial part (like in the case of Siamese twins) or a temporal part (like in the case just depicted). So here is how this case looks under four-dimensionalism :

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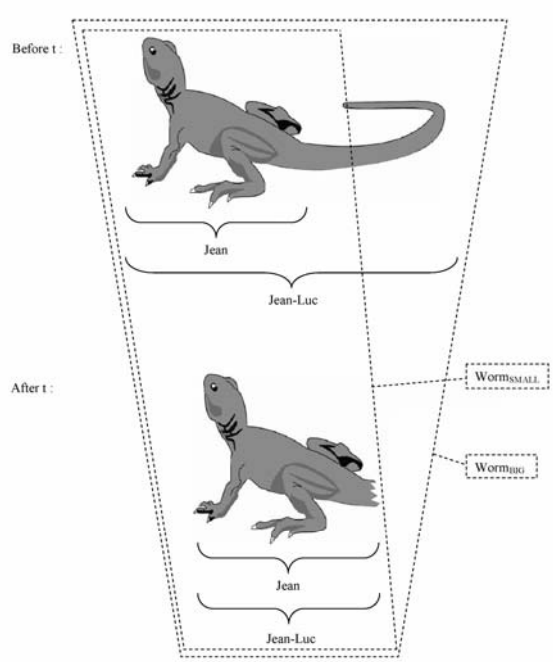


Of course, endurantists also have replies to this objection. Peter Van Inwagen (1981, p. 82) claims that this argument does not yield a problem for endurantism but for the idea that there are 'undetached' temporal parts. Indeed, as a *reductio* not against endurantism but against the existence of undetached parts, the argument can be formulated in exactly the same way we have seen above, and only the conclusion must be modified – (x) will simply become :

(i) So, there is no such thing as Jean-before-t (a spatial 'undetached' part of Jean-Luc).

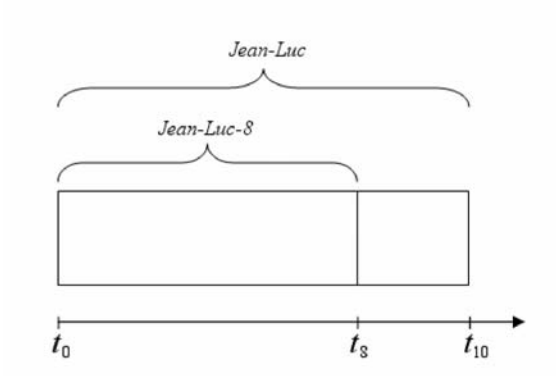
So it seems that it is only a matter of which of the premises one chooses as being the 'absurd' hypothesis in the *reductio*, to save the case of endurantism. But it certainly is not that simple : not any premise can so simply go away. And this is why I believe that Van Inwagen's strategy here fails – for how plausible is it to deny premise (ii) and to claim that Jean-before-t does not exist ? Or, to take a more stringent example, how plausible is it to claim that my hand does not exist – or, as Van Inwagen would put it, that it is not an object ? Such a claim certainly does seem to fly in the face of common sense. And if the only justification for it were to save the case of endurantism here, then the claim would certainly be very ill-motivated. Van Inwagen, of course, has independent reasons for accepting such a claim (see Van Inwagen (1990)), and there also are independent reasons to reject it – the main of these being accepted, and endorsed, by Van Inwagen himself : it

entails a commitment to metaphysical vagueness because it makes composition to be vague. Added to the common sense's incredulous stare, this price one has to pay for the claim that undetached parts of objects are not real existing objects seems to me to make Van Inwagen's reply to the argument against endurantism deeply unappealing.

Another line of reply to this argument could be given by followers of David Wiggins (2001) who recommends a view that permits to embrace coincident entities. While, as in the case of Van Inwagen's reply, accepting genuine coincidence seems to me a remedy that is much worse than the disease, it is not the purpose of this paper to examine this (and other) replies, rather, it will now be interesting to see that a slightly modified version of the argument from undetached parts will not be so easily dismissed by the *four-dimensionalist*.

3. The modal version of the argument

There is a modalized version of the argument from undetached parts that goes as follows.



(i)' Jean-Luc actually lived for 10 years (from t_0 to t_{10}).

(ii)' There is an 'undetached' temporal part of Jean-Luc that existed from t_0 to t_8 – it lived for 8 years only. Let us call it Jean-Luc-8.

(iii)' Jean-Luc could have lived only 8 years. (There is a possible world, W, where Jean-Luc is killed at the age of 8.)

(iv)' Jean-Luc-8 could exist in W with the same temporal extent it actually has.

(v)' So, in the possible world W, Jean-Luc has the same temporal extent as Jean-Luc-8 (as well as the same spatial extent, material constitution, shape, colour, …).

(vi)' So, in W, Jean-Luc is numerically identical to Jean-Luc-8.

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(vii)' Jean-Luc from the actual world is numerically identical to Jean-Luc from W.

(viii)' Jean-Luc-8 from the actual world is numerically identical to Jean-Luc-8 from W.

(ix)' But then, by transitivity of identity, Jean-Luc from the actual world is numerically identical to Jean-Luc-8 from the actual world – which is false.

(x)' So, there is no such thing as Jean-Luc-8 (a temporal part of Jean-Luc).

And so, if there are no temporal parts, four-dimensionalism is false.

This objection was developed by Peter Van Inwagen in (1981), and discussed and answered by Ted Sider's (2001) and Mark Heller's (1990). Sider's response makes use of modal counterpart theory, while Heller appeals to his ontology of conventional objects. Each of those replies has its advantages, and each suffers from different drawbacks. It is the purpose of this paper to offer an alternative to those solutions, an alternative that is directly inspired by the four-dimensionalist's own strategy and so should be at least appealing to those four-dimensionalists who do not whish to embrace modal counterpart theory or Heller's conventional objects.

4. Modal perdurants

The key to the solution is the theory of modal perdurants which is the modal analogue of the four-dimensionalist's 'space-time worms' view : exactly as the four-dimensionalist is an eternalist (temporal realist) who claims that objects (lizards, tables, …) are extended in time and that they exist at different times by having different temporal parts, the friend of modal perdurants is a modal realist who claims that such objects are also extended across possible worlds and exist at different worlds by having different parts there – different modal parts or *world-stages*. Such a theory could thus be called *five-dimensionalism* since it takes objects to be extended across the three dimensions of space, the one dimension of time, and to have also extension across possible worlds – they are aggregates of spatial, temporal, and modal parts. (It is not clear whether possible worlds could play a role of a dimension, even if one were ready to embrace modal realism, so the label 'five-dimensionalism' should be taken as no more than a label.)

Objects like Jean-Luc thus genuinely *persist* across possible worlds. According to this view, Jean-Luc is an individual that stretches across two (or more) different possible worlds – that has parts in two different worlds – and thus, exists at two different worlds. In one of them, W_1, he has, for instance, a big tail, and in the other, W_2, he has a small one – to speak more clearly, he has in W_1 a world-stage that has a big tail and he has in W_2 a world-stage that has a small one – and this is how he manages to have different modal (accidental intrinsic) properties, analogously to the way the four-dimensionalist claims that he has his temporary intrinsic properties. And as in the temporal case, no contradiction can arise here from the fact that those properties (having a big tail, and having a small

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tail) are incompatible since they are had by different things – the different modal parts of Jean-Luc. (The theory thus elegantly avoids any difficulties that are often raised against trans-world identity views.)

But note that this theory is, in a derivative sense, a theory of genuine trans-world identity, since the modal worm Jean-Luc is always, as all objects are, identical to itself. And so there is an object, Jean-Luc, that exists at a world W_1 and that is the very same object, Jean-Luc, that exists at a world W_2 – but one must carefully take notice that the object that exists *in* W_1, namely the W_1-stage of Jean-Luc(-the-whole), is not identical to the object that exists *in* W_2, namely the W_2-stage of Jean-Luc(-the-whole). This is why, while not denying their existence, David Lewis calls such trans-world objects *impossible* individuals since they do not exist wholly in any *possible* world (1986, p. 211)). But this critique is not really one and is only a matter of terminology : one must simply make here a distinction between "existing *at* a world" and "existing (wholly) *in* a world" where for an object to exist at a world, it is enough for it to exist only partly there. Such a distinction is analogous to the four-dimensionalist's claim that objects do not exist wholly at a time but are said to exist at that time by having a temporal part there.

5. Modal perdurants and the Kripke objection

Before examining how this view deals with the modal objection to four-dimensionalism, it is worth noticing first one advantage of the modal perdurants view : it genuinely answers the problem that leads sometimes to rejection of one of its main competitors, that is, modal counterpart theory, which suffers from the famous Saul Kripke's objection[3]. This objection appears because when analyzing *de re* modal statements like "I could have had an accident" it turns out, under modal counterpart theory, that such statements are not really about me since its counterpart-theoretic paraphrase is "In W, there is a counterpart of me that has an accident" – but since a counterpart of me is just not me, and is only someone that resembles me, I could not care less, the objection goes, for what happens to *him* in another possible world. There are many people even in our own world that resemble me, but I do not care for them having an accident in the same way I care about me having one or me possibly having one. And this just shows that counterpart theory goes wrong when explaining why I have such and such modal properties in terms of other people in other worlds having those properties.

Compare to the theory of modal perdurants : the five-dimensionalist's simple proposal here is that I am an aggregate not only of my spatial and temporal this-worldly parts but also of my other-worldly parts. And so it is clear that any modal *de re* claim about me is really, genuinely, about *me* – if I could have had an accident then it is really a part of me, a world-stage of me, that has it. Once more, this parallels what the four-dimensionalist's 'worm-view' claims about my having of temporary properties : I have had an accident because a past temporal part of me

[3] Kripke (1972, p. 45).

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has it (in an atemporal sense of the verb), and I will have an accident iff one of my future temporal parts has one.

All this is very much like the spatial case when I say that I pick up a pen, while of course it is only a spatial part of me, my hand, that performs the job. And more, says the four-dimensionalist, it is a spatial part of a temporal part of you that performs the job. And more, says the five-dimensionalist, it is a spatial part of a temporal part of a modal part of you that performs the job. Look at Jean-Luc : he has a big tail – but, could he have a small tail ? Not under counterpart theory, as the objection tries to show. But, under the modal perdurants view, obviously, *he* (he himself) can.

6. Reply to the modal version of the undetached parts argument

Let us turn now to the modalized objection to four-dimensionalism. As we shall see, it is easily answered on the modal perdurants view, and so this reply could be seen as a motivation for four-dimensionalists to endorse five-dimensionalism (modal perdurants in addition to temporal perdurants) since it helps their own business in the temporal case. Here is the five-dimensionalist's reply to the objection :

The argument succeeds because it presupposes genuine trans-world identity, without modal perdurants. It is first assumed that

(1)' Jean-Luc-$W_{@}$ (Jean-Luc from the actual world) is identical to Jean-Luc-W

and, similarly, that

(2)' Jean-Luc-8-$W_{@}$ is identical to Jean-Luc-8-W

when it is said that the actual Jean-Luc could have had a shorter life and that Jean-Luc-8 could have the same length of life in another possible world (W).

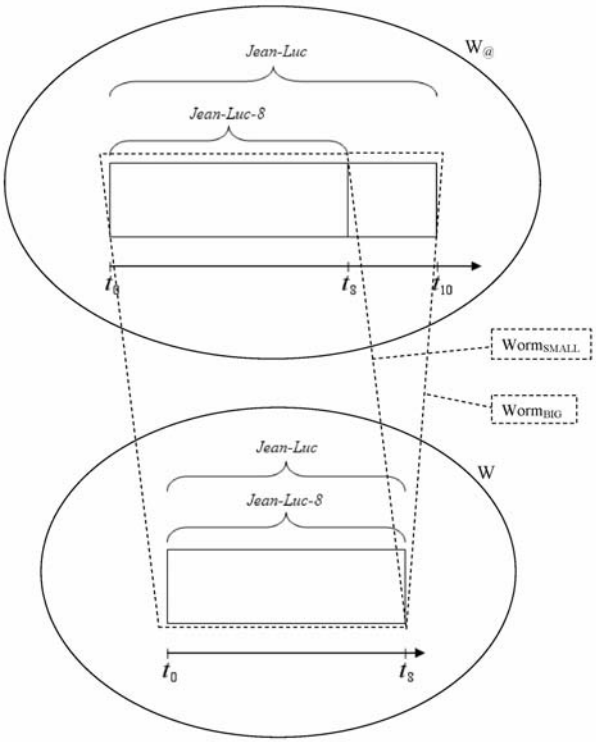
It is then claimed that, obviously, in the actual world (and so in W as well)

(3)' Jean-Luc and Jean-Luc-8 are distinct

and, therefore, in W, there are two different things occupying the same spatio-temporal region, which is unacceptable.

Now, the picture five-dimensionalism provides of this case is the following. There are two modal perdurants – 'modal worms' stretched across two possible worlds (see the figure below). $\text{Worm}_{\text{SMALL}}$ is the modal perdurant that includes Jean-Luc-8 in the actual world and Jean-Luc-8 in W and it is quite clear that those two entities are distinct objects (so (2)' above is rejected) : Jean-Luc-8 in the actual world is nothing but a world-stage of $\text{Worm}_{\text{SMALL}}$ and Jean-Luc-8 in W is just another world-stage of $\text{Worm}_{\text{SMALL}}$, so they are different parts of the same modal perdurant. Similarly for Jean-Luc : there is Worm_{BIG}, the modal perdurant that has

as two different parts Jean-Luc in the actual world and Jean-Luc in W. And again, those two parts are, quite obviously, distinct (so (1)' above is rejected). And since (1)' and (2)' (and thus (vii)' and (viii)' from the argument) are rejected, there is no obstacle to claim that, in W, there is only one (and not two coincident) entity that is a part (a world-stage) of two non-identical modal perdurants – $Worm_{SMALL}$ and $Worm_{BIG}$. In W, Jean-Luc and Jean-Luc-8 are simply one and the same object, and not two that coincide. Indeed, $Worm_{BIG}$ includes the whole of $Worm_{SMALL}$ as a part. So, what looked like a puzzle with coincident entities dissolves as a mere case of part-sharing 'modal worms', exactly as temporal worms can share temporal parts. Here is how the picture looks :



7. The statue and the lump case

The objection from undetached parts is one member of a family. Another member of this family of puzzles that involve coincident entities is the famous case of the statue and the lump of clay. And, here again, there is a modalized version of it that could cause trouble to the four-dimensionalist, and that, again, is easily solved if she makes the further step and embraces five-dimensionalism.

The puzzle appears when one imagines that at t_1 there is a lump of clay that, at t_2, an artist forms into a statue. Let us say that the statue persists until some later time t_3 and is then destroyed (squashed). At a time after its destruction, at t_4, the statue, of course, does not exist anymore but the lump of clay still does. The lump persists from t_1 to t_4 : it existed at t_1 in a certain (let's say cubic) form, then it was shaped into a statue's form and, after the destruction, it was shaped again in some other form. The puzzle arises here because it seems that in the interval of time from t_2 to t_3, the lump of clay and the statue are one and the same object : they have the same form, the same location, they are made up of the same particles. But, if they were the same object, they should, according to the principle of Indiscernibility of Identicals, share all their properties. But this is not the case : the lump of clay has, for instance, the historical property of being cubical at t_1 that the statue has not. So, after all, the statue and the lump of clay are different objects. But then, it seems that we have there two different objects which, from t_2 to t_3, coincide; and this is something to be avoided.

The four-dimensionalist's solution simply exploits the fact that all of the temporal parts of the two objects between t_2 and t_3 are numerically identical and do share all of their properties. The t_2-part of the statue and the t_2-part of the lump, for instance, do not have any properties like "being cubic at t_1" and "not being cubic at t_1" since none of these objects exists at another time than t_2. And so, the solution goes, between t_2 and t_3 there is only one object shared as a part by two space-time worms.

But, here comes the modalized version of the objection, such a solution is probably not entirely satisfactory because even if we accept that the statue and the lump between t_2 and t_3 do not have any different historical properties, they do have different modal properties. For instance, at any time in the interval from t_2 to t_3, the lump *could have survived squashing*, but the statue *could not*. So, after all, there is a reason to believe that, even in the crucial interval of time, there are two different objects, and this would lead us to coincident entities back again. It is good enough that four-dimensionalism dissolves the puzzle as far as historical (temporal) properties are concerned, but it does not help if modal properties are involved.

But it is no trouble for the five-dimensionalist to take her way out of this puzzle, using the same strategy as for the puzzle we have considered above. For, if one is a friend of modal perdurants, there really is no more than one object between t_2 and t_3. And how this allegedly one object can manage to have different modal properties ? Simply because it is part of two (or even more) modal perdurants, simply because it is a part of several objects extended across possible worlds, and because one of those objects, a statue-worm, does not survive squashing in some possible world, while the other, a lump-worm, does. Again, one could use the four-dimensionalist strategy to claim by analogy that the $W_@$-part of the modal perdurant that is the statue and the $W_@$-part of the modal perdurant that is the lump of clay do have all of their intrinsic properties in common – it is only one object. The two modal perdurants simply share it as one of their this-worldly parts.

8. Modal perdurants and modal counterparts

It is worth noticing that one can also deal with this puzzle by the means of modal counterpart theory. As Lewis (2003) claims, there is a multiplicity of counterpart relations that hold between the one and only one object that is there in the actual world between t_2 and t_3, namely the statue-lump, and other-worldly objects, its counterparts. So the this-worldly statue-lump has a counterpart in some possible world that is something that survives squashing "under the counterpart relation that is called to mind when we describe [statue-lump] as a lump, but not under the different counterpart relation that is called to mind when we describe the very same thing as a statue" (Lewis (2003)).

It is no surprise that when modal perdurants can help, counterpart theory is also of good use, for remember that as I have introduced them, modal perdurants are just aggregates of modal counterparts. What the counterpart theorist takes to be different objects existing in different possible worlds, the friend of modal perdurants sees as one big object stretched out across different possible worlds, by having modal parts there. Indeed, Lewis himself claims that the theory of modal perdurants is just an equivalent reformulation of counterpart theory. But before addressing this issue, let us first examine an objection David Lewis raises against modal perdurants with respect to a puzzle similar to those I have just discussed.

9. Lewis' objection to modal perdurants

In Lewis (1986, p. 218-219) and Lewis (1968 (postscript B), p. 41), he raises an objection to modal perdurants based on the discussion of another well-known puzzle case : the case of fission. So let us first make a detour and consider this case in some detail. At t_1, Jean-Luc Picard, captain of Star Trek's starship Enterprise, enters the transport device in order to teleport himself on a planet's surface. Usually, the device is supposed to disintegrate Picard on Enterprise and 'rebuild' him exactly as he was on the planet; but let us now suppose that, due to a malfunction of the mechanism, *two* of them appear on the planet where only one person should have been transported. At some later time t_2, there are then two persons instead of the one that entered the transport room – a plausible way to describe this accident would be to say that the original Jean-Luc Picard has undergone fission. If you don't like Star Trek transport stories, and if you prefer the more usual 'philosophical surgery' examples, think of Jean-Luc at t_1 as entering the Enterprise's sickbay and undergoing an operation that would put one half of his brain in a brainless body and the other half of his brain in another brainless body. Suppose, further, that his original body is annihilated and that both halves of his brain survive the operation, and you'll have a similar result as in the transport accident case : there seem now to be two persons who both have exactly the same right to claim to be the captain Jean-Luc Picard. Again, this seems to be a case of fission, where one person undergoes a certain process and becomes two – let us

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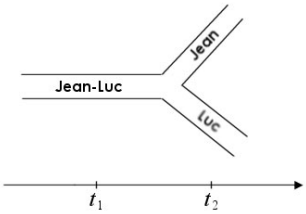
call, from now on, "Jean-Luc" the person before the fission and one of the resulting post-fission persons "Jean" and the other "Luc", as shown on the figure bellow.



As far as Jean-Luc's survival is concerned, there are four possibilities[4] : (a) Jean-Luc did not survive the accident (or the operation) and died, Jean and Luc being then two new accidentally-born persons, or (b) Jean-Luc survived as Jean, and Luc is a new accidentally-born person, or (c) Jean-Luc survived as Luc, and Jean is a new accidentally-born person, or (d) Jean-Luc survived as both Jean and Luc.

I think that (d) is to be preferred. The trouble with possibilities (b) and (c) is that since there is no reason to prefer one to the other (both Jean and Luc, in the transport case as well as in the operation case, have an equal right and the same reasons to claim to be the surviving Jean-Luc), neither of them is attractive – because each choice would simply be *ad hoc*. And let's ask Jean-Luc himself (that is, let's imagine ourselves as Jean-Luc before the accident or the operation, knowing what's going to happen) : would Jean-Luc say that he cares about what will happen to Jean and Luc if we asked him before the fission? Would he fear future pains of Jean and Luc, and hope for their happiness? Would both Jean and Luc be right in claiming Jean-Luc's salary at the end of the month ? The answer to these questions is, I believe, yes – and this is enough to show that Jean-Luc would not consider the fission to be his death, but on the contrary he would think of himself as surviving the 'accident' (so (a) is ruled out). Both Jean and Luc would have Jean-Luc's past experience, skills, memories, and so on. Furthermore, if there hadn't been an accident, and if the transporting mechanism had worked properly – that is if the transport process yielded no more than one person – we wouldn't have any doubts about Jean-Luc's survival and personal identity, and there is no reason for this to change just because another candidate for being Jean-Luc is there. In the surgical case, the same holds : if only one half of the brain had been transplanted and the other destroyed, we wouldn't have any doubts that our patient survived as the individual that has the undestroyed half-brain – but how could the conservation of the other half of the brain change the situation and make that he did not survive?

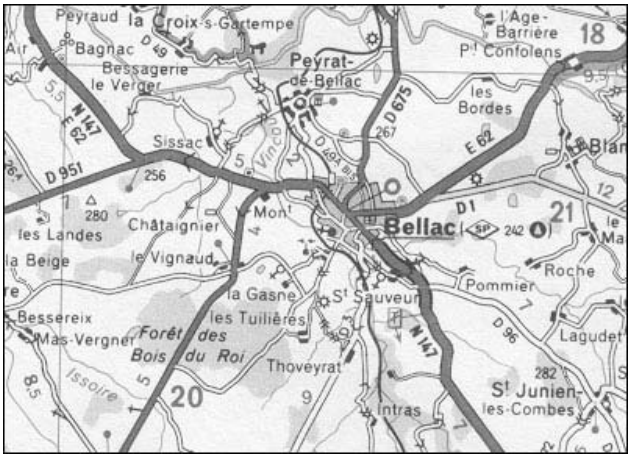
I think these considerations show it to be very plausible that in a case of fission of a person, the person survives as all of the resulting individuals of the fission. But note that this does *not* mean that, after the accident or the operation,

[4] See, for instance, Parfit (1971, p. 375). Parfit says there are only three possibilities but this is just because I develop his second possibility into two, to be more explicit.

there is one person that has the weird experience of being located in two separate bodies, or one person that is bi-located and has no unity of consciousness, rather this case of fission should be conceived of as a case where, after the fission, there are *two* distinct persons which will *from then on* evolve in entirely different ways.

But, if this is right, there is a problem for the endurantist. If Jean-Luc survives as Jean, he is then, according to endurantism, numerically identical to Jean. Likewise, if Jean-Luc survives as Luc, he is numerically identical to Luc. But, by transitivity of identity, we get as a result that Jean is numerically identical to Luc, which is absurd. But the only way to avoid this result would be to endorse the existence of coincident persons; in this case, it would be to endorse the claim that there were *two* persons even *before* the fission occurred. But, surely, this is not something anyone would be willing to accept. Thus, the fission case seems then to provide us with another *reductio* against endurantism.

As in case with undetached parts, the four-dimensionalist has nothing to fear from the fission case – for, according to four-dimensionalism, there is no numerical identity between Jean-Luc and Jean, and Jean-Luc and Luc. Survival, on this view, is not a matter of numerical identity, but rather, as Mark Heller says "for x to survive as y is for x and y to be an earlier and later part of a single object" (Heller (2000, p. 363)). On the four-dimensionalist account, Jean-Luc is then a temporal part of the whole four-dimensional person we could call "Jean-Luc-Jean" and he is also a temporal part of the four-dimensional person we could call "Jean-Luc-Luc". Jean-Luc-Jean and Jean-Luc-Luc are four-dimensional beings that share a pre-fission temporal part – namely, Jean-Luc. What we have here is a simple case of temporal overlap, analogous to a spatial case of a road that forks – and this, certainly, is not an objectionable case. Let us take an example : when I travelled across France this summer, I took a road from Poitiers that, when one arrives in Bellac, forks, as the following figure shows.



The (metaphysically) *one* road that comes from Poitiers, on the left, is referred to by two names "N 147" and "E 62". Then it forks, in Bellac, and its

'post-fission' parts are each called by one of the two names. So, before the road forks, and precisely *because we know that it's going to fork*, the road that goes from Poitiers up to the place where it forks bears the two names of the *two* 'post-fission' roads. But this doesn't mean that there are, metaphysically speaking, two roads (before Bellac), there is only one, in fact there is only one part of a road that is shared by the two roads "N 147" and "E 62". And this point is perfectly well taken by our commonsensical intuitions : despite of the fact that we *do* say that there are *two* roads by giving them two different names, when it needs to be repaired we'll sent equipment that is needed for the repairs of one road only. So, metaphysically speaking, there is only one road, but that road is part of two and this is why it bears two names, and no non-philosopher is ever puzzled by this. And philosophers shouldn't either.

Now, the worm view theorist can provide a solution to the temporal fission case that is analogous to the one we have just seen in the spatial case. Metaphysically, there is only one person at t_1, namely Jean-Luc, but if we knew that a fission were about to occur we could happily claim that Jean-Luc is part of two persons and we could even give those two names, if this were of any use. Because we *do* claim this in the spatial case, and because the worm view theorist takes persistence through time as analogous to extension in space, I claim that it is entirely justified to say that we *would* make the same claim in the temporal case. (If we only have one name like "Jean-Luc" for two non-identical persons, such a name is then, from an atemporal standpoint, ambiguous. But as David Lewis points out, such an ambiguity is perfectly harmless, as long as its two bearers are indiscernible (see Lewis (1983c, p. 64-65)). The need to distinguish the two persons arises only after the fission.)

One could object that there remains a problem with denotation. If, just before the fission, Jean-Luc says "Next week, I'll visit my friend Worf" and if, after that, Jean meets Worf but Luc does not, what should we say about Jean-Luc's statement? Is it true? False? Does it have some other, indeterminate, truth-value, and so are we committed to use a many-valued logic? I think there is a more natural solution at hand : when Jean-Luc seems to speak about one person, his future self, he really speaks about two. The meaning of his sentence has what Mark Heller calls "multiple contents" (Heller (2000, p. 376)). Heller provides us with a nice crossword example (Heller (2000, p. 375)):

This	sentence	contains	exactly	six	words.
sentence					
contains					
exactly					
three					
words.					

In this case, as Heller points out, the word "this" has a multiple content : it denotes the horizontal sentence, *and* the vertical sentence. Once this is accepted, there is no problem in Jean-Luc's case : the "I" in his sentence works similarly – there are two contents, two meanings expressed, one of them being true and the other false.

Now, we can finally state Lewis' objection to the theory of modal perdurants that appeals to the case of fission. The four-dimensionalist's treatment of this case, Lewis claims, sounds awkward and counter-intuitive. As we have seen, following four-dimensionalism, there is metaphysically only one pre-fission individual Jean-Luc, but of course, he, as the pre-fission temporal part we're talking about, is part of two space-time worms that share this part. So even before the fission, and even if there is only one individual, it makes sense to speak about two individuals here. It is awkward, perhaps (not that much, as I have argued), but this is perfectly understandable – after all, none of us usually takes into account such rare and bizarre phenomena as fissions of people. So a bit of counter-intuitiveness is to be expected.

But here comes Lewis' objection, in the case of modality, where we'll have to say pretty similar things if we want to endorse modal perdurants, those 'pathological' cases are everywhere – they are much more numerous than the rare or fictional cases involved in thought experiments like the one above. Take simply the case of Jean-Luc and Jean-Luc-8. It is a modal analogue of the temporal case of fission since there also are two modal perdurants that share one W-part. And so we'll have to ask the same question as before : how many lizards are there in W ? And presumably we'll give the same reply as before : there is, of course, metaphysically only one, but it makes sense to speak of it sometimes as two because it is part of two or more modal perdurants – indeed, it is likely that it is part of an infinity of those. But even if this is perhaps a semantically and metaphysically satisfactory view, it is very awkward and counter-intuitive : I, the person who is writing now, am present here, in my office, but at the same time, I am part of many modal perdurants who are also present here, in a derivative way. There is nothing to object here really, but it is certainly a strange view – and that's why Lewis recommends to accept the existence of such modal perdurants but to reject that we, ordinary people and ordinary objects, are such things. In the temporal case, he claims, we'll be able to defend such views, even though they are quite bizarre, because the puzzles involved are only fictions of philosophers or rare phenomena. But because modality involves such cases everywhere, the view is unbearable in the modal case.

10. Reply to Lewis' objection

In response, I have two things to say. First, things are not as bad as the can look with respect to intuitiveness, since in everyday life, we are mostly interested in what actually happens to us and to actual objects, and so we can happily ignore all the modal perdurants we are part of (thus, usually, "a is F" means "the actual world-stage of a is F", just like if one is a four-dimensionalist, it often means "the

present temporal part of the actual world-stage of a is F"). And second, as Sider also points out (Sider (1996, p. 14-15)), there is no reason why frequency of puzzle cases would matter in their resolution. If one is happy with a solution of a puzzle case, be it fictional or rare, then why wouldn't one be happy with the same solution even in actual and frequent cases ? If a method works, then it works no matter how frequently we must use it.

11. Is the theory of modal perdurants equivalent to modal counterpart theory ?

There is another critique that Lewis addresses to the theory of modal perdurants. The critique is that it just does not provide us with anything new and compelling. In Lewis (1968 (postscript B), p. 41), he says : "I have no objection to the theory of modal [perdurants]. It is an equivalent reformulation of counterpart theory, so it is just as right as counterpart theory itself. It does not offend against Postulate 2, which merely prohibited individuals from being *wholly* in more than one world[5]." Remember that as they were introduced, modal perdurants are aggregates of modal counterparts, and so this is why, I think, Lewis claims that this is no new theory but just another reformulation of counterpart theory. But this is not true. As we have seen, ordinary individuals have parts in several worlds according to the theory of modal perdurants, but not so according to counterpart theory. Also, ordinary names like "Jean-Luc" refer on this view to modal perdurants, but not so according to counterpart theory where they refer to entirely actual individuals – for instance, to what the friend of modal perdurants calls the actual part of the modal perdurant "Jean-Luc". Later, in Lewis (1986, p. 217), he weakens his equivalence claim between the two theories, and acknowledges that, while there is no *metaphysical* difference between the two theories (there is no disagreement on what there is in reality's stock), there is an important *semantic* disagreement, mainly as to what entities are properly called "Jean-Luc", "lizard", and so on.

But there also is another difference that was brought out by Achille Varzi (2001). The difference concerns the way the two theories handle indeterminacy. Let us think of modal sentences like the following :

(a) The lizard Jean-Luc could have been a fire-breathing pink dragon.

Maybe we are not sure whether this sentence is true or not – or simply just suppose that its truth-value is indeterminate.

Counterpart theory will handle the indeterminacy of such statements as a semantic or pragmatic one since the counterpart relation is a relation of similarity, and similarity is vague, so it can be a vague matter whether such and such thing is similar enough to Jean-Luc to be his counterpart or not (is the other-worldly fire-breathing pink dragon similar enough to Jean-Luc? similar in what respects?).

[5] For the Postulate 2, see Lewis (1968, p. 111): Nothing is in two worlds.

Certainly, there is no ontological vagueness here, which speaks in favour of counterpart theory. But still, a worry remains. As Varzi says, the counterpart relation is a primitive of the theory, and it is no good for any theory to have a vague primitive – it means that the theory itself is vague ("a theory built around a vague predicate" (Varzi (2001a, p. 14))), and this can be seen as a serious objection.

But if we take the name "Jean-Luc" to denote the trans-world modal worm instead of the actual part (counterpart) of him, things turn out differently. Statements like (a) will turn out to be indeterminate simply because the singular term "Jean-Luc" is indeterminate – it is indeterminate which modal perdurant is referred to by it. Jean-Luc-in-the-actual-world is a part of many modal perdurants, and if (a) is indeterminate, it is just because some of the candidates to be the referents of "Jean-Luc" contain other-worldly parts that are pink fire-breathing dragons and some do not. As Varzi puts it, "a term is indeterminate insofar as there appear to be many ways of assigning it a referent, all of them compatible with the way the term is used" (Varzi (2001, p. 14-15)). So, depending on the referent of "Jean-Luc", (a) will turn out to be true or false, and it is indeterminate because in our case some admissible referents of "Jean-Luc" have as part a fire-breathing pink dragon, and others do not. This is not always the case, of course, since in some cases the statement turns out to be true under all admissible disambiguations of "Jean-Luc", like in the non-modal claim

(b) Jean-Luc has a tail.

So, in the case of the theory of modal perdurants, indeterminacy is also dealt with as semantic, and not ontological, vagueness but furthermore, it does not make the theory vague – it is a simple case of semantic vagueness like many others. The source of vagueness in the theory of modal perdurants is not the same as in counterpart theory for the former case is just a case of semantic indecision, while the latter is a case of a theory that contains a vague concept as a primitive. As Varzi points out, "Jean-Luc" is, under the theory of modal perdurants, modally vague, exactly as "Mont-Blanc" is spatially (and temporally) vague – because which candidate with such and such spatial and temporal boundaries is to be taken as the referent of "Mont-Blanc" was never defined precisely enough by our linguistic practices. If one wanted to 'precisify' the name "Jean-Luc", under the theory of modal perdurants, one would simply have to make a semantic decision – but not so under counterpart theory, since here it would be necessary to clarify the concept of a counterpart.

So, if this is right, it is not true that counterpart theory and the theory of modal perdurants are equivalent. (Besides, the theory of modal perdurants has perhaps an advantage here over counterpart theory, if one prefers semantic indecision to theoretical vagueness).

12. Unification of modal parts

The preceding discussion clearly suggests the problem of how the different world-stages, or modal parts, of a single modal perdurant are unified to make up a whole. The question is : what's the 'glue' that makes them stick together? Traditionally, in the case of *four*-dimensionalism there are three 'glues' that make the space-time worms stick together: (a) causality, (b) resemblance, (c) spatio-temporal contiguity. I say these are the traditional glues but in fact many four-dimensionalists would probably not endorse at least one of its components. This is the case of four-dimensionalists who endorse the principle of unrestricted mereological composition and who claim, in short, that any mereological sum of anything is a space-time worm. For instance, there is a space-time worm made up of my sandglass today, Jean-Luc's tail on the 9th of February 2003, and all of the tropical fish of the 19th century – and nothing forbids that we name such a worm "Bernard" and refer to it by this name, and quantify over it, like we quantify over 'ordinary' individuals like Jean-Luc, my computer or any particular fish. If we accept such individuals in our ontology, then it is clear that they do not seem to satisfy any of the three components of the traditional glue, or only partially. So this could lead to a first strategy for the *five*-dimensionalist : he could accept the very same principle of unrestricted composition, and simply claim that there is *no* specific glue – any individual composed of any components is as good as any other (as real and existent as any other). The internal glue that unifies a modal perdurant simply needs not have any ontological force.

But while such a view is probably the best a five-dimensionalist can hold, it is likely that it would not sound very satisfactory to many philosophers – for such individuals, even if they exist, are not the *interesting* ones. The interesting individuals are computers, fish, lizards, Jean-Luc, David Lewis, and so on – and the reason they are interesting is precisely because they seem to stick together and stand out from their environment. It is already a tricky issue for the four-dimensionalist to spell out how the space-time worms from the actual world are unified, but not only for the four-dimensionalist – whatever trans-time identity and individuation conditions the endurantist chooses, it will also be available to the four-dimensionalist as the 'glue' that makes the successive temporal parts of familiar objects stick together – but those conditions are not easy to determine. For instance, condition (c) seems to be a reasonable one, until one notices that on the atomic level all of the 'ordinary' individuals that 'stick together' are fairly scattered and are nothing but a huge amount of particles with a lot of space in between (if this simplified picture motivated by physics is right). And in the case of modality, it is clear that (a) is not available either. So it seems that we only have (b) to unify the modal perdurants, if we want them to be the 'interesting' ones. But (b) alone just turns out to be the counterpart relation – and this is why modal perdurants were from the beginning introduced as aggregates of modal counterparts. Maybe this really is satisfactory – after all it serves counterpart theory well, while, as we have seen, it does not make the theory of modal perdurants equivalent to it, and

there are some genuine advantages to the modal perdurant's view. But is there not some better glue one could find?

13. Haecceitism

One way to glue together the modal parts of a single 'interesting' modal perdurant could be to embrace *haecceitism*. This notion was introduced and defined by David Kaplan as follows : "The doctrine that holds that it makes sense to ask – without reference to common attributes and behaviour – whether *this* is the same individual in another possible world, that individuals can be extended in logical space (i.e., through possible worlds) in much the way we commonly regard them as being extended in physical space and time, and that a common 'thisness' may underlie extreme dissimilarity or distinct thisnesses may underline great resemblance, I call *haecceitism*" (Kaplan (1975)). In this excerpt, I take it so that Kaplan means by "thisness" more or less what Alvin Plantinga means by the notion of *individual essence*. An individual essence, for Plantinga, is a property that an individual has, that is an essential property of him, and that he is the only one to have (quantifiers unrestricted). And a property is said to be an essential one to an individual if and only if it is not possible for him to exist and not exemplify this property. So here we are with a notion of a property, and of individual essence, or thisness, that makes one individual to be *this* individual and not some other. And, most importantly, this property is neither equivalent nor supervenient on any qualitative or other properties of the individual that has it – in particular a thisness is not supervenient on similarity. And so, here we have a simple solution to our trouble : a thisness of a modal perdurant that is extended across possible worlds just *is* the glue that makes him stick together – this allows us to say that in different possible worlds *this* individual and *that* individual are just modal parts of the very same trans-world worm. Jean-Luc from $W_@$ will then be said to be part of the same modal perdurant than Jean-Luc from W_1 is, if and only if they both possess the haecceitist property that we could call "Jean-Luc". And the reason why a pink fire-breathing dragon from W_2 is not admissible as a modal part of the same modal perdurant is simply that it does not possess "Jean-Luc-eity".

Now, this solution to the 'glue problem' sounds very nice, but unfortunately I think it does not help us at all. Our original worry was : what makes the different modal parts of a single modal perdurant stick together? Their thisness, is the answer. But now we have a new worry : in virtue of what do the different modal parts of a single modal perdurant possess this thisness (and not some other)? In virtue of nothing, the answer would go (I guess), you are asking a silly question – any individual has the thisness he has simply because he is the individual he is; remember that it was clearly said that having such and such thisness does not supervene on the having of any other properties, so when you ask in virtue of what a thisness is exemplified by some individual, you are in fact asking in virtue of what his other properties, he has the thisness – but that's not the way thisnesses are distributed among individuals, you did not pay attention to what was said.

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The main problem with such a position is that it is a genuinely magical one. Why is this and that the same individual (the modal parts of a single individual)? Because it is, period – says the friend of haecceities. I do not take this to be a bad solution to the problem, I just take it to be an absence of solution that creates an unnecessary mystery and posits obscure properties such as thisnesses. Our original problem was how to discriminate the 'interesting' modal perdurants from all the others, but I just don't see at all how haecceitism could be of any help here.

14. Unrestricted composition

So perhaps, the best glue available to the five-dimensionalist really is the principle of unrestricted composition. The glue that makes stick together different world-stages of a modal perdurant is just a relation that makes only some stuff in other worlds 'accessible' for an individual from a given world[6]. And what this stuff is, is just determined by the constraints one chooses to apply to this relation: for instance, it is not possible for Jean-Luc to be a pink fire-breathing dragon under some definable constraints (perhaps, biological ones), but it is logically possible – so there are different modal perdurants that correspond to different constraints. (David Lewis (see Lewis (1986, p. 234)) uses a very similar strategy to relativize his counterpart relation. According to this view, there is not only one counterpart relation but a plurality of relations – but, according to Lewis, all of them are relations of comparative similarity, it's just that some are under such and such additional constraints, and others are under different additional constraints.) So, in search for a glue for modal perdurants, the present proposal simply amounts here to claim that first, there is an entirely unqualified relation that is not ontologically constrained at all that unifies modal perdurants, and second, that this relation can be qualified, and can be given any constraints we want it to have, to pick out the 'interesting' modal perdurants and discriminate them from the unrestrictedly composed Bernard-like entities. Granted, such qualifications and constraints are human-dependent and have no ontological force at all, but this is only for the good since four-dimensionalists and five-dimensionalists will typically claim that Bernard-like entities are ontologically on a par with, and are every bit as real as 'ordinary' objects that seem to stick together and stand out from their environment, like Jean-Luc.

Intuitively, I find quite compelling the idea that the way we see the world as being cut up into people, lizards, sandglasses, clouds, and so on, is a genuinely human-dependent way, and I can easily imagine there being different intelligent beings in our world that would not cut up the world (the space-time regions filled

[6] In his respect, this strategy is similar to accessibility relations that hold between possible worlds. In short, an accessibility relation is a relation that holds between a world and a set of other worlds that are said to be accessible from it, i.e. that are possible alternatives of the way the world could be under some restrictive conditions – for instance physical laws, logical laws, biological laws, and so on – and this is how one gets a notion of logical, nomic, and biological possibility.

with matter) in the same way we do, maybe because their sensorial apparatus would not be similar to ours, or for other reasons.

15. Conclusion

The modal perdurants view is certainly not a very intuitive one, let alone because it is a modal realist one, and it has its own difficulties. But I hope that this paper shows that a four-dimensionalist could find here resources for dealing with some difficulties for her own view, and that the 'five-dimensionalist' view may very well serve as a good tool in the metaphysics of persistence of material objects through time and across possible worlds.

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The Ontology Suggested by Quantum Mechanics

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Abstract. The limitations of modern ontology are shown when considering the results of quantum mechanics. A possible way to build an ontology for our time is to take into account Peirce's examination of the most basic forms of thought. Once that this has been done, the problem is to find a common conceptual basis. This can be represented by the concept of information. In order to use this concept as the foundation of ontology, we need to find a sufficient general theory of information. Quantum mechanics can be formulated in these terms and can provide such a general framework.

1. Wolff's Ontology

For the following examination a good starting point is modern ontology. Modern ontology was built as a system stemming from what at that time were considered as the first principles of the *whole* science (ontology was in fact considered as the most basic of all sciences, since logic pertained to methodology). Let us take Wolff's *Ontologia*, which is paradigmatic for this examination. In an introduction about the scope and the methodology of this science [§§ 1-26], Wolff points out what seems to him the most important character of ontology: It is a rigorous science that must be based on clear definitions and proofs. In this line, the modern ontology is quite different from the scholastic one. Here, Wolff quotes Leibniz [see for instance Leibniz *PS*, IV, 469]. Thereafter, Wolff presents the two most general principles from which this science must begin. The first is the *principium contradictionis* [§§ 27-55], which was considered as the basis of the whole logic. It is rather a formal principle, and therefore unable to found alone an ontology. For this reason, Wolff, recalling Leibniz' lesson [see *PS*, VI, 612], introduces a second principle that may also cover contingent facts and their relationships, the principle of sufficient reason [§§ 56-78]. In Wolff's words, "with sufficient reason we understand that through which one understands why something happens or is" (*per rationem sufficientem intelligimus id, unde intelligetur, cur aliquid fit*).

Then, Wolff goes to the discussion of impossible and possible [§§ 79-103]. Here, recalling again Leibniz [C, 364], Wolff defines the possible as what does not involve contradiction. Moreover, Wolff distinguish between the concepts of undeterminate and determinate [§§ 104-131]: Undeterminate is a being that can be determined, whereas determinate is any being of which one must affirm all possible predicates (or their negation). From all this, he formulates his definition of *ens*: *Ens* is said to be that what can exist, that is that with which existence does not conflict [§ 134], that is, what is not impossible or what is possible [§§ 132-133]. From the propositions that what exists is possible [§ 170], and what is possible does not necessarily exists [§ 171], Wolff defines existence as the "*complementum possibilitatis*" [§ 174]. From this, Wolff affirms [§§ 225-226] that all that exists is completely determined, that is, it cannot have features that are undetermined [see also Baumgarten 1739, §§ 55 and 148]. This principles is called *omnimoda determinatio*.

There are several reasons for a necessity to revise this framework. I indicate here the three main ones:

(1) It is difficult, after Gödel's theorem [1931], to suppose that one could proceed in a demonstrative way, apart some very tiny domains in logic and mathematics.

(2) After the new discoveries in the 20th-century science, especially due to quantum mechanics, it is difficult to hold a sufficient-reason principle, at least in this generalized form.

(3) According to quantum mechanics, it is difficult to assume that all that exists is completely determined.

Often it is said that Kant's philosophy is a major break in this tradition. However, it is also growingly clear that Kant's criticism partly pertains to this ontological philosophy. As a matter of fact, it is interesting to note that Kant criticized the first but nor the second neither the third feature. In fact, in the first *Critic* he says: "The principle of sufficient reason is thus the ground of possible experience, that is, of objective knowledge of appearances in respect of their relation in the order of time" [B 246]. It is true that Kant says that "all attempts to prove the principle of sufficient reason have ... been fruitless", but he admits that without leading back a given concept to a previous one (even if here it is understood as its cause), the object of experience "would be impossible" [B 811]. About the third feature, it was widely applied in the well-known Kant's criticism of the ontological argument [see Auletta 2004].

2. Quantum Mechanics

In order to critically evaluate modern ontology, it can be interesting to see what are the main lessons of current science. As I have already mentioned, there are reasons

to think that especially quantum mechanics can lead to a new understanding of the foundations of ontology. This is also methodologically interesting to the extent to which this circumstance, if true, shows that also the most abstract sciences, as ontology or mathematics, cannot be completely detached from an empirical ground or at least from a critical confrontation with some empirical sciences and their fundamental results. For this reason, let us first consider what are the fundamental ontological features of quantum mechanics. In my opinion there are three fundamental features that characterize this theory [Auletta 2003; 2006a].

First of all, quantum-mechanical events are not predictable. An event is any result that can be locally and directly experienced. When measuring several quantum systems all prepared exactly in the same state, in general some will pass a certain test and some will not. For instance, suppose to have prepared a lot of photons in a state of polarization at 45 degrees and to let them pass a test represented by a vertical polarization filter. We will discover that in the mean one half will pass and the other one will not. It is possible to formulate an appropriate statistics, but it is intrinsically impossible to increase (even in principle) our knowledge in order to predict if a *single* photon will pass or will not. This intrinsic randomness of quantum mechanics, which does not depend on our subjective ignorance of the physical situation, neither on the number of the involved systems (since it is true for any single system), is what makes impossible to hold a principle of sufficient reason in a generalized form.

Second, in quantum mechanics there are laws, obviously. In particular I recall here the Schrödinger equation. Such a law is deterministic but does not rule directly the properties of a physical system. I understand with "properties" the values that a physical observable (position, momentum, energy, and so on) may have, given a certain state of the system under consideration. Rather, the Schrödinger equations rules the probabilities to obtain determined properties. Formally, any quantum state represents a probability amplitude rather than a complete collection of properties as in the classical case. Moreover, given a quantum system, due to the superposition and uncertainty principles, not all the properties that are abstractly possible can be instantiated together. In fact, the state of a quantum system represents a violation of one of the basic assumption of modern ontology, the *omnimoda determinatio*. For this reason, there does not exist for any quantum state a set of all observables defining the state, but quantum observables are collected in subsets of observables that are reciprocally commutable, that is compatible (whose properties can be measured and instantiated symultaneously).

Finally, given a quantum system (the object system), what properties are instantiated or not depends also on a complex of conditions that are external to the object system, that is on what systems the object system interact with. This is clear in the case of the measurement process, where the possible measurement outcomes depend on two further systems: the measuring apparatus, with which the object system directly interact, and the environment to which both systems are open. Actually, measurement can be seen as an information downloading into the environment such that some properties become instantiated [Zurek 1981-1982; Cini 1982]. However, it is not at all necessary to conceive of this process in subjective

terms. There is in fact a strong difference between the concept of non-intrinsicness of the properties and of their subjectivity. Actually, measurement is a particular case of a wider class of interactions between open systems that are throughout spontaneous and therefore objective.

It is difficult to lose this problem without having recourse to the idea that the initial quantum state somehow represents a form of potentiality relative to the subsequent instantiation of a given subset of properties [see Auletta 2006a]. I define a state as *potential* if and only if it (1) only represents the necessary but not the sufficient condition in order to instantiate the properties which have a non-zero probability to become actual given this state, and (2) can contribute to any actualization of properties only by interacting with other systems that cannot be controlled by the system under consideration.

This is the main weakness of the scaffold of modern ontology: the fact that one passes from possibility to actuality without acknowledgment for the centrality of a potential form of being: It is true that Wolff also speaks of potentiality [1730, §§ 175-178], but for him this only means that there are beings-which can be called potential-that have the sufficient reason of their actual existence in other beings. The latter can be actual or themselves potential. However, sooner or later any potentiality can be reduced to a form of actuality. It is, however, true that, when Wolff speaks of *potentia activa* [1730, §§ 716-725], this is for him a faculty which can produce an act if in the agent there is an original force (*agendi conatus*). However, a faculty is not strictly speaking a potentiality. Is an actual (for instance, acquired) ability to do something. The class of possible things than one can do, given an ability, is potentially infinite. For instance, once acquired the ability to speak Italian, the class of the possible utterances in this language is potentially infinite. This shows that, in the case of a *faculty*, we should rather speak of an active indifference to a lot of possible realizations (in order to distinguish it from a passive form of indifference, for instance as in the case of a brute piece of matter relative to all ways to be shaped, one of Aristotle's preferred examples). On the contrary, the class of *potential* results in the case of quantum-mechanical measurements is often finite or in general confined in specific value ranges, depending on the physical conditions in which the system finds itself and on the experimental arrangement. Similar considerations also apply to systems that are far more complex than quantum-mechanical ones, as in the case of epigenesis for living beings [see Waddington 1974]. Also here, the number of possible epigenetic developments or final results of these processes is limited.

These results of quantum mechanics strongly put in discussion the traditional modern ontology and suggests the necessity to move along a new foundation. Let us now consider what can be done along this way.

3. Peirce's Examination of the Forms of Reasoning

Wolff started from the first logical ("contradiction principle") and conceptual (sufficient reason) principles. The idea that an analysis of thought in its most abstract form can provide a scaffold for ontology is good and perhaps necessary.

However, Wolff's examination is not sufficiently general. In particular, in synchrony with the jump from possibility to actuality, he also sprung from a strictly logical principle (the "contradiction principle") to a principle that has necessarily to do, as Kant points out, with the possible contents of experience. He failed to see that inbetween there is a rich phenomenology, in particular there are many forms of reasoning that cannot be reduced to a deductive form, for not to speaking of being reducible to the principle of non contradiction, but that are neither reducible to a principle ruling directly contingent facts.

Peirce, following Aristotle, engaged himself in this interesting examination [1865; 1866; 1878; Auletta 2005a; 2005d]. According to Peirce, any possible reasoning represents a conceptual (and semiotic) connection between different things. In order to be a reasoning, this connection must have three elements:

1. A law or a rule. Only the lawness can guarantee that a connection is a reasoning. In fact, without such a law, the connection would be a chance relation, but nobody can define as reasoning a relationship grounded on pure chance.

2. A sample, a case, to which this law may be applied. This case can also be a hypothetical or ideal one. Without showing how the law is able to rule at least one case, we had no reasoning but only the repetition in other words of the law. Finally, a law that finds no application range is not a law at all.

3. And a result. In fact, from any reasoning we expect some conclusion, that is, that the application of the law or the rule to the case produces some consequence.

A simple multiplication of any of these three elements adds nothing new at the conceptual level, so that we can limit ourselves to reasoning where only three elements enter. Classically, Aristotle formulated any reasoning in syllogistic form, where three judgements entered. This is not necessary, and judgements are in general a too restrict form of reasoning. However, according to Peirce [1895] in any judgement the subject has the function of a *index* pointing to a given thing (real or ideal, or whatsoever), the predicate has the function of an *icon* by expressing a conceptual content that bears no direct relationship with the thing represented by the subject apart the fact that we find a conceptual connection that Peirce called *symbol*. These three elements are far more general than the constituents of any judgement. In logic symbols are called propositions. Since, on the other hand, judgements, are a good and simple instantiation of propositions or symbols, I shall consider them here as judgements (and take into account reasoning forms that bears an analogy with syllogisms).

If we limit ourselves to apply a given rule to a given sample and obtain a certain result, this is an example of *formal deduction*. An instance is:

RULE	When, without introducing any perturbation, we are able to predict with probability 1 a property about a (distant) system, this property is independent from us and real, that is, throughout objective.
CASE	When predictions of this type fail about a distant system, this, when the conditions remain the same, is due to an action-at-a-distance performed by us when ascertaining this property that perturbs the system.
RESULT	Therefore, systems which are not perturbed by such an action at-a-distance are collections of objective properties.

The first premise above is called the *reality principle* (it is quite general), the second one the *separability principle*, which presents a particular class of systems to which the first principle may be applied or not applied (that is, it cannot be applied to systems that are perturbed by a specific action at-a-distance but to unperturbed systems). This reasoning (which I have summarized very schematically) was actually performed by Einstein and co-workers [1935]. They have also presented a *Gedankenexperiment* by means of which they aimed to prove that quantum-mechanical systems which are not perturbed by such an action at-a-distance cannot be conceived as collections of independent properties, in contradiction with the above result. Since the two premises to Einstein and co-workers seemed to be above any suspicion or doubt, their conclusion was that quantum mechanics shows a fundamental incompleteness in the way it treats physical systems, and that for this reason must be considered a mere statistical theory, i.e., unable to provide predictions about individual systems and their properties.

Now, starting from this example, let us consider what other forms of reasoning are there. In order to obtain other forms, we must change somehow the structure of the above syllogism. This can be done only if we deny the result of the reasoning above. Let's suppose that we are certain of the absolute validity of the rule. Then we have these two premises:

RULE	When, without introducing any perturbation, we are able to predict with probability 1 a property about a (distant) system, this property is independent from us and real, that is, throughout objective.
NEG. OF THE RESULT	However, there are systems that are not perturbed by an action at-a-distance which are not collections of properties that are independent from us.

What can we infer on the basis of these two premises? That there can be non-perturbed systems for which these independent predictions are not possible. This is exactly the inference actually performed by Schrödinger [1935] when trying to answer to Einstein and co-workers' above argument. In other words, the complete inference done by Schrödinger can be schematically depicted as

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RULE	When, without introducing any perturbation, we are able to predict with probability 1 a property about a (distant) system, this property is independent from us and real, that is, throughout objective.
NEG. OF THE RESULT	However, there are systems that are not perturbed by an action at-a-distance which are not collections of properties that are independent from us.
NEG. OF THE CASE	The existence of such non-perturbed systems does not allow the above predictions.

This new property, which resulted from the negation of the case, Schrödinger called *entanglement*, and was probably the most important discovery after the foundations of quantum mechanics in 1925-1927. Entanglement consists in a correlation at-a-distance between quantum systems that does not allow independent predictions. In other words, the properties of a distant system may depend on properties that characterize a system with which we are bound or interact here locally, and this situation does not allow predictions of objective properties about the distant system. I wish to stress that entanglement cannot be interpreted as a form of signal or of communication between the two distant systems. It rather consists in the mere fact that entangled systems do not show independent properties. Schrödinger form of reasoning, where a new result (a new property) is reached through the negation of an assumption that is necessary in a deductive reasoning, was called by Peirce [1878] *abduction*, but was probably already known to Aristotle in the form of hypothetical reasoning. Abduction was congenial to Schrödinger, who made use of it also when examining the problem of the nature of life [1944], especially when prediucting what was later found to be DNA.

Suppose now a further situation. Suppose that we are sure of the case and notwithstanding we find a violation of the expected result. For instance, we have

NEG. OF THE RESULT	There are systems that are not perturbed by an action at-a-distance which are not collections of independent real properties.
CASE	When predictions of this type fail about a distant system, this, when the conditions remain the same, is due to an action-at-a-distance performed by us when ascertaining this property that perturbs the system.
NEG. OF THE RULE	Predictions of the above type do not in general imply any form of reality.

This was the reasoning performed by Bohr [1935] when answering Einstein and co-workers' paper. This does not mean that Bohr assumed explicitly the second premise, but only that he had no reasons for doubting of it, because his starting

point was his experimental evidence, which is necessarily local and says nothing about possible non-local correlations. On the other hand he knew very well, again from experimental evidence, that quantum systems, even if we assume that are not perturbed, cannot be conceived as collection of real properties, and that rather the properties we assign to quantum systems are only contextual to a given experiment (this is the basis of the complementarity principle). Therefore, Bohr was lead to the conclusion that, if in a given experimental context, we obtain a certain result by measuring locally a system and we can also predict with certainty a given result for another distant system, by changing the experimental context we necessarily change the result for the first system and we are no longer authorized to suppose that the above prediction remains true (and this in perfect accordance with the second premise above). In conclusion, to perform predictions *a priori*, without considering experimental contexts, has no relevance in order to determine if certain properties exists independently from us or not. Then, Bohr's conclusion was logically to deny the reality principle. In other words, Bohr extended his experience with quantum systems (the fact that they cannot be in general conceived as collections of real properties) also to distant systems that can be space-like separated. Bohr's reasoning was an *induction*.

Formal deduction, abduction, and induction, are completely different forms of reasoning. Let us say in a schematically way that in any formal deduction the stress is on the rule. On the contrary, in any abduction the stress is on the case, or, better, on the negation of the case, that is on a *new* property that cannot be derived directly from the rule. Finally, in induction the stress is on the empirical instances which lead to the necessity to find a new statistical regularity. In fact, the conclusion of Bohr (that predictions of the above type do not in general imply any form of reality) cannot be directly generalized as a new rule or principle, but only shows that, in certain circumstances, determined systems behave so that, even if one is able to formally predict a certain property, this property is not instantiated.

I have shown sofar that in quantum mechanics both abduction and induction have a citizenship. This is due to the very structure of the theory in which the systems are characterized by an intrinsic uncertainty that forces to perform ampliative reasoning, as both induction and abduction are [see Auletta 2005a]. This does not mean that in quantum mechanics deduction fails. Actually, any time that we can apply the Schrödinger equation we make use of a sort of deduction (that is we infer on a formal plane some general features of the system, also at subsequent or antecedent times). What we cannot do is to reduce abduction and induction to formal deduction. One could be erroneously led to this conclusion by considering the fact that both abduction and induction are ampliative forms of reasoning of which formal deduction can be conceived as the limiting case when we possess a complete knowledge of all properties that we can truly assign to a certain class of systems or of all types of systems that are characterized by certain properties, respectively [Auletta 2005d]. However, this move is not allowed in quantum mechanics and would be not very useful in general, since formal deduction is a too restrict form of reasoning, which, also classically, apply to a small class of cases (I recall here again Gödel's theorem). We can also see, retrospectively, that the principle of sufficient reason of Leibniz and Wolff is

actually an application to the possible experience of a deductive form of reasoning (it applies when we know all conditions, i.e. the sufficient reason, that can determine a certain effect).

4. What is Common to These Forms of Reasoning

The above examination is satisfactory to the extent to which we have provided the most general forms of reasoning which can be applied with success also to quantum mechanics. However, the above examination is not completely satisfactory under other respects. Even if we have enlarged the concept of inference by including all general forms of reasoning, as a matter of fact it is not easy to see the relationship between *forms of reasoning* and what we call an *ontology*. This was actually the main gap in modern ontology, a gap that one tried to fill (in an insufficient way) with the principle of sufficient reason.

The only way to fill this gap is to deepen our examination in trying to find what is the conceptual background of all these form of reasoning, and see if this can throw some light on the foundations of ontology. This problem was perfectly clear to Peirce. In fact, he tried to connect the three forms of reasoning with the structure of signs, in short with semiotics [see Peirce 1894; 1903a-c; 1904]. In other words, he tried to see propositions or inferences as a special case of semiosis, while he considered any *ens* as a sign. In fact, according to Peirce [CP 2.228] any sign has a triadic structure, in particular "A sign, or representamen, is something which stands to somebody for something in some respect or capacity". I shall come back below to this definition. My point here is to stress that this conception shows the right methodology but represents a partial answer.

The *methodology* is right, because Peirce tried to find what are the most general properties of being, and in so doing he pointed out to properties that are common both to material and mental forms of being. In fact, in this ontology thought becomes a *special* case of a more general structure that characterizes any being. This was to a certain extent also the aim of the Middle-Ages philosophy. On the contrary, it is an insuffiency of modern ontology the fact that to the connection between ontology and thought (as well as between ontology and knowledge) was not deserved sufficient attention. On the other hand, it is difficult to found a true ontology that is not able to answer to this question. As it is well known, Leibniz [1686; 1691-93] tried to overcome the Cartesian dualism with the concept of an absolute and intrinsic force, which he applied to any simple being (which he later called *monad*). However, when we again consider Wolff's ontology, we see no trace of this examination. When he speaks of Leibniz' monads [1720, §§ 598-99], he is very cautious. He admits simple forms of beings [1730, §§ 673-702], but he clearly distinguishes between (animal and human) souls and material entities [1720, §§ 892-902]. Materially simple entities are subjected to material forces [1720, §§ 660-669] and, in the *Cosmologia*, are called *atoms of nature* [1731, §§ 185-88]. It is not by chance that, in his *Ontologia* [§§760-61], Wolff struggles with the concept of *gradation*. He is in fact aware that for Leibniz the difference between spiritual and material monads is only a difference of degrees. This very

short (and unsatisfactory) examination shows that for Wolff material atoms and souls share only this common property, to be *simple beings* possessing a *force*, which seems rather a nominal attribution that brings us not very far, at least until one is not able to show what the expansion force of a material body [1720, § 665] and the soul's force may have in common. Moreover, where Wolff says *force*, Leibniz probably intended *energy*. On the contrary, no common property can be found in Wolff, so far I know, between material bodies and thoughts. For both Leibniz and Wolff the latter, by definition, do possess no force: Leibniz said "ideae non agunt. Mens agit" [*PS*, I, 150]. My aim is not to directly reediting a current (and correct) monadology but only to stress why modern ontology does not goes to the foundations of itself.

Let us now go back to Peirce. As I have stressed, Peirce tried to find this new foundation not in the concept of unity or of force and energy (we will see below the insufficency of this concept) but in an universal semiotics. However, semiotics is too much *specific* to be applied to the whole physical world. Quantum-mechanical systems (which were unknown to Peirce) can be surely said subjectively signs, to the extent to which they are signified by human minds and used by human technology, but they can scarcely be defined as signs in themselves. For this reason, we must try another, and more general, solution.

Let us come back to the three general forms of reasoning. All of the three forms of reasoning that we have explored so far are characterized by the attempt at reconciling two different and quite opposite exigencies: (1) a law-like regularity and (2) some specificity or difference. Obviously, the first exigency is expressed paradigmatically by the rule, and, as I have said, the form of reasoning that gives a central place to the rule is deduction. In fact, we have here the maximum of regularity, so that the case is only an application domain of the rule. In general, the most general principles and laws of any science are postulated, that is, often inferred from implicit assumptions that for metaphysical or other reasons are considered more general or fundamental. These laws and principles are then applied. Here, we do not perceive so easily the second exigency. In order, to understand this pole, let us consider the form of reasoning that is to a certain extent quite opposite to formal deduction, i.e., induction. As I have said, here the stress is on the empirical instances, that is on the difference. In fact, these empirical instances are used against a previous law (the conclusion is the negation of the original rule). That is, they are considered an exception or a violation of a given rule. On the basis of the laws that we already know, we expect a certain behaviour or effect, but sometimes our experience shows something new that we could not predict, and this seems to us an anomaly. If the system or the domain where we are acting or that we are reasoning of is sufficiently robust to account for this anomaly as a controlled or temporary random fluctuation, we feel authorized to not dismiss the law or the rule. If, on the contrary, this anomaly grows to a point which seems to endanger the whole building, we feel forced to abandon the law. It is here that abduction comes into play. Abduction tries to build a mediation between these two extremes. In fact, any new property is not a mere derivation from a previous law but it neither represents a simple violation of it. It is the emergence of a new behaviour in the context of already existing laws. In other words, abduction tries to

update old laws in face of new results (obviously, this can also have in the long run feed-back effects on the formulation of the laws themselves). There is no unique or final solution of this problem. In this sense, abduction is an open way of reasoning whose latter consequences in the long run may be seen as inductions or deductions. As a consequence, we also have a very interesting result: Deduction, abduction and induction are the only possible forms of reasoning.

We are then confronted with a triad that represents a generalization of the triad of elements presented above in section 3: (1) A rule, (2) a difference, and (3) a mediation between rule and difference. This triadic nature of inference is the reason of Peirce's attempt at leading the three forms of inference to the nature of sign. In fact, let us take into account the above definition of sign, which I repeat here: “A sign, or representamen, is something which stands to somebody for something in some respect or capacity”. We can see that a sign is a *mediation* for somebody that is bound with an object according to some *rule* but under some specific respect, that is under a *difference*.

There is however a more general way to solve our problem, a way that was unknown at Peirce's time: The most general form to account for this triad is in informational terms. Let me consider this point in short. (1) Any information exchange or information processing presupposes first that there is some difference or discontinuity at the source. Without such a novelty, one would transmit no information at all. For instance, in the sidereal space we look for light or other radiation sources as means for obtaining information about age, distance, chemical composition, and so on, of stars, through which we can perform some inferences about nature and age of our universe. Obviously, if all stars were exactly alike under all respects, nobody would try to obtain this information from their light. (2) Moreover, the possibility of exchanging information presupposes the existence of a channel between receiver and source, otherwise no signal could be received. This represents the regular aspect. Let us consider the eyes as a channel for vision. Obviously, eyes are tuned for certain frequencies of electromagnetic radiation, that enable the brain and the mind to build "images" of the objects. A blind man or woman has not such a channel and therefore cannot receive visible radiation (however, he or she can be instructed about this information through other channels). On the other hand, if the eye would catch light but in an absolute random way, it could not be used as a channel for information. (3) Finally, the signal that we receive from stars is the trade-off between regularity and novelty. As I have already stressed, if the light of a star brings absolutely no new information, then we would take its signal as a mere repetition of something already known. If, on the contrary this signal would be so new that we had no channel for receiving it, we received no information at all. Therefore, the signal that the stars emits and that we receive must be somehow a trade-off between these two extreme possibilities.

In conclusion, what I am suggesting is to consider information as the basic paradigm of a new ontology. However, up to now I have only examined the problem at a rather abstract level. In order to accept information as the basic ontological category, I must exhibit reasons that are grounded in the way our world is. However, classical information theory [see Shannon 1949] is not very apt to give the general theoretical basis we are searching for, because classical

information is a deterministic theory. The limitations of this paradigm, for instance, have become evident in the cognitive sciences, where the model of the brain as a classical computation device has come in crisis [Clark 1997]. For reasons that cannot be explained here, also the possible way out represented by connectionist models does not solve the problem.

Therefore, we must come back to quantum mechanics. In fact, quantum system show a range of behaviours that is far wider than classical systems, and there are reasons to believe that quantum mechanics is the most fundamental theory of our world. Up to now quantum mechanics has been useful for two basic tasks: (1) It has shown why modern ontology was defective and what new ontological problems are posed, and (2) that the most general forms of reasoning find application in its domain. Now, we must show something most important. We will see that quantum systems can be conceived in terms of information and that quantum mechanics itself can be understood as a general theory of information. These two features are sufficient to justify (quantum) information as the most basic ontological category.

5. Quantum Mechanics and Information

Wheeler [1990] was the first scientist to fully understand that quantum systems consist essentially in information. Let us consider for the sake of simplicity a two-level system. Any system of this type can be expanded in term of a basis |0> and |1>, where these two states

1. represent mutually exclusive states and

2. can be linearly combined.

3. Then, it immediately follows that they can be understood as a (binary) code, as, for instance, in quantum computation [see Auletta 2005b].

One could object that in physics several considerations enter, that cannot be reduced to information. For instance, it seems that one cannot have information exchange without some energy expenditure. However, it has been shown that it is not information *processing* that requires energy expenditure but only information *selection* [see Landauer 1961 and 1996; Bennett 1973 and 1982]. So long we do not perform information selections (for instance, by measuring) with quantum systems, they can be conceived as information devices that do not require energy for processing their information. This shows that information is a more general and fundamental "category" than energy (or force).

Another possible objection is that it seems that any information should be embodied in some physical structure. It is however difficult to see what this physical structure at the quantum level should be, apart ... from information!

(1) Quantum systems present the peculiarity that they can code an infinite amount of information but that this information is unaccessible in its totality

[Auletta 2005b]. In general, only a small amount it is through measurement or similar interactions. The rest is irreversibly lost into the environment. For this reason, we must sharply distinguish between the information a quantum system is or represents and the information it can transmit (that one can acquire, which corresponds to the properties that can be instantiated together). Since only the latter can be said *actual* information, we are led to the conclusion that the information a system is or represents is only a form of potential information. This potential information is ruled by the unitary (reversible) quantum transformations that constitute the basic laws of theory. As a consequence of these laws, a quantum system can also become entangled with another quantum system. In establishing this type of non-local correlations, quantum systems constitute a quantum channel, that is an interdependency that allows for forms of communication that are not known classically, for instance teleportation [see Bennett/Wiesner 1992; Bennett *et al.* 1993]. Quantum channels show an important feature that was not completely understood in classical information theory: A channel does not need to be a physical entity but is in general an informational dependency that can be interpreted as *mutual information*, that is as the information that two (or several) systems share. (2) When measuring, as I have already stressed, in general we have a random jump to one of the components of an initial superposition of the system's state (relative to the measured observable). This jump is locally in conflict with the unitary transformations of quantum mechanics, that is with its laws, even if, on a global level, i.e. taking into account the environment (in which the large part of the potential information contained in the initial state of the object system is downloaded), is not. (3) Finally, the dynamic processes themselves (in particular any dynamic process in which systems open to the environment are involved, as it is the case for measurement), is the trade-off between the ideal case of a perfect measurement result and the ideal case of a perfect application of quantum laws. Actually, as it is growingly clear in the last years, when measuring we obtain in general rather intermediate situations, that is situations where the final result is always affected by some "noise" deriving from the initial interference terms [Joos/Zeh 1985; Brune *et al.* 1996]. These terms, though downloaded into the environment, never completely disappears in the object system but always have a ghost presence.

Then, quantum mechanics shows on a far general level the three forms that are typical of information [see Auletta 2006b]: (1) laws, given here by unitary transformations and entanglement, (2) an irreducible difference represented here by the jump to a result of dynamical interactions between open systems, and (3) the dynamical interaction itself as a trade-off between these two features. Notice that quantum laws only rule the interactions between systems on a global level. Locally, events are possible and they can be considered under a certain point of view a "violation" of these laws. The most significant effect of these events is the fact that locally the entropy of the object system (and of the apparatus) strongly increases, whereas the entropy of the whole comprehending object system plus apparatus plus environment may be zero. Then, one could conclude that there are means to control from a global prospective these interactions and therefore to reduce the anomaly of quantum jumps to the regularity of a pure unitary evolution (this is somehow the

aim of the so-called Many-World interpretation of quantum mechancis). This is not the case, because, in order to control the local interaction we should also control and measure the infinity of systems with which the object system is directly or indirectly entangled. In so doing, we would inevitably change the relationships between our object system and the environment, so that we must begin again from the start. This shows a fundamental underdetermination of quantum laws relative to individual events.

Quantum information is not directly a theory of signs but represents in its huge generality the basis for both classical information and semiotics. The indeterminacy of any quantum state (that is, the potentiality of the information it represents) allows for these possibilities [Auletta 2005c-d]. In fact, to a certain extent semiosis can be seen as the reverse of the classical information flow. This consists in a signal that proceeds from past sources to possible future receivers, whereas, in the most simple case of semiosis [see Auletta 2005d], any sign's use or interpretation considers a signal as a sign *of the source*, that is a present signal as a sign of a possible cause. There one goes from the causes to the effects, here from the effects to the causes.

6. Peirce's Trialism

Peirce tried a further generalization of his ontology when, always in accordance with his semiotic paradigm, tried to formulate what seemed to him the most general and abstract categories of our world. His point of departure was Kant's table of categories (in the first *Critic*). In this table, it is evident that any group is divided into three categories. This gave to Peirce the hint to find what was the categorial basis of any possible experience in the most abstract form. In fact, Peirce thought that knowledge (what Kant called *Erfahrung*) was a specific aspect of the more general problem of human experience as such. Making use of Dewey's [1929: 20] words, this enlarged concept of experience must take into account "the existence of ignorance as well as of wisdom, of error and even insanity as well as of truth", so that it will comprehend "the phenomena of magic, mith, politics, painting, and penitentiaries". In this way, such an onotology can be understood to a certain extent as a critical correction of the uncritical aspects of Kant's criticism: To having considered Newtonian science as the paradigm of knowledge, and to having considered knowledge as the paradigm of experience.

Coming back to Peirce's three categories, he called them firstness, secondness, and thirdness [see also Auletta 2005c]. Here, it is not possible to give full account of this theory. However, it can be interesting to shortly consider the main features.

About *firstness*, Peirce said [CP 1.302]: “The idea of First is predominant in the ideas of freshness, life, freedom. The free is that which has not another behind it, determining its actions…”. And [CP 1.357]: “It must be initiative, original, spontaneous, and free; otherwise it is second to a determining cause. […] It cannot be articulately thought: assert it, and it has already lost its characteristic innocence; for assertion always implies a denial of something else. Stop think of it, and it has

flown! What the world was to Adam on the day he opened his eyes to it, before he had drawn any distinctions, or had become conscious of his own existence - that is first, present, immediate, fresh, new, initiative, original, spontaneous, free, vivid, conscious, and evanescent". I stress here that the first is not determined by some previous conditions, exactly as it the case for quantum-mechanical events. About *secondness*, Peirce wrote [CP 1.322]: "The second category that I find, the next simplest feature common to all that comes before the mind, is the element of struggle. […] Now there can be no resistance where there is nothing of the nature of struggle or forceful action. By struggle I must explain that I mean mutual action between two things regardless of any sort of third or medium, and in particular regardless of any law of action". The stress on struggle remains from classical (mechanical) science. The important point, I think, is that secondness implies relations between several things. When we try to understand what relation is, the most general way to express it would be in my opinion *covariance*, where this term covers both the "parallel" and the "antiparallel" case. *Thirdness* is often said to be the middle between the first and the second [CP 1.337].

I think, that even with some conceptual modifications, this framework can be accepted and seen as a possible way to interpret quantum information. We could call firstness as the irreducibility of any *single* event or fact of this world. The law-like behaviour can be seen as a form of *relation*. Any law expresses in fact a covariance. Also entanglement, as a particular form of mutual information, is form of covariance. Finally, the trade-off between these two is *dynamics*.

Another way to express this categorial partition is by starting from a general theory of relations [Peirce CP 1.293, 1.303-332; 3.472-473]: Single events, in their specific anomaly, are monadic. That is, the less we can lead an anomaly to previous laws, the more we consider it impenetrable. On the contrary, any law shows the dependencies of this being with other ones. Any law shows ultimately a covariance, and express for this reason a dyadic relation. Finally, any dynamical mediation between these two first principles is triadic in nature, that is the possible common ground between a monad and a dyad, which can be ultimately a new property, as said above.

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Propertied Objects as Truth-Makers

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I will assume in this essay that a sentence is true because it says what is the case. Accordingly, we can assume that the sentence: (1) '*a* and *b* are both extended', is true because both *a* and *b* are extended. But is *a* and *b*'s being extended fundamental, or can it be explained in terms of some more basic reality? Let us consider two philosophical explanations of this fact: metaphysical realism and trope theory.

1. Metaphysical Realism

The metaphysical realist explains *a* and *b*'s being both extended in the following way. She commits herself to the existence of universals, entities which can be instantiated in more than one object at the same time. She postulates the universal of extension, and claims that universal is exemplified in both object *a* and object *b*. *a* and *b* literally have some *thing* in common. It is their both having the universal of extension in common that explains their both being extended. *a* and *b*'s being both extended is not metaphysically fundamental. It is explained in terms of *a* and *b*'s exemplifying the universal extension.

2. Trope Theory

The trope theorist commits herself to the existence of properties, but denies that properties are universals, things that can be multiply exemplified. Properties, for the trope theorist, are particulars: there is *this* extension in object *a*, which is numerically distinct from *that* extension in object *b*. Such particular properties are called 'tropes'. What explains *a* and *b*'s being qualitatively identical in a certain respect? The extension in *a* and the extension in *b* are numerically distinct, but they exactly resemble each other. The extension in *a* and the extension in *b* are exactly

resembling tropes. What explains the extension in *a*'s exactly resembles the extension in *b*? Nothing. That the two tropes exactly resemble each other is taken to be metaphysical bedrock, not subject to further explanation.

3. Gains and losses between the two theories

Arguably, the move from metaphysical realism to trope theory sacrifices some explanatory value for the theoretical advantage of having a more economical theory. Although trope theorists are still committed to the existence of properties over and above objects, their ontology is still much slimmer than that of the metaphysical realist, in that they only sanction the existence of particulars. The price they pay for this economy is that their explanation stops before that of the metaphysical realist. *A* and *b* are qualitatively identical in a certain respect, and the trope theorist can explain this up to a point: both objects have exactly resembling tropes. But, whereas the metaphysical realist can go on to explain the *qualitative* identity (in a certain respect) of *a* and *b* in terms of the *numerical* identity of the extension of *a* and the extension of *b*, the trope theorist accepts the exact qualitative similarity between the two extensions, to be the metaphysical bottom line. The trope theorist's explanation comes to an end before that of the metaphysical realist.

So the trope theorist has over the metaphysical realist ontological simplicity. But the metaphysical realist has over the trope theorist explanatory depth. Of course, just to describe things thus is not to make a case for either one of these views. More would have to be said about the comparative weights of these theoretical advantages. At the moment, I do not wish to argue for one position or another. But I do want to suggest that, seeing things in this light, there is another position one may take as to the metaphysics of qualitative similarity.

4. Metaphysical primitivism

What we are trying to explain is what it is for *a* and *b* to be both extended. Relative to metaphysical realism, trope theory sacrifices some depth of explanation, in order to give ontological economy. Why not take this move to extremes? A position, which we might boldly call, 'metaphysical primitivism', holds the following. That *a* and *b* are both essentially extended objects is metaphysically fundamental, there is no deeper explanation. Just as the trope theorist takes the extension of *a*'s exactly resembling the extension of *b* to be the bottom line, not subject to further explanation, so the metaphysical primitivist takes *a* and *b*'s being essentially extended to be the bottom line. There is no further explanation for the fact that both objects are extended, because we have reached bedrock and our spade has turned.

Presumably, relative to metaphysical realism, metaphysical primitivism would have the same advantages and disadvantages as trope theory, only to a greater extent. Just as trope theory falls short of metaphysical realism in explanatory power, so too will metaphysical primitivism fall short of the

explanatory power of trope theory. Trope theory has *some* explanation of why both objects are extended: they both instantiate exactly resembling tropes. But it has no deeper explanation of what it is for two tropes to exactly resemble each other. This is where its explanatory power comes to an end. The metaphysical primitivist cannot even explain to the degree that the trope theorist does. The metaphysical primitivist simply accepts that *both objects being essentially extended* is a primitive, not subject to further explanation. It has zero explanatory power.

I guess strictly speaking metaphysical primitivism is not really an explanation at all. It offers no explanation of the phenomenon in question. It is nevertheless surely a coherent position on the metaphysics of qualitative identity. One might think it is not a very *good* position, in that it fails to explain anything at all. But for now my only concern is to secure its position on the map.

On the up side, just as trope theory is more economical than metaphysical realism, metaphysical primitivism will be more economical still. Trope theory has the theoretical advantage over metaphysical realism that it is not committed to universals of any kind. Nevertheless the trope theorist's peculiar kind of explanation does involve him investing in an extra kind of particular over and above objects: the trope. The metaphysical primitivist is committed neither to universals, nor to tropes. She is committed to nothing other than *a* and *b* themselves. This is clearly a highly economical position.

5. Is metaphysical primitivism consistent with truth-maker theory?

One might imagine the following objection to metaphysical primitivism:

> *If a metaphysician wants to hold that it is true that two objects are extended, then she is obliged to cough up sufficient truth-makers to buy this truth. The mere existence of object a and object b cannot by itself add up to the truth of the sentence: 'a and b are both extended'. The existence of a and b is sufficient for the truth of the sentence: 'a and b exist'. But surely for a and b not only to exist, but to be extended, some other truth-maker is required.*

No truth-maker theorist suggests that truth-makers must correspond one-to-one with the sentences which they make true. The existence of scarlet will serve as truth-maker both for the sentence, 'There exists scarlet', and the sentence 'There exists red'. So if the truth-maker theorist is to have a problem with the mere existence of *a* and *b* being sufficient truth-makers for both '*a* and *b* exist' and '*a* and *b* are extended', then he must offer some strict criterion on what can and cannot count as a truth-maker for a given truth.

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David Armstrong suggests that the mere existence of truth-makers ought to *necessitate* the truth of the sentences which they make true. He calls this view 'truth maker necessitarianism':

> *P* (a proposition) is true if and only if there exists a T (some entity in the world) such that T necessitates that *p* and *p* is true in virtue of T.

But for the metaphysical primitivist, *a* and *b* are, necessarily, extended objects. That they are extended is not a matter of their being involved with some other entity, a universal or a trope. It is simply primitive that *a* and *b* are both (necessarily) extended. Given that *a* and *b* are necessarily extended, then the mere existence of *a* and *b* necessitates their being extended. Even on the truth-maker necessitarian's grounds, the mere existence of *a* and *b* is sufficient truth-maker for the sentence: '*a* and *b* are extended'.

> *There is some sleight of hand going on here. To say that a and b are necessarily extended is just to say that a and b necessarily instantiate the property of extension, which involves necessitating the existence of the property of extension. The claim that a and b are necessarily extended just sneaks in the property of extension (which then serves as a truth-maker for (1)) without anyone noticing.*

For the metaphysical realist or the trope theorist, for *a* and *b* to be necessarily extended *just is* for *a* and *b* to necessarily instantiate the property of extension, which involves necessitating the existence of the property of extension. But for the metaphysical primitivist, it is the other way round. If it is true to say that *a* and *b* instantiate extension (I will address this issue presently), then this is because *a* and *b* are necessarily extended. That *a* and *b* are necessarily extended is the metaphysical bottom line.

But does the metaphysical primitivist have the right to this primitive fact? It is useful here to compare metaphysical primitivism with trope theory. The kind of primitivism the metaphysical primitivist accepts is not different in kind, although perhaps in degree, to the kind of primitivism the trope theorist accepts. Just as trope theory accepts that two extensions' exactly resembling each other is not a matter of their being involved with some other entity (e.g. a universal of resemblence), but is primitive, so the metaphysical primitivist holds that the fact that *a* and *b* are essentially extended is not a matter of their being involved with some other entity (e.g. a universal or a trope), but is primitive. Just as the trope theorist holds that the mere existence of two extensions necessitates their resembling each other and hence is sufficient truth-maker for the truth that the two tropes exactly resemble each other, so the metaphysical primitivist holds that the mere existence of the two objects necessitates their both being extended and hence is sufficient truth-maker for the truth that *a* and *b* are extended. If trope theory is consistent with truth-maker theory, then so is metaphysical primitivism.

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Of course, it might still be objected that the greater degree of primitiveness that metaphysical primitivism accepts, and its correspondingly greater failure to explain, shows that metaphysical primitivism is a bad theory. But it is merely the *coherence*, not the theoretical value, of metaphysical primitivism which I am currently endeavouring to show.

6. Do metaphysical primitivists believe in properties?

But one might still have doubts about whether metaphysical primitivism is a coherent view. I have already suggested that one theoretical advantage of metaphysical primitivism is its lack of ontological baggage. It is committed only to the particular objects *a* and *b*. It denies the existence of properties, in this case extension. Nevertheless it accepts that *a* and *b* are both (non-reductively) extended. But surely to accept that both *a* and *b* are (non-reductively) extended *just is* to accept that extension is present in the world?

To put it more broadly, the metaphysical primitivist believes that objects are *propertied,* but denies that this commits her to the existence of properties. But is it not the case that to believe that objects are propertied *just is* to believe that properties are present in the world? Is it not straightforwardly contradictory to accept that objects are propertied, but deny the existence of properties? To accept that objects are in ways, but to deny the existence of those ways?

There is an innocent sense in which metaphysical primitivists believe in properties. If all it means to say that extension exists is that some objects are extended, then in this sense the metaphysical primitivist believes in extension. In general, if all it means to say that properties exist is that objects that are propertied exist, then in this sense the metaphysical primitivist believes in properties. But properties in this sense are no addition in being to the propertied objects on which they supervene. The ultimate truth-makers for sentences asserting their existence are not properties, but particular propertied objects. In so far as metaphysics is interested in what entities a certain view offers as minimally sufficient truth-makers for all true sentences in a certain domain, and in so far as it equates the ontological commitments of a given view with such minimally sufficient truth-makers, metaphysical primitivism is a nominalist position. In an important sense, metaphysical primitivism is not committed to the existence of properties.

Compare again to trope theory. If all it means to believe in the existence of similarities between tropes is to believe that tropes are (non-reductively) similar, then in this sense trope theorists believe in the existence of resemblances between tropes. But resemblances in this sense are no addition in being to the tropes on which they supervene. Just as there is an innocent sense in which metaphysical primitivists believe in properties, so there is an innocent sense in which trope theorists believe in resemblances. But there is an important sense in which trope theorists are not committed to resemblances: for the trope theorist resemblances do not serve as ultimate truth-makers. Just as the trope theorist is not ultimately committed to the existence of resemblances, so the metaphysical primitivist is not ultimately committed to the existence of properties.

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7. What is it to be the metaphysical bottom line?

Metaphysical primitivism holds that ultimately the world is a world of propertied objects. This is the metaphysical bottom line. Propertied objects are the ultimate truth makers for true property attributions. But what is it for a certain description of reality to be ultimate, to be the metaphysical bottom line? What is it for certain entities to be the 'ultimate truth-makers'? So far I have relied on an intuitive understanding of this, but it would not be unreasonable for a reader to demand clarification on this point.

To a large degree we can rank descriptions in terms of the notion of asymmetric supervenience. The complete physical description of the world is more basic than the complete biological description of the world in the sense that the truth of the former guarantees the truth of the latter but not vice versa. God need only fix the physical facts to guarantee the truth of the complete biological description of reality. In this sense the physical facts are the ultimate truth makers for the biological truths, and the biological facts are no addition in being to the physical facts. But this form of ranking is inapplicable when the supervenience between two descriptions of reality is symmetrical. This is exactly what we find in the present case.

Take the following two sentences:

(1) '*a* and *b* are both extended'.
(1*) '*a* and *b* both have the property of extension'.

There is no doubt that these two sentences are logically equivalent, i.e. the two sentences mutually entail each other, and so the notion of supervenience will be no use in deciding whether 1 or 1* is the more basic description: both descriptions supervene on each other.

Nevertheless, it is surely a legitimate question which of the two descriptions is the more *metaphysically perspicuous* description of the reality they both describe. Take the following two sentences:

(2) 'There are families'.
(2*) 'There are individuals who are related to each other'.

Just as with (1) and (1*), (2) and (2*) are logically equivalent. Nevertheless, it seems clear that (2*) is the more metaphysically perspicuous description of the reality both sentences describe. (2*) offers us an account of what it is, metaphysically, for there to be families; (2) does not offer us an account of what it is, metaphysically, for individuals to be related to each other. There exist families in virtue of there being individuals who are related to each other, and not vice versa. The existence of individuals that are related to each other is the ultimate ontological reality in virtue of which both sentences are true. Even though there is a sense in which (2) and (2*) 'mean the same thing', i.e. they both assert the same

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state of affairs to obtain, (2*) is the more metaphysically perspicuous representation of that state of affairs.

Do then families exist? Does the fact that we can paraphrase (2) for the more metaphysically perspicuous (2*) mean that we can define families out of existence? Of course there is a perfectly legitimate sense in which families exist. Families exist if people are related to each other, and people are related to each other. But as philosophers deciding what we are ontologically committed to, we ought to be interested in existence in a more restricted, admittedly technical, sense. Those entities which exist in this more restricted sense are those which appear in the most metaphysically basic, most metaphysically perspicuous, description of reality.

Choosing the more metaphysically perspicuous description from between (2) and (2*) is straightforward. Choosing the more metaphysically perspicuous description from between (1) and (1*) is more controversial. That is why the ultimate reality of qualitative identity is a philosophical issue, whilst the ultimate reality of families is not. The metaphysical realist holds that (1*) is the more metaphysically perspicuous description: *a* and *b* are extended in virtue of *a* and *b* exemplifying extension. The metaphysical primitivist holds that (1) is the more metaphysically perspicuous description: *a* and *b* exemplify extension in virtue of *a* and *b*'s being extended. In a philosophically important, admittedly stipulative sense, the metaphysical realist, but not the metaphysical primitivist, is not committed to properties.

8. How does metaphysical primitivism fair with contingent properties?

One might imagine the following objection:

> *You have cleverly picked the sentence under consideration to your advantage. When it comes to explaining qualitative identity of essential properties, metaphysical primitivism arguably has sufficient truth-makers. Given that a and b are necessarily extended, the mere existence of a and b is sufficient truth-maker for the truth of the sentence: (1) 'a and b are extended'. But what of sentences referring to ways in which a and b happen to be, e.g. (3) 'a and b are spherical'. Given that a and b are (we can suppose) only contingently spherical, the mere existence of a and b does not necessitate the truth of (3). There are possible worlds, and presumably other times in the actual world, where a and b exist, and yet (3) is false. According to truth-maker necessitarianism, then, a and b are not sufficient truth-makers for (3). To get the truth of (3), we must add (at least) the existence of the property of sphericity. But then if we are going to end up investing in properties at some point along the way, we might as well invest in them to start off with. The demand for theoretical uniformity would suggest that we ought to give the same account of contingent property possession as we give of necessary property possession. It would be deeply unsatisfying if we were to give*

> *one account of that it is for a and b to be extended, and another account of what it is for a and b to be spherical. But this would seem to be what the metaphysical primitivist ends up with.*

It is of course true that if *a* and *b* are contingently spherical, their mere existence will not necessitate the truth of (3). But if we can make sense of the notion of an object being *necessarily* primitively propertied in a certain way, such that its being propertied in that way is fundamental and cannot be explained in more basic terms, such as its being related to a property (and I see know reason why we should not, the propertied object seems to me a much more intelligible entity than the universal or the trope), then surely we can make sense of the notion of an object's being *contingently* primitively propertied in a certain way, such that its being propertied in that way is primitive and cannot be explained in more basic terms, such as its being related to a property. There seems to be no essential connection between a fact about an object being *primitive*, and its being *necessary*.

It seems then that we can make sense of *a* and *b*'s being primitively contingently spherical at a certain time and world. If *a* and *b* are primitively contingently spherical at time t and world w, then they will surely be sufficient truth-makers for the truth of (3) at t in w, even though, strictly speaking, the mere existence of *a* and *b* does not necessitate the truth of (3). If metaphysical primitivism can provide truth-makers for necessary truths, then it can provide truth-makers for contingent truths.

I conclude therefore that metaphysical primitivism is a coherent position on the metaphysics of qualitative identity, which is consistent with truth-maker theory. I now turn to the question of whether metaphysical primitivism is a *good* theory, as well as a coherent one.

9. Is metaphysical primitivism a good theory?

As I have already claimed, metaphysical primitivism is a highly economical theory. Unlike metaphysical realism, it is not committed to the existence of universals. Unlike trope theory (and metaphysical realism) it is not committed to the existence of properties. It does not even require something so ontologically cheap as a set. Metaphysical primitivism looks like the cheapest position on the market. I take it that all thing being equal, we ought to go for the more economical theory. If all things are equal, metaphysical primitivism will clearly be the view we ought to go for.

But we have also noted that there is a considerable worry that primitivism scores very badly on explanatory value. Indeed, strictly speaking, primitivism fails to be an explanation at all. It accepts that object *a* and object *b* are both spherical. But it offers us literally no account of what this entails: of what it is, metaphysically, for two objects to be spherical. Metaphysical primitivism is the most economical theory, but it is also seems to be the least explanatorily powerful.

But the 'failure' of primitivism to give a substantial explanation is only significant if there is something that needs explaining, something the other theories

do better. But it is far from clear to me that there is anything to explain here. Let us call the fact that *a* and *b* are spherical (at the present moment) 'F' (I take the existence of this fact to supervene on the existence of the two spherical objects). There are of course many explanations of F that the primitivist is quite entitled to give. She might give a causal/historical explanation of this fact, perhaps involving ball making factories. Perhaps, in principle if not in practice, she could give a story in completely physical terms to the effect that, given the laws of nature and the initial conditions of the universe, it was (naturally) inevitable that F would obtain. In this sense, the primitivist can explain F as much as anyone else.

Alternatively, someone might need some explanation of F, the fact the *a* and *b* are spherical, because they do not fully understand the sentence (3) '*a* and *b* are both spherical'. Perhaps they lack a clear understanding of the meaning of the word 'spherical', and in this sense do not know what it is to be spherical. In this case what is required is not a metaphysical explanation, but a geometrical definition: an object is spherical when all the spatial points on its surface are equidistant from its centre. But this is the kind of explanation which the metaphysical primitivist has as much a right to give as anybody else.

But let us say that I have sufficient explanations in both these areas. I understand what it is for *a* and *b* to be spherical, in the sense that it is for both of them to have all the spatial points on their surfaces equidistant from their centres. I also know how *a* and *b* got to be spherical: they were shaped in a factory in Dundee. It is not clear that there is anything left here to explain about what is going on metaphysically. It is not clear that the 'explanation' that the metaphysical realist, or the trope theorist, or the more extreme nominalist, is offering, is one that is needed.

The opponent of metaphysical primitivist may still shake her head, and claim that although there is something that everyone understands about what it is to be spherical, or to be propertied in general, there is still something of which we are ignorant: some explanatory gap which the metaphysician needs to fill. But then it seems to me that the onus is on the opponent of metaphysical primitivism here to tell us exactly what is still left to be explained. In what sense do we understand what it is for *a* and *b* to be spherical simply by (fully) understanding the sentence: '*a* and *b* are spherical', and in what sense is there something still left to be explained?

Until this is done, I think we ought to assume that the metaphysical explanations that the metaphysical realist and the trope theorist give about what it is for two objects to be spherical are redundant. The weird entities, tropes, universals, which they invest in along the way are a waste of money. If this is correct, then metaphysical primitivism, the only account which does not try to explain what need not be explained, the only account which does not spend money where none is needed, is the correct view of the metaphysics of qualitative identity.

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Towards a Working Trope Ontology

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Abstract. In the present paper, I argue that a one-category ontology, based upon individual property-instances (tropes) only, is to be preferred. I show that the traditional alternatives have insurmountable problems, and formulate trope ontology in such a way that it can solve them, and also overcome the difficulties on the basis of which criticisms are usually moved against tropes themselves. Lastly, I show that a specific understanding of tropes can be extracted from contemporary physics.

1. Ontological possibilities

1.1 The general categories

Traditionally, philosophers have interpreted reality by helping themselves to a number of basic categories, first formulated by Aristotle as an exhaustive representation of the elements of reality:

	Particulars	**Universals**
Said of a subject	A) Individual properties (Socrates' greekness)	B) General properties (Greekness)
Subject of predication	C) Individual substances (Socrates)	D) Genera and species (Man, Animal)

Categories A) and B) refer to properties. Realists about universals contend that A) is derivative on B), that is, that we have only one, say, 'redness', and that every case of a red thing is a case of an instantiation of the *same* universal at a particular space-time location (or in a particular material *substratum*). Nominalists deny this, and rather take B) as derivative on A), in the sense that only individual property-instances exist, and universals are only conceptual/linguistic entities.

Category D) is the category of natural kinds. As argued by Russell, this category is reducible to that of universals (B)): one can modify the term, say, 'man' and obtain the property of 'being a man', which belongs to category B).

Lastly, C) is perhaps the most intuitive category: it defines those individual, concrete, space-time located, entities in which properties inhere, and which we usually treat as subjects in our language.

Given these four fundamental categories, one can move onto an analysis of the way they are put to use with a view to formulating one's ontological assay. The traditional view has it that everything that exist is a concrete particular (C)) that instantiates one or more universal properties (B)).

1.2 Substrata and properties

This view is commonly associated to the names of Aristotle and Locke.

Aristotle argued for the need for a '*substratum*' bearing property-instances on the basis of the simple idea that attributes are necessarily attributes of something that is the subject of predication and is ontologically prior to them.

He made a clear distinction between 'matter' and 'form', and took the former to be the fundamental 'stuff' constituting material reality. Anything in the universe is composed of a quantity of matter, which is 'qualified' by the particular form(s) that inhere(s) in it. Properties, which Aristotle identified with this latter element, are dependent on matter, i.e., they need a material support and cannot subsist without it. But, at the same time, they are essential in order for the subject to actually be something, in the sense of *existing as* a specific kind of thing rather than another. In short, matter needs form in order to be qualified, and form needs matter to perform its very role of defining the 'way in which things exist'.

The intuition that this structure mirrors the ontological structure of the world surely appears *prima facie* correct, especially if one considers that our language clearly has a substance-attribute structure.

Nevertheless, well-known problems immediately arise for any view that postulates a *substratum* over and above properties. On the one side, these difficulties are basically related to the *ontological nature* of this *substratum*. On the other, there is an *epistemological* difficulty.

Starting with the ontological issue, the point is that there appears to exist an internal inconsistency in the notion of a *substratum*. If something is property-less, it has to be so simple that it cannot actually perform any 'ontological work'. So the very idea of a basic property-less *substratum* seems inconceivable.

This sort of argument can be rejected, though, by claiming that 'property-less' here only refers to a lack of *empirical* properties. When defining a *substratum* as property-less, one is in fact already attributing a property to it. Even a bare particular has, thus, a number of *metaphysical* features. Being property-less, in particular, means that empirical intrinsic properties *cannot be attributed to the substratum yet*.

One remaining difficulty is, though, that if *substrata* exist as individuators, they must have a least one metaphysical feature, namely, that individuality that distinguishes the complexes they participate into from all the rest of the entities

that exist. Even though, as I have just argued, this is not in itself contradictory, one might legitimately suggest that the same move can be implemented in ways that do not require the postulation of Aristotelian matter: for example, by putting individuality in the property instances directly;[1] or, alternatively, in the space-time points at which universal properties are instantiated.

Let's now turn to the epistemological issue. Russell ([1912, Ch. 5] and [1917]) suggested that it is plausible to endorse a '*Principle of Acquaintance*' setting empiricist constraints on what we should believe. The gist of the Principle of Acquaintance is that if something is not directly known (or, *a fortiori*, knowable), but only indirectly reconstructed, then we can cast doubts on its existence; an obvious corollary is that whenever the role this mysterious something is supposed to play in one's ontology can be equally performed by one or more different categories that are instead knowable, we should prefer the alternative scenario.

Substrata appear indeed to be known only indirectly ('by description') and thus to be dispensable in one's ontology. Locke argued that all we know about things in the world are properties, so when it comes to defining the substance acting as the *substratum*' of these properties, the latter inevitably looks like an 'I don't know what' that can't *in principle* be known directly. Some writers (among the most recent ones, LaBossiere [1994]) try to answer that this is not the case, and we actually *get to know substrata without any violation of the Principle of Acquaintance*: we simply have to count among our experiences the acquaintance with the things' individuality and with the properties' belonging to an entity rather than another, and being unified in it; but all these experiences are such that through them we directly get in contact with the things' *substrata*.

However, such arguments fail, as they cannot be used to prove that we know substrata by acquaintance, but merely *suggest an inference to the best explanation*, i.e., an '*abductive*' inference *from* the obvious fact that we experience distinct individuals around us and that we can tell one thing's properties apart from the other things' properties, to the existence of *substrata*. Surely, though, other explanations, not referring to bare particulars as *substrata*, are logically possible. Since this is the case, one incurs in the well-known problems that affect all abductive arguments, the crucial point being that they can be only presented as best explanations from a specific point of view and under certain substantial subjective assumptions.

In other words, by weakening the -clearly untenable- initial claim that we know *substrata* by acquaintance, and trying to show that their existence can be inferred from the available data about reality as something necessary, one is simply left *assuming* this existence, failing to provide the necessary support to dualist (*substrata* plus properties) ontologies, at least until all the alternatives are proven not to be viable. For obviously reference to *substrata* not appearing necessary to other ontologists, the latter will surely reject the idea that recurring to them actually constitutes the best explanation.

[1] Which is what trope ontologists do.

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A spirited defence of the notion of *substratum* against the kind of doubts raised by the Lockean idea that it is a mysterious 'I don't know what' has been launched in the last thirty years or so by Loux (see his [1978]). He argued that substances cannot be conceived of apart from their properties and that Aristotle himself, while distinguishing matter and form, clearly held that they only exist together. In the world, that is, there only are substances as unions of *substrata* plus a number of essential properties.

This might well be correct from the point of view of the interpretation of Aristotle,[2] and even plausible from the ontological point of view, but it hardly counts as an argument in favour of *substrata*. If they are unable to exist without properties, and we only experience substance+properties compounds, why should we include *substrata* in the metaphysical inventory at all? To really make sense of Loux's *substance ontology*, with the word 'substance' intended as referring to concrete individuals *intrinsically provided with properties*, one seems forced to move towards a more radical nominalism that coherently takes the whole property/property-bearer distinction as a purely conceptual one, without an ontological counterpart. Such a monist ontology must be, however, distinguished from the dualist *substrata*+properties one that we are currently discussing, and will be examined below.

It seems thus fair to say that the *substrata*+properties option appears pretty weak. The available alternatives to it, that we now need to examine, are the following:

(i) Properties as universals only, or

(ii) Particulars with qualities but without properties as a separate category, or

(iii) Properties as particulars only.

For the time being, the analysis made in this section allows us to formulate a first *desideratum* for any consistent ontology. Even though *substrata* ontologies are not entirely convincing, they address an issue that cannot be avoided and that any plausible ontology certainly has to take into account. This is the issue regarding the 'unity' of individuals as possessing properties; that is, the fact that we experience complex individuals with many properties and are able to identify them, distinguish them from others and also tell properties of one of them apart from those of other individuals. In short, a *desideratum* that any viable ontology has to satisfy is:

DES1: the preferred ontological account has to explain what it means to say that property *P* is a property of individual *x* or, in other words, that individual *x* as a unitary complex *exemplifies* property *P*.

This said, let's now move on to an analysis of the possible alternatives, namely options i), ii) and iii) above.

[2] This seems to be, indeed, the best interpretation of category C) above as it appears in Aristotle's writings.

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1.3 Universals only? The bundle theory

The first view, which is known as the '*bundle theory*', is perhaps the most common alternative to the *substratum* ontology.

What we call universals are nothing but Plato's '*forms*' or '*ideas*'. Thinking about the multiplicity of similar things and the various degrees of (im)perfection of things, Plato reached the conclusion that our world is an imperfect copy of the true Reality, which is a realm of perfect entities that exist separately.

The basic argument for this conclusion is the well-known *one-over-many argument*, that can be formulated as follows:

(1) No F particular is F in virtue of itself

(2) Whenever many Fs are F, they are F in virtue of having some one thing, the F, predicated of them

(3) The F is predicated in the same way of all the particular Fs

(4) Therefore the F is something besides particular Fs, and separated from particular Fs

(5) Since the F must be available for predication and is the most perfect case of F-ness, it is everlasting

(6) Whatever is a one-over-many, separated, and everlasting is a form

(7) Therefore the F is a form

Plato's realm of the ideas contains, therefore, unique perfect '*exemplars*' that all worldly particulars 'participate into' or 'imitate'. Endorsing such a metaphysical picture allowed Plato to explain both similarity and difference across multiplicity, that is, the reason why the same property can be predicated of many subjects, and at the same time be exemplified in different ways by different subjects, and yet remain the same property.

In the XX century, Russell revamped such arguments in favour of realism about universals, also arguing that universals are necessary as truth-makers.

In his [1940], he eventually came to the view that every individual substance is just a bundle of universals. He realised that the role played by the particular 'property-bearers' that he previously took to be indispensable and necessary -mainly to account for numerical difference-, could actually be reconstructed by deriving particulars from universals. The numerical difference of what appear to be distinct concrete particulars is simply to be accounted for in terms of the bundles of universals being instantiated at different space-time locations. Therefore, given an ensemble of different space-time points, everything else can be explained by recurring to universals only.

Adopting this view of properties, however, causes well known problems: if properties are conceived of as entities actually existing in a heavenly world (more generally, to use the technical definition, as *ante res* universals) an explanation of

the relation between what we experience and such things must be given. But this explanation seems difficult to formulate.

Aristotle made it clear that there must be a relation between Platonic ideas and the things which exist in our world and exemplify them: but these two distinct kinds of things cannot 'communicate' without such a relation being made possible by a third entity, at the same time numerically different both from the idea and its exemplifications and qualitatively similar to both. This latter similarity must then be explained in turn, and one thus needs to introduce two more similarity- (i.e., participation-) relations, and so on *ad infinitum*.

This argument, known as Aristotle's '*third man*' argument, generally applies to any realism about universals which takes the latter to exist independently of their instantiations (*ante res*). To provide the classical example, think about the universal 'man' or 'being a man':

(1) If *x* is a man, *x* is a man by participating in the form of 'manhood', which we can call level-1 manhood, and is a paradigm of which *x* is a likeness

(2) Paradigms and their likenesses are similar to one another

(3) Therefore, if *x* participates in level-1 manhood, *x* is similar to it

(4) If any two things are similar to one another, they are similar because they participate in some one form

(5) Therefore, if *x* and level-1 manhood are similar to one another, there is some one form in which they both participate, in virtue of which they are similar

(6) Therefore there is another form of manhood, level-2 manhood

(7) This process must be necessarily reiterated generating higher and higher-order 'manhoods'

In the light of this, Aristotle recommended the rejection of *transcendental*, *ante res* universals and argued that they should instead be understood as *immanent* (*in rebus*). That is, universals are real entities, but they don't have to –indeed they cannot- be distinct from their instances; instead, they are simply to be identified with the latter.[3]

A bundle theory supported by immanent realism about universals appears promising, as it provides a simple and economic ontology in which similarity, difference and the dynamics of linguistic predication are readily accounted for.

However, there actually are some difficulties for this view.

[3] Note that this means that what we referred to above as Aristotelian 'forms' are immanent universals; this is compatible with the idea, which is a crucial *tenet* of Aristotelian philosophy, that everything that exists is individual. Armstrong [1978] is a contemporary advocate of *in rebus* realism about universals, universals necessarily exist as particularized in 'states of affairs'.

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Universals are assumed to have the peculiar feature of 'ubiquity', i.e., multiple instantiation of one and the very same entity. This is exactly what allows to straightforwardly explain what makes different things exactly similar; but it also appears to have very undesirable consequences as soon as we carefully consider what is known as the Principle of the Identity of the Indiscernibles.

The Principle of the Identity of the Indiscernibles states that if two objects have all their properties in common, then they are the same entity.

Formally,

$$\forall x \forall y (\forall P (Px \leftrightarrow Py)) \rightarrow (x=y) \qquad \text{(PII)}$$

According to PII, individuality implies distinction: if two entities are to count as two, there must be some feature which distinguishes one from the other. Therefore, PII can be used to 'pick out', as it were, individuals by pointing at (some of) their properties.

But now, if we suppose to have two identical bundles, we will immediately conclude that they actually are the same bundle: the fact that universals are identical (numerically) across their instances, and that nothing else exists, cannot but imply that multiplicity is simply an appearance, for there are no individuators of the specific (bundles of) property instances.[4] Therefore, the bundle theorist appears to be committed to the necessary truth of PII. O'Leary-Hawthorne, for one, tries to argue that this doesn't contradict the possibility of a plurality of qualitatively identical things; he says that the conceivability of Black's universe[5] just proves that "the following possibility is a very genuine one: There is a bundle F, G, H [etc.] five feet from itself and nothing else" [1995, 193].

However, there seems to be an ambiguity here, in that PII is the principle according to which exact similarity implies numerical identity, so it is at least unclear how it can be said to be true in spite of the possibility envisaged by O'Leary-Hawthorne. It looks as though a clear distinction between repeatable universals and unique instances must be postulated. But then, why exactly are universals taken to be real entities? The main reason seems to be, as we said above, that realism about universals explains similarity straightaway. A second desideratum for any alternative ontology thus naturally emerges.

DES2: the preferred ontological account has to explain similarity between different individuals.

[4] Obviously one could refer to *substrata* or space-time points in order to differentiate the qualitatively identical bundles, but we are assuming a 'radical' version of the bundle theory here, not parasitic on anything else.

[5] Black [1952] hypothesised a counterexample to the PII. He imagined a universe constituted by two identical spheres with the same properties F, G and H... (same chemical composition, same diameter, same temperature etc.) located five feet from each other and nothing else. This universe is such that the spheres have all their properties -monadic and relational (including space-time position, which is necessarily defined in terms of relations to the 'other sphere')- in common. By PII, this should entail that they are the same entity. But the two spheres were assumed to be numerically different, so we allegedly have a counterexample to PII.

Together with DES1, DES2 would also guarantee the ability of one's preferred ontological account to present the posited basic entities as truth-makers for all our meaningful sentences, as required by Russell and Armstrong.

1.4 Quinean nominalism

As seen above (option ii)), an alternative, but equally reductionist possibility, is that of an ontology with individual concrete particulars only.

This is the sort of nominalism supported, or at least suggested, by Quine, according to whom the concept of property does not point towards an autonomous ontological category, but only defines the specific *respects* in which concrete particulars (which are all that exist) resemble to/differ from each other.

Let's see how Quine gets to the formulation of this peculiar variety of nominalism.

In his [1948] and [1953], he offers a simple logical argument to the effect that there is no need to posit entities corresponding to attributes and relations, for the use of an abstract noun need not have any ontological consequences. In agreement with his famous claim that 'to be is to be the value of a variable', he points out that abstract nouns generally appear in our common language outside the scope of existential and/or universal quantifiers, and this implies that using them doesn't require any ontological commitment. In other words, if we say that, for example, there is an individual *x* with property *P*, in logical form

$\exists x(Px)$,

we are committed to the existence of *x*, and possibly of individuals of the same type, but not to the existence of *P*. The meaningfulness of a term simply doesn't entail that it has a reference.

Quine also emphasises the relativity of our linguistic schemes, the availability of different schemes of quantification, and the consequent impossibility of deriving one unique ontology from the analysis of language. But there surely is one 'privileged' linguistic structure, and with respect to it he recommends to look carefully and derive all and only the necessary ontological consequences. Once we do so, as we have just seen, radical nominalism appears immediately plausible. What it entails is that we should be strongly 'deflationist' regarding properties. Any abstract noun can be replaced by a predicate expression wich is either true or false of any individual without the noun designating anything. It is only the individual itself that must be assumed to exist. So, for example, all white things are white because, say, 'that is white' applies to each and every one of them, and this without reference to anything like a universal 'whiteness' if not in thought and language.[6] Properties reduce to set-theoretic constructions, intersections among sets of individuals.

Now, this surely makes sense, but further analysis shows that there are some problems. Talk of 'respects', or 'aspects' seems to require an internal complexity in things. If concrete individuals were unanalysable simples, non-decomposable

[6] Quine can then accept Russell's and Armstrong's point that universals are required as truth-makers, and yet give them a purely instrumental value.

unities, it wouldn't be possible to identify their *different* aspects, and talk about them in different ways. But, on the other hand, if they are complexes, it appears natural to analyse them in terms of the respects themselves, i.e., to decompose them into subparts.

To put it slightly differently, there seems to be an unresolved tension between the reductive attempt to make do with simple concrete particulars only, and the idea that these can meaningfully be subjects of various sorts of predication, even in spite of the fact that properties do not exist. If particulars have many aspects, they have to be analysable, and so the need for a non-reductive analysis becomes manifest; if they are really simple, they shouldn't have different aspects.[7]

One might respond to this difficulty in two ways:

One way is to relax the requirement that abstract individuals (properties) be completely dispensed with and say that they only exist in a concrete individual as their subject, but remain a distinct ontological category. But this amounts to nothing else than the Louxian-Aristotelian conception of substance we have already considered which, as we have seen, collapses onto the dualist (i.e., not nominalist about properties anymore) *substrata*+properties framework.

Another possibility is that of biting the bullet and accepting radical nominalism in a 'reductivist' variant. That is, one might try to argue that what we normally take to be individuals can indeed most of the time be analysed in terms of simpler components; but this analysis still identifies concrete individuals as ontologically fundamental. Namely, those that have only one respect. At this point, though, the doubt naturally comes to mind that if we go as far as looking for basic concrete individuals that ultimately coincide with only one 'respect', distinguishing Quinean nominalism from ontologies with properties only is just a matter of terminological preference. *Why not directly take the 'respects' themselves, that is, the properties, as basic?* After all, properties are the only 'stuff' that we know and the idea that they are necessarily dependent on something else might just be an appearance due to our logico-linguistic structures.

The above discussion regarding the bundle theory, though, has shown that this reduction cannot be performed in terms of universals. It appears thus natural to go for option iii), namely to *take individual property-instances as the fundamental ontological category*. This option will be examined in the next section. Before doing so, let's formulate one further *desideratum*:

DES3: the preferred ontological account has to explain what the basic, most simple individuals are, and explain everything in terms of these.

[7] Note that what we said about *substrata* and the possibility of defining them as property-less simples despite their possessing a number of metaphysical features cannot be repeated successfully here. For the aspects we are interested in here, and in relation to which we talk about concrete individuals, are empirical aspects.

2. Tropes

The idea that property-instances are particulars appeared in the work of Stout in the '20s (see Stout's [1921] and [1923]), and has been later advocated as the basis for a one-category ontology, most notably, by Williams (see Williams [1953]) and Campbell (see Campbell [1981]). According to these latter authors, only individual property instances, usually called '*tropes*' following Williams' usage of the term, exist. If this is the case, abstract particulars become the basic building blocks of all that exists, and concrete particulars turn out to be derivative on them. This is the ontology I want to defend in the present essay. Even though it seems quite a natural choice as soon as one undertakes the sort of careful ontological analysis performed in the previous sections, trope ontology still has to face some difficulties that seem to emerge.

2.1 Alleged lack of autonomy of tropes

One criticism normally moved against property-based ontologies rests upon the conviction that properties are not independent entities. If properties are by definition things which get predicated of something else, they cannot -*in principle*- be autonomous. Therefore, the mere analysis of the structure of our language should suffice to show that the whole project of constructing a one-category ontology based upon properties is destined to fail.

It is though easy to respond that this is a mistake based upon the entrenchment of the dualist substance-property paradigm into our way of conceiving of, and speaking about, reality; and that, in general, ontological conclusions should not be drawn on the basis of language. As proven, for example, by arguments like Quine's on the possibility of existentially quantifying over entities in ways different from the usual one, the use of a certain language-structure doesn't entail anything necessary from the ontological point of view.

The fundamental thing that must be emphasised is that in a trope ontology property-instances, i.e., those entities that are usually defined as abstract particulars, become autonomous entities. That is, they remain abstract in the sense that they exist in complexes from which we separate them by an act of abstraction;[8] but they become, at the same time, concrete, in the sense that they don't exist as, say, Platonic ideas, or concepts in the mind, or '*possibilia*' that must be exemplified by some subject. Rather, they are what actually constitutes the material world.

This said, it is immediately clear that, unlike Aristotelian-Lockean *substrata*, that are unknowable in principle, tropes are a plausible candidate for occupying the position of basic building blocks of material reality exactly because they are exempt from the 'epistemic accessibility problem' that *substrata* ontologies have. Not only tropes are not unknowable, but the tropes' nature is *fully* exhausted by

[8] This is the reason why Campbell sticks to the definition of 'abstract particulars'.

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their causal efficacy,[9] i.e., by their knowable manifestation. Therefore, one has no difficulties in immediately identifying what can, and ought to, be taken as ontologically basic.

One might reply that this can perhaps be the case for some properties, but others cannot be so understood, for it makes no sense, for example, to speak about a concrete autonomous 'shape' trope, or 'colour' trope, as they must be necessarily be understood as inhering in something else which is the material 'thing', and cannot possibly be independent. All the attempts to show that, on the contrary, even the more abstract kinds of properties can be reified as tropes are rather unconvincing. However, a trope ontologist can perfectly accept this.

It is one of my central contentions that not all properties are tropes in the ontologically strong sense, i.e., that we don't have to maintain that, say, a particular shade of blue is an irreducibly basic material element of reality. Rather, I believe some kind of reductionism is very plausible here, and *only some of the properties we ascribe to things are 'truly real'. The others are simply constructs out of the former*. Differently put, while some properties are indeed tropes and, therefore, as material building blocks of reality, compose all the rest, other properties are not equally primitive and are, instead, analysable in terms of structures of tropes as their simpler components. Obviously, only the former category is what a trope theorist must take as basic.

Let's analyse our example of something being of some particular kind of colour. For a thing to be of a certain colour, it means that that thing reflects light waves in a particular way. But the quality of this reflection is related to the kind of surface that thing happens to have. And this, in turn, is reducible to the structure of the set of molecules the thing is composed of and, therefore, to the specific arrangement of particles that constitutes the thing. But particles can in turn analysed in terms of their properties (as tropes). It is thus evident that analysis immediately brings us to the most basic physical level: this is the level at which one ought to look for those entities that can be properly called tropes.

Tropes are thus only a subset (the fundamental subset) of the set of all properties. More specifically, a distinction has to be made between:

1. TROPES: tropes proper, which are to be identified with the basic elements of material reality. What these exactly are, it is the role of physics to clarify;

2. D-PROPERTIES: derivative properties, that is, complex structures of basic tropes that are not primitive individuals as their constituents are. These still count as individual properties, for no universal has to be postulated; therefore, one can call them tropes even though, strictly speaking, they are not tropes. Properties like colour are D-properties.

[9] In a way that is unachievable for traditional substance ontologies, and Quinean radical nominalism, which take what we know about things to be empirically accessible 'aspects' of more fundamental subjects.

At least some entities that we traditionally define as abstract particulars are therefore actually basic real individuals that we can take as independent of everything else and actually constituting all other things.[10]

2.2 Relations between tropes

Traditionally, trope theorists have postulated two basic relations between tropes, to be understood in simple set-theoretic terms. There exist, they say, sets of *compresent* tropes,[11] and sets of (exactly) *similar* properties. The former are the individual objects that populate the universe; the latter correspond to universals, understood in purely nominalistic terms.

This immediately attracted criticisms against the theory, that allegedly appears parasitic on those universals it purports to do away with.

Supporters of universals indeed typically claim that trope theorists cannot postulate these two specific relations (of compresence and similarity) without acknowledging that they must be universals. This seems particularly clear in the case of similarity. For example, if one were allowed to abstract from all the accidental features of, say, an actual triangle and consider it as representative of the geometrical properties of all triangles, one would still have to explain why it is a triangle. Either it resembles all other triangles, or it is a prototype to which all other things we can call triangles resemble. In both cases a relation of resemblance is constitutive of the very class of entities we are talking about. But this relation cannot be a particular, otherwise the same problem we had for triangles applies for instances of resemblance. The question is the same in both cases: what is it that makes the actual instances all instances of the same monadic property or relation? The relation of resemblance has, therefore, to be a universal. But once we (are forced to) admit one universal, then we might as well accept universals *tout court*.

The notion of compresence can be turned against the nominalist in the same way, and also appears to have one further problem. In order for two property-instances to be compresent, there must exist an already defined spatial (or spatio-temporal) setting enabling to define location and, with it, the very notions of 'presence' and 'compresence'. This amounts to endorsing space-time substantivalism, according to which space-time has an ontological status on its own, and exists independently of (and prior to) the existence of everything else. Yet, endorsing a specific view on the ontology of space-time on the basis of a preference regarding a different, and not obviously related, ontological issue

[10] One could reply that there are non-supervenient properties, i.e., properties which have real causal effects in the world but cannot be reduced to monadic tropes (like EPR-like correlations in quantum mechanics, or biological properties). I believe that, if this is the case, the basic point to be made is that such properties are to be taken as primitive *emergent tropes* and not, as it might seem appealing initially, as D-properties which we are unable to analyse. In other words, trope ontology doesn't require radical physicalist reductionism.

[11] In this light, for a property to be exemplified just means that it is part of a bundle of tropes that exist together.

appears contentious. Moreover, we aim to formulate a one-category ontology which is not parasitic on other metaphysical assumptions.[12]

Let's then see what can be said about compresence and resemblance from a trope-ontological point of view.

2.2.1 Compresence

As for compresence, various analyses have been put forward in order to make sense of it by reducing it to something else that can be more easily handled in a trope context.

Simons [1994], for example, suggested replacing it with Husserl's *foundation relation*. Husserl says that an entity *t* is founded on another entity *s* if *s*'s existence is necessary for *t*'s existence. And *s* and *t* are *directly foundationally related* if each one is founded on the other. Tropes, Simons claims, can be such that given a collection of them, each one is foundationally related to every other in the collection and nothing else. Bundles of foundationally related tropes, he calls '*foundational systems*' and identifies with the fundamental constituents of physical reality: objects, Simons maintains, are just entities that can be partitioned into elements forming a foundational system (these objects he calls '*integral wholes*').

While, according to Simons, the foundation relation allegedly "serves to bind things into a unity without requiring any further glue" [1994, 559], it is difficult to see why the relation of foundation should be any less problematic than that of compresence. Saying that two tropes necessarily *exist together* because they are directly foundationally related just begs the question, because the relation of direct foundation is itself explained in terms of reciprocal dependence, i.e., joint existence. Moreover, if we only refer to a notion of reciprocal dependence for existence, why should it be limited to individuals that *exist together at the same space-time point*? Since it is a fact that we take space-time localized complexes as individuals, and this is the specific reason why a relation of compresence seems to be required, Simons has to show that such a relation can not only be substituted by but also reconstructed in its entirety from foundation relations. Unfortunately, he doesn't appear to be able to do so.

In general, if we say that trope *t* and trope *u* are compresent because of a third trope/universal *c* they are in relation with, then we have to explain the relation between *t*, *u* and *c* in turn. And if, as it seems sensible, we don't want to talk about compresence again, we have to say that *t* and *u instantiate c*. The problem of explicating this notion of instantiation, though, is likely to have the same structure of the initial problem: either instantiation is primitive, but then we might as well take compresence as primitive; or it is analysed, but then it has to be taken as a further trope *c*', and its relation between *t*, *u*, *c* and *c*' must be explained in turn. It is clear that, by so proceeding, one ends up incurring in a vicious regress structurally similar to the infamous Aristotelian 'third man' argument against Plato's transcendental realism about Forms.

[12] Remember the comments made above when discussing the bundle theory.

Some would argue that all this shows that individual *substrata* are required. LaBossiere [1994], among others, contends that the only way to avoid the infinite regress is by acknowledging the existence of bare particulars endowed with the power to glue many tropes together and give them unity. Nonetheless, considering the arguments against bare particulars mentioned in section 1, it is at least unclear why one should take the postulation of the *substratum* as less *ad hoc* and more explanatory than that of primitive compresence. In particular, one might legitimately ask why it shouldn't be the case that also the relation between the *substrata* and their properties (as compresence, or as some form of 'tie') needs an explanation.

I believe that all this is sufficient to show that if s/he takes any extra relation among tropes ontologically seriously, no strategy will be of any avail fo the trope theorist. In this, Russell was substantially correct. I think, though, that it is not only possible to avoid ontological commitment as to the reality of the relation in question; *it is plainly wrong to suppose that the relation must be hypostatised*.

A common argument, presented among others by LaBossiere in the paper just referred to, and reminiscent of some Platonic passages, has it that if something has a real power, then it is real; in our case, since relations between tropes have certain causal consequences, they must be real entities. And this has to be the case, in particular, for relations that bind tropes together in unitary complexes. This argument is invalid, though, because obviously if we postulate a bind*er* then it must be real. But we can just postulate a bind, a relation between things, and this by no means entails that we have to believe in a further entity. After all, if John and Jack hug each other, we can say that they constituted a clear and tight relation, but by no means do we have to hypostatize the hug as a third real entity. In sum, there is a difference between the case in which the relation holds and that in which it doesn't, but the difference is *identifiable with* the relation itself as an arrangement of individuals, and not actively *determined by* it as something ontologically autonomous.

Admittedly, the hug analogy cannot be considered enough to solve the problem of compresence; but it is meant here to support the idea that the right way to go is to adopt a 'deflationary' strategy, in which no new entities are introduced beyond tropes to explain their being compresent. To put it simply, I believe that a sort of Quinean account of the ontological commitment forced on us by our language- and thought-structures can be applied to the analysis of the notion of compresence. Doing so makes it plausible the idea that compresence is just this: the matter of fact consisting in two things existing together, which we tend to reify in our language, but by no means needs to be a real entity on its own to be understood. A third category of properties appears thus to emerge:

3. C-PROPERTIES: Non-material properties, like compresence and other conceptual/logical properties. These exist in our minds, i.e., have a *conceptual* status, and are used by us to represent the relations between tropes (and

derivative properties). But they by no means need to be hypostatized.[13] Given the fact that the nominalist about universals can accept that universals exist in the mind, s/he can happily intend these C-properties as universals.

Getting back to compresence, let's see what we mean when we take it as a c-property expressing a simple matter of fact about tropes.

If space-time substantivalism is correct, then space-time points have primitive *thisness* and identify uniquely the property-instances: in this scenario, compresence expresses the fact that many tropes exist at the same space-time point.

In the case of relational space-time, instead, it would be the relations between tropes as fundamental individuals that would constitute space-time, and not the other way around. Compresence would mean coexistence of two (or more) individuals also related to other individuals spatio-temporally in the same way; *but without coexistence presupposing a spatio-tmeporal 'stage'*.

Lastly, even a recently developed 'third way', seeing space-time as a structural feature of physical reality which is neither derivative nor primitive, seems congenial to the trope ontologist's perspective. Sunny Auyang, (in various places, such as [1995], [2000] and [2001]) has rejected the whole substantivalism-relationalism dichotomy in the philosophy of space-time on the basis of the fact that the fibre bundle formalism, which is widely used in contemporary physics, allows to re-construct the role of space-time without taking a stance in the metaphysical debate regarding its nature. By analyzing the implementation of this formalism[14] in physical theory, she concludes that *space-time can be taken as absolute and objective but not substantival*. More specifically, the indexing function of space-time can be preserved without postulating space-time as a basic primitive 'stage' for the physical fields as 'actors'. Space-time does indeed make local fields distinct, but it doesn't 'support' them in any physical sense. Hence, space-time and the physical fields are *reciprocally related*, and cannot exist without each other: the space-time manifold which 'contains' and 'structures', on the one hand, and the concrete fields which get 'contained' and 'structured', on the other, actually 'come to existence' together.[15]

What this implies for our framework, is that, again, compresence can be derived from basic matters of fact about tropes and their arrangements. Two tropes would not co-occur at the same location but rather, by existing together, contribute to the 'coming into existence' of the space-time structure that allows for the *epistemic* individuation of them as being compresent.

Besides one's personal preferences, the fundamental point to be made here is that, unlike what happens for other ontologies, in the trope theory I am defending *any* understanding of space-time can be interpreted in such a way that compresence is explained away without the need to hypostatize it as a real universal.

[13] This is clearly argued already by Price [1953].

[14] Which is basically a generalization of the Cartesian product.

[15] In this sense structuralism differs from the relational views.

2.2.2 Similarity

Let's move on to the similarity relation. As we have seen while discussing the realist views on properties as universals, similarity is said to necessarily be a universal.

Campbell [1981] objected that resemblance could simply be taken as an internal relation between tropes, that is, as something that supervenes on the tropes and is not, consequently, an ontological addition to them. Without an explanation, though, Campbell's position goes dangerously close to assuming the supervenience of similarity in an ad hoc manner. However, a plausible explanation can be provided.

It just requires a sort of '*reversal of perspective*'. Similarity, that is, should not be taken as a primitive feature of things from which other features -including, most notably, sameness of causal efficacy- derive. Rather, *it is to be explained in terms of the of causal efficacy of things, where it is this latter element that becomes fundamental*. After all, in general, we don't get to know things, acknowledge their similarities and then observe or conclude for equality of causal powers. We rather group things together on the basis of their properties, the causal powers each one of them possesses separately from everything else, and *on the basis of the grouping* say that they are similar. For example, we don't assert that two things are of the same colour because they have the same property. That would be uninformative. Rather, two things are said to be (exactly) similar in colour because they stimulate our eye-structure in (more or less) the same way; whenever our eye has a certain reaction when hit by reflected light, we call the something which reflects that type of light by the same colour-name. More specifically, two things have (exactly) similar colour if they have significantly analogous (identical) causally efficacious surface structures. Obviously, defining in turn similarity of structures shouldn't be too difficult and is likely to be done by appealing to sameness of constituents (tropes) and geometrical-topological features.

So the basic similarities are those among the causal powers of the basic tropes, and all the rest can be reconstructed on the basis of these. And note that this is by no means pure conventionalism: the groups of things we identify are obviously dependent on our ways of perceiving things and analyzing their causal properties, but these latter are objective and mind-independent. In other words, our similarity classes are linguistic/conceptual artifices through which, nevertheless, we express the classifications and divisions we find in nature, which are not determined by us.

Bearing in mind that tropes do not HAVE properties but ARE properties, we can reduce similarity to a simple fact about the way basic individuals (and constructions thereof) are. The primitiveness of this 'way of being' is unproblematic and easily understandable, and similarity appears thus satisfactorily

explained.[16] In my terminology, similarity appears to be a C-property that, albeit based on mind-independent matters of fact, exists as such in our minds only.

2.3 Other objections to trope ontology

2.3.1 The boundary problem

The '*boundary problem*' amounts to the idea that if tropes are everything there is out there, given one of them we are immediately committed to an infinity of tropes. For example, consider this white sheet of paper. The whiteness of it is a single trope, or so it appears to be: this seems rather uncontroversial as soon as we distinguish clearly this whiteness from that, say, of the stool over there. But what happens if we tear this sheet in two parts, each one of these in two again and so on? It looks like we get as many white tropes as the number of pieces we tear the original sheet into, without any actual multiplication of the original stuff. We surely have the means, from a linguistic point of view, to identify each white piece of paper unequivocally. But should we say, from the ontological point of view, that we can tear a trope into pieces? If this was the case, then tropes wouldn't be simple basic constituents any more. On the other hand, if we deny this, we have to say that the multitude of tropes was already there and we didn't really have one trope to begin with.[17]

Perhaps, if we accepted the existence of *substrata* on top of tropes, we might say that dividing the initial *substratum* into pieces, we multiplied the 'bits' of Aristotelian matter, and that the form of whiteness which inhered in the initial unitary *substratum* was correspondingly multiplied into many white forms. Apart from the fact that one might formulate a 'boundary problem for substrata' (how divisible is matter? What are the basic indivisible bare particulars?), since I have already presented reasons not to endorse a substance-attribute view based upon bare particulars as property-bearers, my strategy will be different. As I see it, non-fundamental properties, those D-properties that I distinguished from tropes above, are indeed conventionally individuated, and can therefore be divided into parts without ontological inflation; *but tropes cannot*.

The whiteness of this sheet of paper is just a non-fundamental D-property (not, strictly speaking, a trope), to be conceived of as the by-product of a complex structure of simpler ontic units (tropes), which are the only realities that count as individuals in a strong, ontological sense.[18]

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[16] Price [1953] already claimed that resemblance can be understood as a primitive notion in the sense of a formal relation that merely express our ways of ordering things and concepts on the basis of the way things are out there.

[17] The boundary problem appears to apply diachronically as well, that is, in time as well as in space. Even if we experience something as perduring, that is, as having a continuous existence in time, we can still identify subparts of it, for example that thing's colour for a period of time which is only a part of its entire life. Is it one trope on its own, or just a part of a bigger trope coinciding with the entirety of the thing's colour throughout its existence?

[18] See the discussion above of how colours as D-properties are to be analysed in terms of physical structures of basic tropes.

Therefore, no ontological proliferation takes place, for the basic tropes are unchangeable and indivisible, and what changes when we, for example, tear a sheet of paper into pieces, is just the relations between basic tropes (and with these, all the properties and relations that are derivative on basic tropes). Thus, for the really fundamental constituents of reality there simply is no boundary problem; for the derivative ones, the alleged problem is not actually a problem at all.

2.3.2 Are additional relations/universals needed?

In his [1988], Hochberg tried to refute nominalism about universals by elaborating further on the criticism (that we have already presented) according to which any trope ontology must acknowledge the existence of some basic universal relations.

According to what Hochberg calls 'moderate' nominalism, only particular quality-instances exist but we must also postulate a universal 'similarity tie'. Hochberg's idea is that it turns out that even in this weakened form –accepting that similarity is a universal and trying to maintain that all the rest is just made out of tropes - nominalism is untenable, for other universal relations must be postulated beyond similarity. He formulates a complex argument in the paper, but I will focus on one specific difficulty that he raises.

According to Hochberg, nominalism cannot deal adequately with problems posed by the treatment of *order in relational facts*. Anything like aRb (think about, say, the relation of particular a being to the right of particular b) is explained by the realist by saying that we have two facts aI[…] and bS[…], where […] stands for the complex aRb. In other words, we can explain any relation between two individuals, and the specific role played by each individual in it, by simply claiming that there is a further relation between the individual and the original relational complex. In a realist setting, this is enough, for by invoking relations of a higher logical type (I and S) to explain a given relation (R) one makes a coherent 'logical leap': further relations might be needed in order to explain, say, the relation between R and S, but as an explanation of the specific ordering of the initial whole, nothing else is needed. In other words, one doesn't have a *vicious* regress because at each level a satisfactory explanation can be provided.

The nominalist cannot do this, because only particulars are involved all along and thus all the relations are of the same logical order. The complex aRb is, therefore, just a sum of individual entities, where *R* is of the same kind as a and b. But from such a group of particulars no ordering can emerge and thus one will never be able to distinguish the complex aRb and the complex bRa. In short, it seems that, since it puts all entities on a par, and specifically takes them to be all particulars, the trope ontologist's nominalist framework simply lacks the elements to provide the explanation sought, which appears instead to require some sort of 'ontological asymmetry'.

Given what I said in the previous sections, a reply to Hochberg is, however, easily found. The mistake Hochberg makes is that of attributing to the nominalist the idea that all properties are tropes in the sense that they are basic non-further-analysable ontic units. While this might have been an assumption for many trope theorists, trope ontology, as I argued already, doesn't need to say this at all. The

properties mentioned by Hochberg, in particular those determining order in relational facts, just *describe* basic facts about the world, i.e., about tropes and their 'organisation' into complexes, without, for this reason, being ontologically on a par with them, i.e., without *determining* these facts. Therefore, the trope ontologist can happily accept them as universals, while consistently denying them physical reality.

2.3.3 The alleged complexity of tropes

Moreland [1985] and, later, Mertz [1996] have argued that each trope has at least two distinguishing aspects, one that makes it a trope, and thus makes it resemble other tropes, and another that makes it the particular it is (its *thisness*). But if *thisness* is what identifies a trope, the other aspect, its nature, must be repeatable (i.e., 'being a trope' must be conceived of as a universal). Alternatively, a trope's nature has itself a *thisness*, which makes the trope own two distinct *thisnesses*. Moreover, the new individuator must be distinguished from the individuated nature, and this can only be accomplished by introducing a further *thisness*, immediately creating a vicious infinite regress. Therefore, tropes cannot be simple.

Armstrong [forthcoming] formulates this very same criticism in terms of truth-makers. He says that even though it is an epistemic possibility that different truths about a trope x are made true by it as a simple entity, given the complexity in the set of truths about x one should expect x to be complex as well. Namely, if different things can be meaningfully said of a given trope, it is very plausible that the trope has different aspects, different features that make different assertions about it (or in which, at any rate, a predicate corresponding to it appears) true or false. But if tropes are complex and not simple, their alleged ontological primitiveness is undermined.

To this, I reply that if one accepts, as Armstrong himself does, that truth-making theory rejects the idea of a 1-to-1 correlation between truths and truth-makers, there is no need to consider trope theory weakened in any sense. If it is a possibility for a simple trope to be a truth-bearer for a number of truths, implausibility claims do not imply anything ontological until further independent arguments against the simplicity of tropes are provided.

One might reply by recalling that Quinean nominalism appeared in need of some modification in a previous section exactly because it tried to keep both ontological simplicity of concrete particulars and multiplicity of their aspects. But the case of tropes is actually different. While concrete particulars were presented as having many aspects in the sense of possessing a number of *physical* properties, a trope as I have defined it surely has many ontological aspects, but *only one* physical manifestation. It is indeed only to a multiplicity of metaphysical features of tropes that Moreland and Mertz refer. But all theses metaphysical aspects are numerically identical, that is, it is by simply being the trope it is that a trope counts as one entity, is similar to other tropes in its nature, is distinct from other things and so on; and it is this very same thing that causally affects other entities, interacts with other tropes and so on. In other words, it is simply a mistake to take each of the aspects we are talking aobut as a distinct individual property of the

given trope, for they are merely identified by a *distinction of reason*, i.e., by conceptual analysis.

In particular, tropes are ontologically simple units, provided with what is commonly defined as primitive *thisness*, that is, the peculiar *transcendental* property that determines the things' individuality. Following the scholastic distinction between *properties* and *modes*, the latter must be intended as a *mode of being*, that is, as a basic way of existing, which is irreducible to anything else.

Every trope thus has an empirical content (its nature) and individuality as a metaphysically primitive mode of being. The two aspects, though, do not imply a complexity of constituents.

2.4 An assessment

In comparison with the other ontologies examined in the previous section, it seems fair to say that trope ontology, formulated along the lines suggested, appears to fare much better, and to be exempt from the crucial problems of the former.

The desiderata for a working ontology set out in the previous chapter were:

DES1: the preferred ontological account has to explain what it means to say that property *P* is a property of individual *x* or, in other words, that individual *x* as a unitary complex *exemplifies* property *P*.

DES2: the preferred ontological account has to explain similarity between different individuals.

DES3: the preferred ontological account has to individuate what the basic individuals are, and explain everything in terms of these.

What I said about compresence as coexistence and about similarity as derivative on the individual tropes' causal efficacy appears to be enough to overcome the difficulties represented by DES1 and DES2. An individual object is a set of coexisting tropes, each one of which is exemplified by the complex itself, and that together determine the very physical fact of their compresence. Numerically distinct individuals can be similar due to similarity in their properties, where the latter are ultimately grounded in the basic tropes' causal efficacy. But the tropes can be exactly similar while being numerically different, so there is no problem whatsoever in explaining the fact that *two different* things are exactly *similar*.[19]

What is now left to examine, is the nature of the basic tropes. That is, in order to satisfy desideratum DES3, we have to look at physics more closely and identify the basic constituents of material reality.

3. Tropes as field-parts

In this section, I will briefly suggest the way in which an ontology of tropes can be formulated which is consistent with what contemporary physics tells us.

[19] This suggests that in a trope ontology the Principle of the Identity of the Indiscernibles is false.

Since our basic question is 'what are the basic constituents of reality?', it is natural to examine quantum physics, which surely is our best current guess as to the nature of the micro-world. Without entering into a detailed interpretation of quantum theory, one can surely say that it implies that many things that we take for granted in the classical domain described by Newtonian mechanics do not hold anymore, at least not in an obvious way. And this is particularly true as far as individuality and ontology are concerned.

Some very important figures among the founding fathers of quantum mechanics (for example, Born [1926]) thought that the notion of an individual particle had to be given up in the light of the new theory. This is basically because, while in the classical domain individuals are identifiable by making reference to their qualities, this is not the case in the quantum realm and, moreover, classical statistics is replaced by a different sort of statistics at the quantum level.

Let's analyse these claims in more detail, beginning with the issues of individuation. It is a traditional metaphysical issue whether individuality as the matter of fact consisting of an entity being numerically distinct from all the others can be equated to *empirical*, *qualitative* difference, or not.

If this were the case, one should be able to tell two things apart by referring to at least one quality[20] that one thing has and the other doesn't have, or to some property which is differently exemplified in the two things. Formally, the converse of PII, that is,

$$\forall x \forall y((x \neq y) \rightarrow \exists P((Px \& \neg Py) \text{or} (Py \& \neg Px)) \qquad \text{(CPII)}$$

must hold.

Now, Leibniz thought that it is impossible for two things to be identical and only differ in their space-time locations. This is not the case in classical mechanics, in which absolutely identical particles are surely a possibility. Nevertheless, space-time locations do indeed act as a differentiating factor, for particles are assumed to be impenetrable and consequently always differ at least as regards their space-time properties. This appears to be enough to state that numerical identity can be inferred from qualitative difference (discernibility).

In quantum mechanics, on the contrary, *even space-time location can be shared among two or more 'indistinguishable' particles* (i.e., particles of the same kind that also happen to have all their state-dependent properties in common), and it is thus necessary to deny PII and refer to individuality as a primitive metaphysical fact in order to defend the idea that particles are individuals.

Turning now to the statistics, all this is rendered more difficult by the fact that quantum particles do not follow classical statistics, but behave in a peculiar way. The usual example is that of two particles with two accessible states, analogous to a 'two balls and two-boxes' classical system: while classically we have four possible states (balls 1 and 2 in box A, balls 1 and 2 in box B, ball 1 in

[20] Of course, these have to be qualitative, empirical properties that, at least in principle, we should be able to detect.

box A and ball 2 in box B, ball 2 in box A and ball 1 in box B), in quantum mechanics we have only three, for the two 'one ball in each box' cases count (as can be proven experimentally) as one.

Given this, and the above evidence regarding absolute indiscernibility, many commentators followed the founding fathers and took the basic entities of quantum reality as non-individuals (among the suggestion there have been events,[21] processes,[22] quanta,[23] factors[24] etc.). Nevertheless, it is to be clearly stated that this is not a necessary step, for what one has here is a sort of *ontological underdetermination*, whereby the choice between non-individuality and individuality as primitive *thisness* (also called 'transcendental individuality' on the basis of the fact that it is taken to be independent of anything empirical) is still open.

As we already mentioned, it is possible to simply break the link between numerical identity and qualitative indiscernibility suggested by PII and its derivates, and take individuality to be a primitive, transcendental, metaphysical fact. As for the peculiarity of quantum statistics, moreover, it is not enough to break the ontological underdetermination, for it is sufficient to say that the peculiar nature of it is not due to the nature of the entities involved, but rather to the existence of constraints on the number of states which are actually accessible to the particles. It can be seen that, once we give a proper description of the physically accessible (not only logically conceivable) states, the right probabilistic weights are attributed. Certainly, taking particles as non-individuals would allow to avoid defining these restrictions and thus having surplus mathematical structure in the theory not corresponding to anything actually physical.[25] But whether this is preferable or not just has to do with methodological strategies for theory choice and interpretation that don't seem enough to determine ontology.

The story, however, doesn't end here. As it is well-known, quantum mechanics is not relativistic, and in order to obtain a coherent and unified account of all the physical phenomena, physicists had to shift from quantum mechanics to quantum field theory (QFT) which is instead relativistic. Despite its name, QFT is not obviously a field theory, in the sense that even though the mathematical formalism of it describes mathematical fields, the ontological counterpart of the theory is not uniquely determined by this. In particular, quantum field theory can still be intended as a theory about particles, just by taking it as a 'refinement' of traditional quantum mechanics with particles. Nevertheless, various arguments have been offered that allegedly show that an interpretation of QFT as a theory of individual particles is untenable, which would solve the above ontological underdetermination between non-individuality and transcendental individuality.

[21] See Bartels [1999], who proposes an ontology inspired by Davidson's ideas on events and causal individuation (Davidson [2001]).

[22] Whitehead [1929], followed by Stapp [1977] and [1979], and Seibt [2002].

[23] Teller [1997].

[24] Simons [2002].

[25] This is what Redhead and Teller [1991] do.

These arguments can be grouped in two sets:

1. Arguments related to the nature of the vacuum: QFT describes some states as states with no particles ('vacuum states') but these are not totally empty, and actually turn out to be the *locus* of physical activity (the vacuum has energy greater than zero; and a moving observer in the vacuum experiences a 'thermal bath' of particles –the 'Unruh effect'-);

2. No-go theorems on the localizability of particles:[26] relativistic QFT is unable to describe particles as localized in finite regions of space-time, for doing so would violate basic relativistic requirements like the impossibility of superluminal speed; moreover, the Reeh-Schlieder theorem states that local measurements can never distinguish n-particles states from the vacuum.

Without entering in the details, 1) and 2) seem to suggest that sticking to individual particles by attributing primitive *thisness* to them is logically possible but not in harmony with what is actually described by the theory. Putting it very roughly, 1) shows that there probably is something more fundamental then particles, that exists when no one particle does. While 2) amounts to the fact that a modification of relativistic QFT able to keep both localizable particles and an agreement with Einstein's theories is (if not impossible) particularly hard to find.

Accepting these results means to take QFT as a theory about fields as continuous media of interaction, so confirming the idea that, throughout the history of science, the notion of field gained progressively more and more credibility basically because it represents an answer to the doubts raised by the Newtonian idea of action-at-a-distance without physical transmission of energy.[27]

Now, while I take this is as sufficient to conclude that the underdetermination is broken if it is intended as an underdetermination between classically intended particles and something else, I want to suggest that it is not sufficient to further conclude that we have to give up individuality of the fundamental constituents altogether. Given the arguments summarized in 1) and 2), the alternative between non-individuality and transcendentally given individuality, *is resolved in favour of the former only provided that we equate the possibility of transcendental individuality with an ontology of classical or quasi-classical particles. But this is by no means necessary for, as we have argued at length, there are many ways to identify the basic particulars of an ontology.*

Taking the evidence coming from quantum physics as enough to choose a completely new ontology is thus certainly unwarranted, and based upon an incomplete knowledge and/or assessment of the metaphysics involved. Since radically revisionary ontologies without individuals are less intuitive, less in keeping with our experience of the world and not exempt from problems,[28] I would

[26] See Redhead [1995], Malament [1996] and Halvorson and Clifton [2002].

[27] Historical reconstructions have been offered by many authors (most notably, Hesse and Cao), but I would recommend Lange [2002].

[28] See, for example, the discussions in Kuhlmann et al. [2002].

rather see a successful attempt to formulate an ontology still based upon individuals as very welcome.

But a way to formulate such an ontology is readily available at this point: it is, not surprisingly, the trope ontology I have defended in the previous sections.[29] My contention is that the basic entities constituting material reality are tropes, to be identified as the field-intensities described by the formalism of QFT. These intensities are the values of the physical fields at specific space-time points (or in 'smeared' regions), and are all that is needed to exhaustively represent the physical world and what happens in it.

In his [1983], Redhead suggested that in QFT particles are a logical possibility but are reduced as '*ephemerals*' in a continuous flux of change and physical activity determined by creation and annihilation events. Here, I am suggesting to split the notion of individuality from that of particles more clearly, and take these *ephemerals* as tropes. These can be thought as 'quasi-occurrents', in the sense that they tend to exist at space-time instants, and even though they can have a longer life, surely they are not continuants, i.e., they are not re-identifiable in time after new creation and/or annihilation events, for QFT simply forbids talk of a continuous history of a specific particle and rather sees particles as epiphenomena.

As I have shown in section 2, taking tropes as the basic constituents of reality puts individuality directly in the property-instances and thus resists the derivation - via PII- of numerical identity from qualitative sameness. In our case, two particles can be constituted by exactly similar tropes, and yet be numerically distinct because their tropes are. The above arguments from indistinguishability are thus rejected. And so are those having to do with statistics, against which we just need to refer to the abovementioned state-accessibility constraints.

Moreover, reminding that our tropes are not the traditional particles' properties, but the fundamental intensities of physical fields, we also resist the consequences of arguments related to the vacuum and to the theorems against localizable particles. As for the former, we just need to say that whatever level of energy one has which is not nothing, that points to the presence of a field-trope. Therefore, if level-0-energy still corresponds to physical events, it entails that, while there might not be particles, there are indeed tropes. As regards the latter, even more simply, one just has to state that the various arguments formulated in the

[29] An account of tropes in terms of fields was offered by Campbell [1990, Ch. 6], who suggested that we should understand the universe as made of a number of basic physical fields. There are, says Campbell, six fields, four to be identified with the four basic physical forces, one matter field and a space-time field. These fields are all-pervading entities which fill (and are compresent with each other in) every point of space(-time). However, one might object that postulating fields as primitives means going back to substance ontology, for we have now assumed as basic entities fields (or 'field tropes') with different point-like intensities, viz., substances with properties. This seems at odds with the absolute simplicity initially postulated for tropes, for all the different intensities Campbell talks about become 'aspects' of the same big trope. Moreover, Campbell makes the unwarranted assumption that fields coexist with space-time at all points, which is at least contentious from the perspective of contemporary physics.

literature rule out the localizability of particles as traditionally intended, while no problem whatsoever arises if one thinks in terms of field-intensities.[30]

Contemporary physics thus seems to support trope ontology in a clear and strong manner, which emerges as soon as we implement a careful analysis of QFT and its interpretation against the background of a non-superficial understanding of the metaphysical concepts and ontological alternatives involved. This allows to satisfy DES3, that is, the last desideratum for a viable ontology that I have formulated in section 1: tropes are (quasi-)point-like intensities of the basic physical fields, and everything else is constituted by structures of them.

4. Conclusions

Trope theory has more or less recently emerged as a simple, economic and plausible ontology. In the first section, I have explained why this is so, by reviewing the traditional debate about ontology and showing why alternative accounts meet with difficulties that make them unsatisfactory. In section 2, I have argued that trope theory can indeed be formulated in a way that solves the problems of the other accounts. Yet, trope theory has encountered problems and difficulties itself, and this made it interesting but nothing more. The traditional problems, though, I have argued in the remainder of the section, can be responded, basically by distinguishing between what is really fundamental and what is derivative in one's ontology, and by re-conceptualizing crucial notions like compresence and similarity. The last big issue, that is, the identification of the basic tropes, can be clarified by referring to contemporary physics. Doing so, I have argued, supports trope theory and also sheds light on interpretative problems affecting the philosophy of the physical theories themselves. It seems then that trope ontology is more than a promising framework, and rather turns out to be a consistent and useful account, that puts ontology and physics together in a harmonic way. I hope all this will stimulate further work devoted to the subject, and more support being provided to trope ontology in the future.

[30] One might object that the results rule out localizability of individuals *tout court*. To this, I would react in one of two ways: either by pointing out that, the theorems crucially require the number of individuals to be the same across time, which is a requirement for particles but not for tropes intended as continuously created and annihilated individuals as manifestations of a field. Or by claiming that all these results only affect the way in which we can apply and interpret the localization operators in our theories, not ontological facts about localizable individuals. This latter claim going along the lines of a differentiation between the epistemic and the ontological consequences of relativity, the second strategy is more contentious. Moreover, if it turned out to be plausible, it might be generalized and used for a defence of traditional particles too. Therefore, I would emphasise that tropes as basic individuals simply do not have the same features (most notably, a fixed number in time) as traditional particles.

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The Structure of Commonsense Knowledge

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Abstract. Several decades of research in natural language understanding (NLU) has made it quite clear that most language comprehension tasks require the storage of, and reasoning with, massive amounts of background (henceforth, *commonsense*) knowledge. Predictably, the unavailability of this ontological structure that reflects our commonsense view of the world has often translated into difficulties in formulating the proper semantic framework for a number of phenomena in natural language.

In this paper we will first argue that the structure of commonsense knowledge must be discovered rather than invented. Furthermore, and inline with an insightful observation which Hobbs (1985) made few years back, we will demonstrate that many of the challenges in the semantics of natural language can be reduced to near triviality, if semantics is performed on top of a strongly-typed ontological structure that reflects our view of the world and the way we *talk* about in ordinary language. Subsequently, we discuss the nature of this ontological structure and argue that natural language, which is the best known theory of our (shared) commonsense knowledge, should itself be used as a guide in discovering this structure. The ultimate goal of this work is the development of a logic with content, and 'ontological content' in particular, as Cocchiarella (2001) has convincingly argued for in recent years.

1. Introduction

In *Logic and Ontology,* Cocchiarella (2001) convincingly argues for a view of *"logic as a language"* in contrast with the (now dominant) view of *"logic as a calculus"*. In the latter, logic is viewed as an "abstract calculus that has no content of its own, and which depends on set theory as a background framework by which such a calculus might be syntactically described and semantically interpreted." In the view of *"logic as a language"*, however, logic has content, and "ontological content in particular." According to Cocchiarella, a logic with ontological content

necessitates the use of type theory (and predication), as opposed to set theory (and set membership), as the background framework. An obvious question that immediately comes to mind here is the following: what exactly is the nature of this strongly-typed ontological structure that will form the background framework for a new logic that has content?

In our opinion, part of the answer lies in an insightful observation that Hobbs (1985) made some time ago, namely that difficulties encountered in the semantics of natural language are due, in part, to difficulties encountered when one attempts to specify the exact nature of the relationship between *language* and the *world.* While it has not received much attention, the crucial point that Hobbs makes is the observation that if one "assumes a theory of the world that is isomorphic to the way we *talk* about it" (emphasis added), then "semantics becomes very nearly trivial". This intimate relationship between language and knowledge, or, more specifically between natural language understanding (NLU) and knowledge representation and reasoning (KR&R), can be demonstrated by several examples. As a first illustration, consider the following:

(1) *John visited a house on every street*
(2) *John advertised a house on every street*

From the standpoint of commonsense, most readers would find no difficulty in the following reading for (2),

(3) $(\exists x)(\text{HOUSE}(x) \wedge (\forall y)(\text{STREET}(y) \wedge \text{ON}(x,y) \rightarrow \text{ADVERTISED}(j,y)))$

implying the advertising of the same house on several streets. However, the same is not true in (1), as one can hardly conceive of a single house physically existing on several streets. Thus, while a wide scope a, implying a single house is quite plausible in (2), the more plausible reading in (1) is

(4) $(\forall x)(\text{STREET}(x) \wedge (\exists y)(\text{HOUSE}(y) \wedge \text{ON}(x,y)) \rightarrow \text{VISITED}(j,y))$

implying several houses, making (1) read something like 'on every street, John visited some house'. Lacking syntactic or semantic explanations, we argue that such inferences must be a function of our commonsense knowledge of how the on relation between houses and streets is typically manifested in the real world (in the possible world we actually live in!). This process is even more complex due to the fact that different individuals may have different scope preferences in the same linguistic context, as the experiments of Kurtzman & MacDonald (1993) suggest. While it has been argued that such problems do not always require the storage of and reasoning with vast amounts of background knowledge (see Saba & Corriveau, 2001), other linguistic comprehension tasks clearly do. For instance, consider the resolution of `He' in the following:

(5) *John shot a policeman. He immediately fled away.*
(6) *John shot a policeman. He immediately fell down.*

Clearly, such references must be resolved by recourse to commonsense knowledge – for example, that, typically, when shot(x,y) holds between some x and some y, x is the more likely subject to flee (6), and y is the more likely subject to fall down (5). Note, however, that such inferences must always be considered defeasible, since quite often additional information might result in the retraction of previously made inferences. For example, (6) might after all be describing a situation in which John, a 7-year old who was shooting a bazooka, fell down. Similarly, (5) might actually be describing a situation in which the policeman, upon being slightly injured, tried to flee away, perhaps to escape further injuries!

Computationally, there are clearly a number of challenges in reasoning with un-committed (or 'underspecified') logical forms, and this has indeed received considerable attention by a number of authors (e.g., see Kameyama, 1996, and the excellent collection of papers in van Deemter & Peters, 1996). However, the main challenge that such processes still face is the availability of this large body of commonsense knowledge along with a computationally effective reasoning engine. While the monumental challenge of building such large commonsense knowledge bases was indeed faced head-on by a few authors (e.g., Lenat & Ghua, 1990), a number of other authors have since abandoned (and argued against) the 'knowledge intensive' paradigm in favor of more quantitative methods (e.g., Charniak, 1993). Within linguistics and formal semantics, little or no attention was paid to the issue of commonsense reasoning at the pragmatic level. Indeed, the prevailing wisdom was that a number of NLU tasks require the storage of and reasoning with a vast amount of background knowledge (van Deemter, 1996), and that has led some to conclude that these approaches were `highly undecidable' (e.g., Reinhart, 1997).

In our view, both trends were partly misguided. In particular, we hold the view that (*i*) language 'understanding' is for the most part a commonsense 'reasoning' process at the pragmatic level; and (*ii*) the 'understanding as reasoning' paradigm, and the underlying knowledge structures that it utilizes, must be formalized if we ever hope to build scalable systems (or, as John McCarthy likes to say, if we ever hope to build systems that we can actually understand!). In this light we believe the work on integrating logical and commonsense reasoning in language understanding (Allen, 1987; Pereira & Pollack, 1991; Zadrozny & Jensen, 1991; Hobbs, 1985; Hobbs *et al.*, 1993; and more recently Asher & Lascarides, 1998; and Saba & Corriveua, 1997) is of paramount importance[1].

Much of this work is directed towards formulating commonsense inferencing strategies to resolve a number of ambiguities at the pragmatic level. Although it has been shown (see e.g., Saba & Corriveau, 2001) that these inferences do not always require the storage of and reasoning with a vast amount of background knowledge, it is clear that a number of tasks do require such a knowledgebase.

[1] Outside the domain of NLU, other pioneering work such as that of (McCarthy 1980), was also done in the same spirit, namely to integrate logical and commonsense reasoning.

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Indeed, substantial effort has been made towards building ontologies of commonsense knowledge (e.g., Lenat & Ghua, 1990; Mahesh & Nirenburg, 1995; Sowa, 1995), and a number of promising trends that advocate ontological design based on sound linguistic and logical foundations have started to emerge in recent years (e.g., Guarino & Welty, 2000; Pustejovsky, 2001). However, a systematic and objective approach to ontological design is still lacking. In particular, we believe that an ontology for commonsense knowledge must be *discovered* rather than *invented*, and thus it is not sufficient to establish some principles for ontological design, but that a strategy by which a commonsense ontology might be **systematically** and **objectively** designed must be developed. In this paper we propose such a strategy.

We should note here that while we do not claim we have achieved our ultimate goal, we believe that we are on the right path towards discovering (as opposed to inventing) 'the' structure of commonsense knowledge, and that such a systematic and strongly-typed structure will solve a number of problems that we have wrestled with for a number of years. We therefore believe that our work, while not final in many respects, is worthy of presenting to the NLP and AI research communities, hoping that others can pickup where we left off, by identifying errors, filling in some gaps, completing the picture along the way.

2. Language and Knowledge

Our basic strategy for discovering the structure of commonsense knowledge is rooted in Frege's conception of Compositionality. According to Frege (see Dummett, 1981, pp. 4-7), the sense of any given sentence is derived from our previous knowledge of the senses of the words that compose it, together with our observation of the way in which they are combined in that sentence. The cornerstone of this paradigm, however, is an observation that has not been fully appreciated regarding the manner in which words are supposed to acquire a sense. In particular, the principle of Compositionality is rooted in the thesis that \our understanding of [those] words consists in our grasp of the way in which they may figure in sentences in general, and how, in general, they combine to determine the truth-conditions of those sentences." (Dummett, 1981, pp. 5). Thus, the meanings of words (i.e., the concepts), and the relationships between them, can be reverse-engineered, so to speak, by analyzing how these words are **used** in everyday language.

This simple idea forms the basis of our strategy in discovering the structure of commonsense knowledge: what language allows one to say about a concept, tells us a lot about the concept under consideration. In other words, the nature of these concepts can be discovered from the manner in which they are **used** in everyday language. To illustrate this point further, consider the following examples, which (Montague, 1969) discussed in addressing a puzzle pointed out to him by Quine:

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(7) $[\![\textit{John painted a unicorn}]\!] = (\exists x)\big(\text{UNICORN}(x) \wedge \text{PAINTED}(j,x)\big)$

(8) $[\![\textit{John found a unicorn}]\!] = (\exists x)\big(\text{UNICORN}(x) \wedge \text{FOUND}(j,x)\big)$

The puzzle Quine was referring to was the following: both translations admit the inference $(\exists x)(\text{UNICORN}(x))$ – that is, both sentences imply the existence of a unicorn, although it is quite clear that such an inference should not be admitted in the case of (7). According to Montague, the obvious difference between (7) and (8) must be reflected in an ontological difference between *find* and *paint* in that the extensional type $(e \rightarrow (e \rightarrow t))$ both transitive verbs are typically assigned is too simplistic. Montague was implicitly suggesting that a much more sophisticated ontology (i.e., a more complex type system) is needed, one that would in fact yield different types for *find* and *paint*. One reasonable suggestion for the types of *find* and *paint*, for example, could be as follows:

(9) $\textit{find} :: \ (e_{\mathsf{Animal}} \rightarrow (e_{\mathsf{Entity}} \rightarrow t))$

(10) $\textit{paint} :: \ (e_{\mathsf{Human}} \rightarrow (e_{\mathsf{Representation}} \rightarrow t))$

Thus instead of the flat type structure implied by $(e \rightarrow (e \rightarrow t))$, the types of *find* and *paint* should reflect our commonsense belief that we can always speak of some Animal that found something (i.e., any Entity whatsoever), and of a Human that painted some illustration, or as we called it here a Representation. Before we proceed, however, we point out that throughout, we will use this Font for concept types in the ontology, and this FONT for predicate names. Thus, x::LivingThing means x is an object/entity of type LivingThing and $\text{APPLE}(x)$ means the predicate or property APPLE is true of x. Note, further, that in a flat-type system, where there is one type of entity, e, the expression $(\exists x)\big(\text{UNICORN}(x) \wedge \text{FOUND}(j,x)\big)$ is equivalent to the typed expression

$$(\exists x :: \mathsf{Entity})\big(\text{UNICORN}(x) \wedge \text{FOUND}(j :: \mathsf{Entity}, x)\big)$$

- that is, John, the unicorn, and any other object are all objects of type Entity. With this background, the proper translation of (3) and (4) and the corresponding inferences can now be given as follows:

(11) $[\![\textit{John painted a unicorn}]\!]$
$= (\exists x :: \mathsf{Representation})(\text{ELEPHANT}(x) \wedge \text{PAINTED}(j :: \mathsf{Human}, x))$

(12) $[\![\textit{John painted a unicorn}]\!]$
$= (\exists x :: \mathsf{Entity})(\text{ELEPHANT}(x) \wedge \text{FOUND}(j :: \mathsf{Human}, x))$

Eureka! Adding a rich type structure to the semantics seems to have solved Quine's puzzle, as the correct inferences can now be made: if John *found* a unicorn,

then one could indeed infer that an actual unicorn exists[2]. However, the *painting* of a unicorn only implies the existence of a representation (an illustration) of something we call a unicorn! Stated yet in other words, (12) implies that a unicorn Entity (including perhaps a unicorn Toy) exists, while (11) implies a unicorn Representation exists. There are two points that this discussion intends to emphasize: (*i*) is the need for a rich type structure to solve a number of problems in the semantics of natural language; and (*ii*) that this type structure is actually systematically discovered by an analysis of how ordinary language is used to talk about the world. This opinion will (hopefully) be further supported by our discussion of nominal compounds.

3. Nominal Compounds and Ontological Categories

The semantics of nominal compounds have received considerable attention by a number of authors, most notably Kamp & Partee (1995), Fodor & Lepore (1996), and Pustejovsky (2001), and to our knowledge, the question of what is an appropriate semantics for nominal compounds has not yet been settled. Recall that the simplest (extensional) semantic model for simple nominal constructions is that of conjunction (or intersection) of predicates (or sets). For example, assuming that $\text{RED}(x)$ and $\text{APPLE}(x)$ represent the meanings of *red* and *apple*, respectively, then the meaning of a nominal such as *red apple* is usually given as

(13) $[\![red\ \ apple]\!] = \{x \mid \text{RED}(x) \wedge \text{APPLE}(x)\}$

What (13) says is that something is a *red apple* if it is red and *apple*. This simplistic model, while seems adequate in this case (and indeed in many other instances of similar ontological nature), clearly fails in the following cases, all of which involve an adjective and a noun:

(14) *former senator*
(15) *fake gun*
(16) *alleged thief*

Clearly, the simple conjunctive model, while seems to be adequate for situations similar to those in (13), fails here, as it cannot be accepted that something is *former senator* if it is *former* and *senator*, and similarly for (15) and (16). Thus, while conjunction is one possible function that can be used to attain a compositional meaning, there are in general more complex functions that might be needed for other *types of ontological categories*. In particular, what we seem to have is something like the following:

[2] Of course, in such a type system we would have $\text{Rational} \supset \text{Animal}$ and therefore John, an entity of type Rational, can be the subject of FOUND which expects an entity of type Animal.

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(17) $[\![red\ \ apple]\!] = \{x \mid x \text{ is } \textsc{red} \text{ and } x \text{ is } \textsc{apple}\}$

(18) $[\![former\ \ senator]\!] = \{x \mid WasButIsNotNowA\_\textsc{senator}(x)\}$

(19) $[\![fake\ \ gun]\!] = \{x \mid LooksLikeButIsNotActuallyA\_\textsc{gun}(x)\}$

(20) $[\![alleged\ \ thief]\!] = \{x \mid IsNotNowButCouldPossiblyTurnOutToBeA\_\textsc{thief}(x)\}$

It would seem, then, that *different ontological categories* require *different semantic functions* to compute the meaning of the whole from the meanings of the parts. In fact, the meaning (intension) of some compound might not be captured without resorting to temporal and/or modal operators. For example, we argue that the following are reasonable definitions for the concepts *fake, former* and *alleged*:

(21) $(\forall x{:}\mathsf{Physical})\Big(\textsc{fake}(x) \equiv_{df} \lambda P\Big[(\exists y{:}\mathsf{Physical})\big(\neg P(x) \wedge P(y) \wedge \textsc{similar}_{\{\textsc{shape},\textsc{size}\}}(x,y)\big)\Big]\Big)$

(22) $(\forall x{:}\mathsf{Role})\Big(\textsc{former}(x) \equiv_{df} \lambda P\Big[(\exists t)\big((t < now) \wedge P(x,t) \wedge \neg P(x,now)\big)\Big]\Big)$

(23) $(\forall x : \mathsf{Role})\Big(\textsc{alleged}(x) \equiv_{df} \lambda P\Big[(\exists t)\big((t > now) \wedge \neg P(x,now) \wedge \Diamond P(x,t)\big)\Big]\Big)$

That is, 'fake' applies to some concept P as follows: a certain physical object x is a fake P *iff* it is not a P, but it actually is similar (in certain respects) to something else, say y, which is actually a P. On the other hand, what (22) says is the following: a certain x is a former P *iff* x was a P at some point in time in the past and is not now a P. Finally, what (23) says is that something is an 'alleged' P *iff* it is not now known to be a P, but could possibly turn out to be a P at some point in the future.

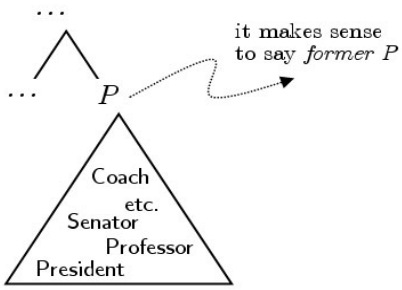
It is interesting to note here that the intension of *fake* and that of *former* and *alleged* was in one case represented by recourse to possible worlds semantics (the case of (22) and (23)), while in (21) the intension uses something like structured semantics, assuming that $\textsc{similar}_{\{A_1,A_2,\ldots,A_n\}}(x,y)$ which is true of some x and some y if x and y share a number of important features, is defined. What is interesting in this is that it suggests that possible-worlds semantics and structured semantics are not two distinct alternatives to representing intensionality, as has been suggested in the literature, but that in fact they should co-exist. Moreover, note that the representation of the meaning of *fake* given in (21) suggests that *fake* expects a concept which is of type Physical Artifact, and thus something like *fake idea*, or *fake song*, for example, should sound meaningless, from the standpoint of commonsense[3]. Second, the representation of the meaning of *former* given in (22) suggests that *former* expects a concept which has a time dimension, i.e. is a temporal concept.

[3] One can of course say *fake smile*, but this is clearly another sense of *fake*. While *fake gun* refers to a gun (which is of type Artifact) that is not real, *fake smile* refers to a dishonest *smile*, or a *smile* that is not genuine.

Finally, we should note that the ultimate goal of this type of analysis is to discover the ontological categories that share the same behavior. For example, conjunction, which as discussed above is one possible function that can be used to attain a compositional meaning, seems to be adequate for all nominal constructions of the form [A N] where A is a physical property (such as *red, large, heavy,* etc.) and N is a physical object (such as *car, person, desk,* etc.), as expressed in (24)[4].

(24) $[\![A\ \ N]\!] = \left\{x \middle| A_{\mathsf{PhysicalProperty}}(x) \wedge N_{\mathsf{PhysicalThing}}(x)\right\}$

Similarly, an analysis of the meaning of *former*, given in (15), suggests that there are a number of ontological categories that seem to have the same behavior, and could thus replace P in (22), as implied by the fragment hierarchy shown below.



A question that must be answered now is the following: assuming (22) is proper logical formulation of our commonsense understanding of what *former* means, then what is the logical counterpart of the fact that 'it does not make sense' to say *former father*? As we show in appendix A, the fact that 'it does not make sense to say *former father*' translates into logical contradiction.

4. Intensionality and Compositionality

In addition to nominal compounds of the form [$Adj\ Noun$], the problem of nominal compounds in the case of noun-noun combinations has also been problematic, and, according to Weiskopf (forthcoming), this is largely due to the multitude of possible relations that are usually implicit between the two nouns. To illustrate, consider the following:

(25) $[\![brick\ house]\!] = \left\{x \middle| x\ \ is\ a\ \text{HOUSE}\ \ that\ is\ \textbf{made of}\ \ \text{BRICK}\right\}$
(26) $[\![dog\ house]\!] = \left\{x \middle| x\ \ is\ a\ \text{HOUSE}\ \ that\ is\ \textbf{made for}\ \ a\ \text{DOG}\right\}$

[4] While (24) states that some adjectives are intersective, (24) does not say anything about the meaning of such adjectives. See appendix B for such a discussion.

Thus, while a *brick house* is a HOUSE 'made of' BRICK, a *dog house* is a HOUSE that is 'made for' a DOG. It would seem, then, that the relation implicitly implied between the two nouns differs with different noun-noun combinations, and they have generally been considered as posing a serious challenge to the general program of compositional semantics in the Montague (1974) tradition. However we believe that this argument is fallacious, in that the problem is not due to compositionality, but (again) is due to the non-existence of a rich-type structure that reflects differences in the ontological nature of certain entities in the seam tics. In particular, assuming the existence of a strongly-typed ontology might result in identifying a handful of patterns that can account for all noun-noun combinations. For example, it would seem that MADEOF is the relation implicit between all $[N_1\ N_2]$ combinations whenever N_1 is an object of type RawMaterial (which in turn is a subtype of Substance) and N_2 is an object of type Artifact:

(27) $[\![N_{\mathsf{RawMaterial}} N_{\mathsf{Artifact}}]\!]$
$$= \lambda P \lambda Q\ \{x :: \mathsf{Artifact} \mid P(x) \wedge (\exists y :: \mathsf{RawMaterial})(Q(y) \wedge \textsc{madeOf}(x,y))\}$$

The following are some example instances of (20), denoting the meanings of *brick house, wooden spoon, plastic knife* and *paper cup*, respectively:

$$[\![brick\ \ house]\!] = \{x :: \mathsf{Artifact} \mid \textsc{house}(x) \wedge E_1\}$$
$$[\![wooden\ \ spoon]\!] = \{x :: \mathsf{Artifact} \mid \textsc{spoon}(x) \wedge E_2\}$$
$$[\![plastic\ \ knife]\!] = \{x :: \mathsf{Artifact} \mid \textsc{knife}(x) \wedge E_3\}$$
$$[\![paper\ \ cup]\!] = \{x :: \mathsf{Artifact} \mid \textsc{cup}(x) \wedge E_4\}$$

where

$$E_1 \equiv (\exists y :: \mathsf{RawMaterial})(\textsc{brick}(y) \wedge \textsc{madeOf}(x,y))$$
$$E_2 \equiv (\exists y :: \mathsf{RawMaterial})(\textsc{wood}(y) \wedge \textsc{madeOf}(x,y))$$
$$E_3 \equiv (\exists y :: \mathsf{RawMaterial})(\textsc{plastic}(y) \wedge \textsc{madeOf}(x,y))$$
$$E_4 \equiv (\exists y :: \mathsf{RawMaterial})(\textsc{paper}(y) \wedge \textsc{madeOf}(x,y))$$

Note, further, that other subtypes of Substance and other subtypes of Artifact might require a relation other than MADEOF. For example, consider *bread* and *knife*, which are specific types of Substance and Artifact, respectively:

(28) $bread :: \mathsf{FoodSubstance} < \mathsf{Substance}$
(29) $knife :: \mathsf{Tool} < \mathsf{Implement} < \mathsf{Instrument} < \mathsf{Artifact}$

While *knife* combines with a raw Substance (Material), such as *plastic, bronze, wood, paper*, etc. with the relation MADEOF, it combines with a

FoodSubstance with the relation USEDFOR. Consider the following compounds of a FoodSubstance (which is a specific type of Substance) and some Instrument (which is a specific type of Artifact):

$$[\![butter\ \ knife]\!] = \{x :: \mathsf{Instrument} \mid \text{KNIFE}(x) \wedge E_1\}$$
$$[\![coffee\ \ mug]\!] = \{x :: \mathsf{Instrument} \mid \text{MUG}(x) \wedge E_2\}$$
$$[\![cereal\ \ box]\!] = \{x :: \mathsf{Instrument} \mid \text{BOX}(x) \wedge E_3\}$$

where

$$E_1 \equiv (\exists y :: \mathsf{FoodSubstance})(\text{BUTTER}(y) \wedge \text{USEDFOR}(x,y))$$
$$E_2 \equiv (\exists y :: \mathsf{FoodSubstance})(\text{COFFEE}(y) \wedge \text{USEDFOR}(x,y))$$
$$E_3 \equiv (\exists y :: \mathsf{FoodSubstance})(\text{CEREAL}(y) \wedge \text{USEDFOR}(x,y))$$

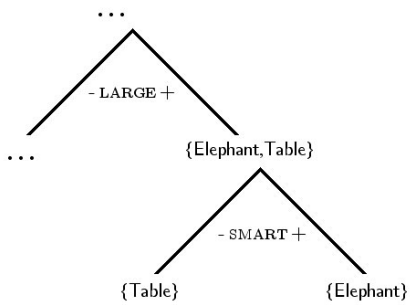
Although we cannot dwell on such details here, we should point out that since the purpose of an object of type Instrument (or more specifically, Tool) is to be *used for* something, the specific type of usage would in turn be inferred from the specific Instrument/Tool (e.g., *cutting* in the case of a *knife, holding* in the case of *mug*, etc.) Finally, we should note that while the analysis we have been conducting in this section might shed some light on the difficult problem of nominal compounds, this is not the only linguistic tool that can help us discover the structure of commonsense knowledge. In fact, we can discover another piece of the puzzle by an analysis of adjectives and verbs that may or may not apply to nouns, which we discuss in the following section.

5. Type Inferences in Formal and Natural Languages

We first start this discussion by introducing a predicate $app(p,c)$ which is taken to be true of a property p and a concept c *iff* "it makes sense to speak of the property p of c". As an example, consider the following two sets of adjectives and nouns:

(30) $P = \{\text{LARGE}, \text{SMART}\}$
(31) $C = \{\mathsf{Table}, \mathsf{Elephant}\}$

A quick analysis of the predicate $app(p,c)$ on the four adjective-noun combinations yields the following: $app(\text{LARGE},\mathsf{Table})$, $app(\text{LARGE},\mathsf{Elephant})$, $app(\text{SMART},\mathsf{Elephant})$ and $\neg app(\text{SMART},\mathsf{Table})$. That is, while it makes sense to say 'large table', 'large elephant', and 'smart elephant', it does not make sense to say 'smart table'. This kind of analysis yields the structure shown below.



First it must be pointed out that the above structure was *discovered* and not *invented*! Note also that a number of inferences can now be made, for example that whenever it makes sense to say 'smart' of some object c then it also makes sense to say 'large' of c. This kind of analysis is not much different from the type inferencing process that occurs in strongly-typed, polymorphic programming languages. For example, consider the linguistic patterns and the corresponding type inferences shown in table 1 below.

Pattern	Type Inference
$x + 3$	x is a Number
REVERSE(x)	x is a Sequence
INSERT(x,y)	x is an object; y is Sequence of x objects
HEAD(x)	x is a Sequence
EVEN(x)	x is a Number

Table 1. Linguistic patterns and their corresponding type inferences.

From $x + 3$, for example, one can infer that x is a `number` since numbers are the \kinds of things" that can be added to 3 (or, for the expression '$x + 3$' to make sense, x must be a Number!) In general, the most generic type possible is inferred (i.e., these operations are assumed to be polymorphic). For example, all that can be inferred from REVERSE(x) is that x is the generic type Sequence, which could be a List, a String (a sequence of Characters), a Vector, etc. Note also that in addition to actions (called functions or methods in programming lingo), properties (truth-valued functions) can also be used to infer the type of an object. For example, from EVEN(x) one can infer that x is a Number, since lists, sequences, etc. are not the kinds of objects which can be described by the predicate `even`. This process can be more formally described as follows:

1. we are given a set of concepts $C = \{c_1,\ldots,c_m\}$ and a set of actions (and properties) $P = \{p_1,\ldots,p_m\}$

2. a predicate $app(p,c)$, where $c \in C$ and $p \in P$ is defined such that the action (or property) p applies to (makes sense of) objects of type c.
3. for each property $p \in P$, a set $C_p = \{c \mid app(p,c)\}$, denoting all concepts c for which the property p is applicable, is generated.
4. a concept hierarchy is then systematically discovered by analyzing the subset relationship between the various sets generated.

To illustrate how this process (systematically) yields a type hierarchy, we consider a small set of concepts $C = \{\mathsf{List}, \mathsf{String}, \mathsf{Set}\}$ and a set of properties (or actions) $P = \{\text{EMPTY}, \text{MEMBEROF}, \text{SIZE}, \text{TAIL}, \text{HEAD}, \text{REVERSE}, \text{TOUPPERCASE}\}$ that may or may not be plausibly applied to a certain concept. Shown in figure 1 below is a number of sets that are generated by $app(p,c)$ (fig. 1a) and the concept hierarchy implied by the subset relationship among these sets (fig. 1b).

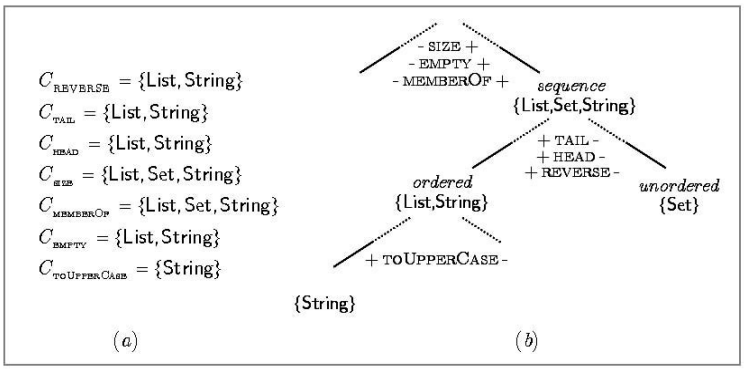


Figure 1. Sets generated by $app(p,c)$ (a) and the structure implied by these sets (b).

Note that each (unique) set corresponds to a concept in the hierarchy. Equal sets (e.g. C_{TAIL} and C_{HEAD}) correspond to the same concepts. The label of a given concept could be any meaningful label that intuitively represents all the sub-concepts in this class. For example, in figure 1b `sequence` was used to collectively refer to sets, strings and lists. Note that there are a number of rules that can be established from the concept hierarchy shown in figure 1b. For example, one can state the following:

(32) $(\forall c)(app(\text{REVERSE}, c) \supset app(\text{SIZE}, c))$
(33) $(\exists c)(app(\text{SIZE}, c) \wedge \neg app(\text{REVERSE}, c))$
(34) $(\forall c)(app(\text{TAIL}, c) \equiv app(\text{HEAD}, c))$

Here (32) states that whenever *it makes sense* to *reverse* an object c, then it also makes sense to ask for the *size* of c. This essentially means that an object to

which the *size* operation can be applied is a parent of an object to which the *reverse* operation can be applied. (33), on the other hand, states that there are objects for which the *size* operation applies, but for which the *reverse* operation does not apply. Finally, (34) states that whenever it makes sense to ask for the *head* of an object then it also makes sense to ask for its *tail*, and vice versa.

Finally, it must be noted that in performing this analysis we have assumed that $app(p,c)$ is a Boolean-valued function, which has the consequence that the resulting type hierarchy is a strict binary tree. In fact, this is one of the main characteristics of our method, and has led to two important results: (*i*) multiple inheritance is completely avoided; and (*ii*) by not allowing any ambiguity in the interpretation of $app(p,c)$, lexical ambiguity, polysemy and metaphor are explicitly represented in the hierarchy. This is discussed below.

6. Language and Commonsense Knowledge

The work described here was motivated by the following two assumptions: (*i*) the process of language understanding is, for the most part, a commonsense reasoning process at the pragmatic level; and (*ii*) since children master spoken language at a very young age, children must be performing commonsense reasoning at the pragmatic level, and consequently, they must posses all the commonsense knowledge required to understand spoken language[5]. In other words, we are assuming that deciding on a particular $app(p,c)$ should not be controversial, and that children can easily answer questions such as those shown in table 2 below.

Note that in answering these questions it is clear that one has to be coconscious of metaphor. For example, while it is quite meaningful to say *strong table, strong man,* and *strong feeling*, it is clear that the senses of *strong* in these three cases are quite distinct. The issue of metaphors will be dealt with below. For now, all that matters, initially, is to consider posing queries such as $app(\text{SMART},\text{Elephant})$ – or equivalently, questions such as 'does it make sense to say *smart elephant*?', to a five-year old. In replying to such questions we claim that $app(v,c)$ is binary-valued – that is, while it could be a matter of degree as to how smart a **certain** elephant might be (which is a **quantitative** question), the **qualitative** question of whether or not it is meaningful to say `smart elephant' is not a matter of degree.

[5] Thus it may very well be the case that *everything we need to know we learned in kindergarten*!

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Query	Does it make sense to say...
app(WALK,Elephant)	elephants walk?
app(TALK,Elephant)	elephants talk?
app(SMART,Elephant)	elephants are smart?
app(SMART,Mountain)	mountains are smart?
app(SCREAM,Book)	books scream?
app(HAPPY,Sugar)	happy sugar?

Table 2. Deciding on a particular app(v,c) from the standpoint of commonsense.

With this background we now show that an analysis of how verbs and adjectives are used with nouns in everyday language can be used to *discover* the structure of commonsense knowledge:

- Select a set of adjectives and verbs, $V = \{v_1, \ldots, v_m\}$
- Select a set of nouns $C = \{c_1, \ldots, c_n\}$
- For every pair (v_i, c_j) where $v_i \in V$ and $c_j \in V$ generate a set of concepts $C_i = \{c \mid app(v_i, c)\},\ 1 \le i \le m$
- Build a hierarchical structure by analyzing the subset relationship between all sets $C_i \in \{C_1, \ldots, C_m\}$

As an initial example, consider the set of verbs $V = \{$MOVE, WALK, RUN, TALK, REASON$\}$ and the set of nouns $C = \{$Rational, Bird, Elephant, Shark, Animal, Ameba$\}$. By repeatedly applying $app(v,c)$ the following sets can be generated:

(35)
$$
\begin{aligned}
C_{\text{MOVE}} &= \{\text{Rational}, \text{Animal}, \text{Bird}, \text{Elephant}, \text{Shark}, \text{Ameba}\} \\
C_{\text{TALK}} &= \{\text{Rational}\} \\
C_{\text{REASON}} &= \{\text{Rational}\} \\
C_{\text{THINK}} &= \{\text{Animal}\} \\
C_{\text{WALK}} &= \{\text{Rational}, \text{Bird}, \text{Elephant}\} \\
C_{\text{RUN}} &= \{\text{Rational}, \text{Bird}, \text{Elephant}\}
\end{aligned}
$$

First we note that while some decisions could `technically' be questioned (say by a biologist) our strategy was to simply consider the question from the point of view of commonsense. In deciding on a particular $app(v,c)$ we considered poising

to a five-year old queries such as *do elephants fly, do they run, do they talk*, etc. This initial process resulted in sets such as these:

(36)
$$
\begin{aligned}
&+L_2- = \{\text{DEVELOP}, \text{FORM}\}\\
&+L_3- = \{\text{EVOLVE}\}\\
&+L_4- = \{\text{LIVE}, \text{DIE}, \text{BORN}, \text{GROW}\}\\
&+L_6- = \{\text{BRANCH}\}\\
&-R_6+ = \{\text{MOVE}, \text{TRAVEL}\}\\
&-R_7+ = \{\text{SLEEP}, \text{REST}, \text{EAT}, \text{DIGEST}, \text{BLEED}, \text{HURT}, \text{THINK}\}\\
&+L_8- = \{\text{SIT}, \text{JUMP}, \text{WALK}, \text{RUN}\}\\
&-R_9+ = \{\text{ROAR}, \text{SCREAM}, \text{CRY}, \text{YELL}\}\\
&+L_{10}- = \{\text{TALK}, \text{THINK}, \text{REASON}, \text{SUPPOSE}, \text{ASSUME}, \text{REFLECT}\}\\
&-R_{11}+ = \{\text{REASON}\}
\end{aligned}
$$

A subset analysis on these sets results in the hierarchy shown in figure 2 below. Note that some powerful inferential patterns that can be used in language understanding are implicit in the structure shown in figure 2. For example, what does not think does not hurt (L_7), what walks also runs (L_8), anything that lives evolves (L_3 and L_4), etc.

Note that according to our strategy every concept at the knowledge- (or commonsense) level must 'own' some unique property, and this must also be linguistically reflected by some verb or adjective. This might be similar to what Fodor (1998, p. 126) meant by "having a concept is being locked to a property". In fact, it seems that this one way to test the demarcation line between commonsense and domain-specific knowledge, as domain-specific concepts do not seem to be uniquely locked to any word in the language. Furthermore, the property a concept is locked to (e.g., the property THINK of Rational) is closely related to the notion of *immutability* of a feature discussed in (Sloman *et al*, 1998), where the immutable features of a concept are those features that collectively define the essential characteristics of a concept.

7. Towards a Strongly-Typed Meaning Algebra

If Galileo was correct and *mathematics is the language of nature*, then Richard Montague (see the paper on ELF in (Thomasson, 1974)), is trivially right in his proclamation that *there is no theoretical difference between formal and natural languages*. Moreover, if Montague is correct, then there should exist a formal system, much like arithmetic, or any other algebra, for concepts, as advocated by a number of authors, such as Cresswell (1973) and Barwise (1989), among others. What we are arguing for here, is a formal system that explains how concepts of

various types combine, forming more complex concepts. To illustrate, consider the following:

(37) *artificial* :: NaturalKind → Artifact
(38) *flower* :: Plant
(39) *flower* :: Plant ⊃ LivingThing
(40) *flower* :: Plant ⊃ LivingThing ⊃ NaturalKind
(41) *artificial flower* :: Artifact

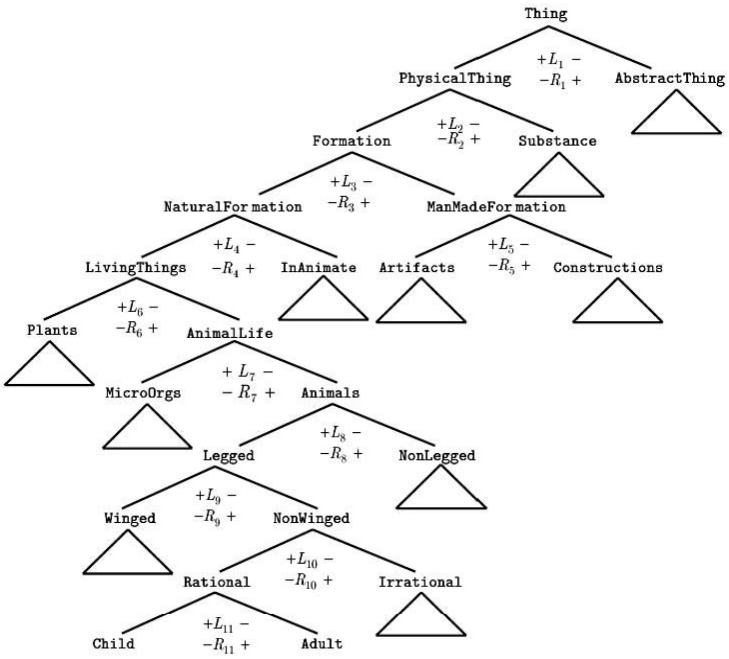

Figure 2. An **Adult** is a **Physical**, **Living Thing** that is FORMED, it GROWS, it DEVELOPS, it MOVES, it can WALK, RUN, HEAR, SEE, TALK, THINK, REASON, etc.

What the above says is the following, respectively: *artificial* is a function that takes a NaturalKind and returns an Artifact; a *flower* is a Plant; a *flower* is a Plant which is a LivingThing; a *flower* is a Plant, which is a LivingThing, which in turn is a NaturalKind; and, finally, an *artificial flower* is an Artifact. Therefore, '*artificial c*', for some NaturalKind c, should in the final analysis have the same properties that any other Artifact has. Thus, while a *flower*, which is of type Plant, and is therefore a LivingThing, grows, lives and dies like any other LivingThing, an *artificial flower*, and like any other Artifact, is something that is manufactured, does not grow, does not die, but can be assembled, destroyed, etc.

The concept algebra we have in mind should also systematically explain the interplay between what is considered commonsense at the linguistic level, type checking at the ontological level, and deduction at the logical level. For example, the concept *artificial car*, which is a meaningless concept from the standpoint of commonsense, is ill-typed since Car is an Artifact, and Artifact does not unify with NaturalKind – neither type is a sub-type of the other. The concept *former father*, on the other hand, which is also a meaningless concept from the standpoint of commonsense, escapes type-checking since *father,* which is a Role, is a type that *former* expects as shown in (35) below.

(35) $\mathit{former} :: \mathsf{Role} \rightarrow \mathsf{Role}$

However, as we show in appendix A, the fact that the concept *former father* is meaningless, while it escapes type-checking, is eventually caught at the logical level by resulting in a contradiction. Thus what is meaningless at the linguistic level should be flagged at the type-checking level, or, if happens to escapes type-checking, such as *former father,* it should eventually result in a logical contradiction at the logical level (see appendix A concerning *former father*). The picture we have in mind therefore can be summarized as shown in figure 3 below.

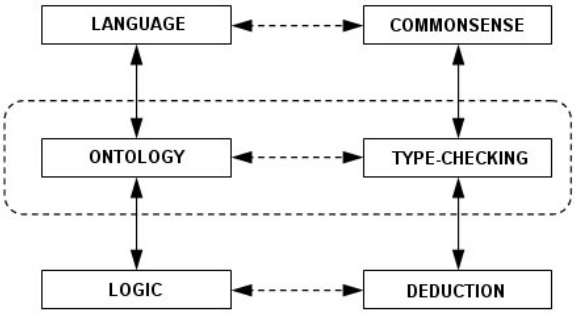

Figure 3. The interplay between language, ontology and logic.

The strongly-typed concept algebra that we envision is one that would also explain why removing the middle noun form (36), for example, changes the subject considerably while the same is not true in (37):

(36) *computer book sale*
(37) *information management system*

That is, what is the nature of the ontological categories that behave similar to *book* and what are those that are similar to *management*? This specific problem is crucial for such tasks as subject-based information retrieval.

Finally, a remark must be made about the distinction between analytic and synthetic knowledge. What we have been suggesting in this paper is a process that would hopefully lead to the *discovery* of a strongly-typed ontology of commonsense knowledge, along with a strongly-typed concept algebra. While this is a challenging Endeavour in its own right, alone this will do little to the construction of natural language understanding systems, unless an inferencing strategy that utilizes this ontology is properly formulated. While the ontology provides the synthetic knowledge that an NLU system might need, an NLU system must clearly use quite a bit of analytic knowledge. The following are two typical examples of what we mean by analytic knowledge that an NLU system would need:

(38) $(\forall \mathrm{P})((\exists \mathrm{C}_1)(app(\mathrm{P},\mathrm{C}_1)) \supset (\forall \mathrm{C}_2)((\mathrm{ISA}(\mathrm{C}_2,\mathrm{C}_1)) \supset app(\mathrm{P},\mathrm{C}_2)))$

That is, any property P that applies to (or makes sense of) some concept C_1, also applies to (or makes sense of) any other concept C_2 of its subtypes. Clearly there are numerous other such rules that should ultimately be added.

8. Concluding Remarks

In this paper we have argued that many problems in the semantics of natural language are due to a large gap between semantics (which is an attempt at understanding what we say in language about the world) and the way the world is. This seemingly monumental effort can be grossly simplified if one assumes, as Hobbs (1985) observed some time ago, a theory of the world that reflects the way we talk about it. We have shown here that assuming such a theory (i.e., such a strongly-typed ontology of commonsense concepts) reduces certain problems to near triviality. Discovering such an ontological structure is clearly another matter. Clearly, however, since natural language is the best known theory of our (shared) commonsense knowledge, analyzing natural language and the way we talk about the world is the best avenue to discovering the nature of this ontological structure. Subsequently, we have argued here and presented a new approach to the systematic design of ontologies of commonsense knowledge. The method is based on the basic assumption that \language use" can guide the classification process. This idea is in turn rooted in Frege's principle of Compositionality and is similar to the idea of type inference in strongly-typed, polymorphic programming languages. The experiment we conducted shows this approach to be quite promising as it seems to have answered a number of questions simultaneously. In particular, the approach seems to (*i*) completely remove the need for multiple inheritance; (*ii*) provide a good explanation for polysemy and metaphor; and (*iii*) suggest a good model for the semantics of nominal compounds.

Much work remains. Indeed, in addition to discovering the nature of this strongly-typed ontological structure and the corresponding (meaning) algebra, a

natural language understanding would ultimately systems require a powerful defeasible inferencing strategy to resolve a number of problems at the discourse level. With that much still ahead, it would seem that Turing might have had it right all along – understanding spoken language might, after all, be the ultimate test of machine intelligence.

Appendix A

Using the logical formulation of the meaning of *former* given above, we show here how the concept '*former father*' translates into a logical contradiction.

First, we reiterate the meaning of 'former' in (1). In (2) we state the fact that the role type Father has an essential temporal property, namely that once someone is a father they are always a father. The deductions that follow should be obvious.

1. $(\forall x{:}\mathsf{Role})\Big(\textsc{former}(x) \equiv_{df} \lambda P\Big[(\exists t)\big((t<now) \wedge P(x,t) \wedge \neg P(x,now)\big)\Big]\Big)$
2. $(\forall x)\Big((\exists t_1)\big(\textsc{father}(x,t_1) \supset (\forall t_2)\big((t_2 > t_1) \supset \textsc{father}(x,t_2)\big)\big)\Big)$
3. $(\exists t)\big((t<now) \wedge \textsc{father}(x,t) \wedge \neg\textsc{father}(x,now)\big)$ (1) applied on FATHER
4. $(t < now) \wedge \textsc{father}(x,t) \wedge \neg\textsc{father}(x,now)$ EI of (3)
5. $\textsc{father}(x,t)$ $\wedge$ – elimination of (4)
6. $(\exists t_1)\big(\textsc{father}(x,t_1) \supset (\forall t_2)\big((t_2 > t_1) \supset \textsc{father}(x,t_2)\big)\big)$ UG of (2)
7. $\textsc{father}(x,\mathsf{u}) \supset (\forall t_2)\big((t_2 \geq \mathsf{u}) \supset \textsc{father}(x,t_2)\big)$ EI of (6)
8. $(\forall t_2)\big((t_2 > t) \supset \textsc{father}(x,t_2)\big)$ (5), (7) and MP
9. $(t_2 > t) \supset \textsc{father}(x,t_2)$ UG of (8)
10. $(t < now)$ $\wedge$ – elimination of (4)
11. $\textsc{father}(x,now)$ (9), (10) and MP
12. $\neg\textsc{father}(x,now)$ $\wedge$ – elimination of (4)
13. $\perp$ (11) and (12)

Appendix B

In (24) above we stated that the meaning of some adjectives, namely those expressing a physical property of a PhysicalThing are intersective. Specifically, we argued that for constructions of the form [A N] where A is a physical property (such as *red, large, heavy*, etc.) and N is a object of type PhysicalThing (such as *car, person, desk*, etc.), the meaning of [A N] can be obtained as follows:

(1) $[\![A\ \ N]\!] = \left\{x \middle| A_{\mathsf{PhysicalProperty}}(x) \wedge N_{\mathsf{PhysicalThing}}(x)\right\}$

Note here that the above expression is not a statement about the meaning of any particular adjective. Instead, (1) simply states is that some adjectives, such as *large, heavy*, etc. are intersective.

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Thus, in $[\![large\ table]\!] = \{x \mid \text{LARGE}(x) \wedge \text{TABLE}(x)\}$, for example, it is assumed that the meaning of *large*, namely the predicate LARGE(x) has been defined.

Although the semantics of such adjectives is not our immediate concern here, it must be pointed out that semantics of such (intersective) adjectives, which are presumably the simplest, can be quite involved, as these adjectives are very context-sensitive – clearly the sense of '*large*' in '*large elephant*' is quite different from the sense of '*large*' in '*large bird*'. Assuming the existence of a predicate, $\text{TYPICAL}_{\text{A}}(x)$, which is true of some object x and one of its attributes A, if x is a typical object with respect to A, then the meanings of such adjectives as *large* and *heavy*, for example, could be defined as follows:

(1) $$large \Rightarrow (\forall x : \mathsf{PhysicalThing})\Big(\text{LARGE}(x) \equiv_{df} \lambda P\Big[P(x) \wedge (\exists y : \mathsf{PhysicalThing})\big(P(y) \wedge \text{TYPICAL}_{\text{SIZE}}(y) \wedge \text{SIZE}(x, s_1) \wedge \text{SIZE}(y, s_2) \wedge (s_1 >> s_2)\big)\Big]\Big)$$

(2) $$heavy \Rightarrow (\forall x : \mathsf{PhysicalThing})\Big(\text{HEAVY}(x) \equiv_{df} \lambda P\Big[P(x) \wedge (\exists y : \mathsf{PhysicalThing})\big(P(y) \wedge \text{TYPICAL}_{\text{WEIGHT}}(y) \wedge \text{WEIGHT}(x, w_1) \wedge \text{WEIGHT}(y, w_2) \wedge (w_1 >> w_2)\big)\Big]\Big)$$

What (1) and (2) say is the following: that some P object x, is a *large (heavy)* P, *iff* it has a SIZE (WEIGHT) which is larger than the SIZE (WEIGHT) of another P object, y, which has a typical SIZE (WEIGHT) as far as P objects go. It would seem, then, that the meaning of such adjectives is tightly related to some attribute (large/size, heavy/weight, etc.) of the corresponding concept. Thus, such adjectives, while they are intersective, they are context-dependent, in that they get their meaning fully specified only in the context of a specific concept.

There are other types of adjectives, which are sometimes referred to as subjective, which can be even more complex (e.g., *famous, important, obvious,* etc.), and their treatment is certainly beyond the scope of this paper. What we simply wanted to highlight in this discussion is the importance of this kind of linguistic analysis in discovering the nature of various ontological categories, which is the underlying strategy in our approach to the discovery of the structure of commonsense knowledge.

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Second Order Logic, Intended Models and Ontology

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Abstract. The article considers some results on second order logic and intended models. The incompleteness of second order logic and other related formal results are interpreted in philosophical and ontological terms.

1. From first to second order

The article assumes that the reader has a basic knowledge of first order predicate calculus both from a syntactic point of view (e.g. definition of terms and formulas, rules of deduction) and a semantic one (concepts of interpretation, model, logical consequence). It also assumes standard definitions of the meta-theoretical properties of calculus and formal theories.

In first order logic it is possible to quantify on first order, or individual, variables. For that matter, these variables are the only ones in our alphabet. However, there is nothing to prevent us from increasing the set of non logical signs to include a set of higher order variables (which we can denote with X, Y, Z, and indices if necessary). In this case, we have two formation rules for higher order quantified formulas (if α is a formula and X is a free predicate variable, then $\forall X\alpha$ and $\exists X\alpha$ are formulas) and the corresponding introduction and elimination rules for predicate quantifiers. What difference does it make if we adopt these new signs? We may answer that the changes are numerous and very profound, at both the theoretical level (in respect to the structure itself of language) and the meta-theoretical one (considering the language itself as an object and analyzing its properties).

The first feature brings a remarkable increase in expressive power. Imagine formalizing a sentence like the following:

(a) If two individuals share the same properties then they are identical.

It is clear that we have to quantify on two kinds of objects – individuals and properties – and it is evident that the expressive resources of first order language are not sufficiently powerful for the purpose.[1] The formal translation of (a) is nevertheless easily achieved if we pass to second order language:

(a*) $\forall x \forall y(\forall X(Xx \leftrightarrow Xy) \rightarrow x=y)$

Note that (a*) is a part of Leibniz's identity principle. In second order logic it is possible to define identity. Indeed

(b) $a=b \Leftrightarrow \forall X(Xa \leftrightarrow Xb)$

Two individuals (a and b) are identical if and only if they share all properties. We can, moreover, introduce function variables (*f*, *g*, *h*,…) and state, for example, the existence of the identity function

(c) $\exists f \forall x(f(x)=x)$

2. Semantics and structures

Given certain items in the syntax of second order calculus, we can proceed with the definition of semantics. What kind of interpretation of predicate variables can we provide? First, consideration of the concept of structure for second order language is useful when analysing certain features of the L-structure which supports the interpretation of first order language.

An L-structure is defined on the basis of a particular (instance) example of predicate language L. The object support of the L-structure (A) is the domain of quantification for individual variables belonging to L. In our language, predicate constants are interpreted on particular objects fixed in L-structures. More formally:

. $S = \langle A, \overline{P}_1 \ldots \overline{P}_n \rangle$

For example, let $L_{N<}$ be the language of the order structure of natural numbers. The corresponding L-structure would be:

$S_{N<} = \langle N, \overline{P}_1 \rangle$

Where N = {0, 1, 2, 3, …} whereas $\overline{P}_1$ corresponds to sign <.

[1] It is very important to refer to two different kinds of 'entities'. As a matter of fact, it is possible to adopt first order logic when dealing with complex objects like sets and properties. However, it is essential that these objects be the only ones in the theory.

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The structure of the second order logic semantics (hereafter S_{II}) now appears different. In this case, we must handle two kinds of variables and must therefore fix two object domains, one for individual variables, the other for predicate variables. This is not entirely true, however, because higher order domains are generated from the domain of individuals. The structure contains a support for individuals completely analogous to that presented for first order. It is then necessary to have a domain for all properties and relations of individuals. A property can be defined as a particular set: the set of individuals which possess that property. Likewise, but with a higher degree of complexity, a dyadic relation is a set of ordered pairs, a triadic relation is a set of ordered triplets and so on. To specify the domain of all predicate variables, we must consider all subsets of the object support A, all subsets of ordered pairs obtained from the elements of A, and so on. If A is the object support of S_{II}, then P(A)[2] will be the domain of monadic predicate variables, $P(A \times A)$ will be the domain of dyadic predicate variables, and so on. Synthetically, we can state that, for all n,

$$S_{II} = \langle A, \wp(A) \cup \wp(A \times A) \cup \wp(A \times \ldots \text{ n times } \ldots \times A) \rangle$$

It is clear at this point that the 'generative' process of higher order domains starts from the domain of individuals. Once A has been fixed, the definition of $P(A^n)$ can be obtained using well known set-theory procedures, and this requires the construction of all possible relationship combinations. The extensional approach adopted consists in defining a relation as a certain set (of ordered n-tuples). Intuitively, this may seem rather strange. Of course, properties and relations define sets, but are they completely reducible to sets? The property 'to be green' defines the class of all green objects, but if we find a predicate coextensive with 'to be green' then we are forced to admit that these two properties are extensionally identical. This is the main perplexity raised by the extensional approach. We can imagine two properties of Euclidean triangles: equilaterality and equiangularity. Intuitively, these two properties are different: one is the feature of having all sides of equal length, the other is the feature of having all angles of equal width. However, every equilateral triangle is equiangular, and vice versa. In regard to the structure of S_{II}, equilaterality and equiangularity will be defined as the same subset of P(A): they will therefore be indistinguishable. We can formally write the extensional requirement thus:

$$(\text{est})(\text{om}\, \overline{x}_1 \ldots \overline{x}_n)\ (\text{om}\, \overline{P}^n)\ (\text{om}\, \overline{Q}^n)\ (((\overline{x}_1 \ldots \overline{x}_n) \in \overline{P}^n \Leftrightarrow (\overline{x}_1 \ldots \overline{x}_n) \in \overline{Q}^n) \Leftrightarrow \overline{P}^n = \overline{Q}^n)$$

Another distinctive feature of the structure of S_{II} is the maximality condition. Because of this property, the structure displays all possible combinations of attributes (properties and relations). S_{II} is thus maximal or full. The difference between full structures and L-structures becomes clear: in the former case variables

[2] $\wp(X)$ denotes the power set of X, that is, the set of all its subsets.

are constituted by all possible combinations of the attributes of the structure's object support; in the latter, particular sets are fixed as denotations of predicate constants. Comment is required on the cardinality of structure. As said, the cardinality of a structure is established by the cardinality of its object support. In the first order case, if A is countable then all structures will be countable because every P^n is defined as a subset of A. Matters change, and very profoundly, in the case of S_{II}, for if A is countable then P(A) is uncountable. More precisely, if the cardinality of A is $\aleph_0$ (as in the case of natural numbers) then the cardinality of P(A) is $2^{\aleph_0}$. Therefore, although the full structures are initially defined from the object support, they display an increase in cardinality. To summarize, the maximality property and the power set operation are two aspects of the same phenomenon present in full structures.

However, the interpretation just presented is not the only one available. It is possible to impose restrictions on the higher order domains and thus obtain less dense structures. These particular structures are said to be *general*; essentially, the maximality requirement is therefore eliminated. Not every possible combination of attributes is in the structure. It is evident that the power set operation is not used as a criterion for 'construction' of the domain for predicate variables. The canonical (or full) structure discussed above can be considered a limiting case of the general structure, where the full structure coincides with a general structure in which the range of predicate variables coincides with the entire power set. Of particular interest among general structures (or non standard ones, according to the fashionable terminology) are Henkin structures. Henkin's ground-breaking idea was to consider only attributes that are nameable in the language as existing. Hence, with X denoting the attributes of structures, and P the predicate variables, it holds that

(om X) (ex P) (I(P) = X)

that is, for every attribute in the structure there is a predicate sign. Clearly, not all possible combinations of attributes can be described in the language, with the consequence that the Henkin structure is less dense than the standard one. This becomes apparent if we consider the cardinality of sets. Our language is always countable, whereas the cardinality of the set of all subsets of a countable set (as in the set of individuals) is uncountable, as shown above. It immediately follows that if only nameable attributes exist, their number will be at most countable and, therefore, a portion of the power set.

Henkin semantics enable us to obtain interesting results: for example, that there exists a completeness theorem for higher order logic with respect to this semantics. To understand this finding, we must reflect on the fact that the expressive increase in second order logic is due to the canonical interpretation of predicate quantifiers. As we shall see in the next paragraph, only with standard semantics can we obtain characterizing mathematical results as interesting as categoricity; if we interpret a second order language with Henkin semantics we gain completeness of calculus but we lose (as with first order) categoricity. Simply put, second order logic with non standard semantics is only apparently of higher

order. The Henkin interpretation is nothing but a sortal interpretation of higher order language. More specifically, the non standard structure is:

$S_H = \langle A, P^1 \ldots P^n \rangle$

where A is again the object support of the structure, P^1 is a meta-theoretical letter for a set of subsets of A, P^2 is a set of subsets of ordered pairs, and so on. When P^1 coincides with P(A), P^2 with $P(A \times A)$, and so on, then the Henkin structure is identical to the full structure.

From a philosophical point of view, it is interesting to note that non-standard structures highlight a fundamental issue in the philosophy of logic and in formal theories in general. There exists a 'gap' between sets defined as beginning from, in a certain sense, constructive operations and admission of the existence of 'large' sets, such as the set of all subsets. From a perspective *à la* Henkin, *esse est construi* since it makes no sense to postulate the existence of something which cannot be expressed by language. Cocchiarella, for example, uses Henkin semantics as a formal device with which to enhance conceptualism. Predicates are cognitive structures, and quantification of these entities is only possible because we are operating within a kind of 'construed' structure. In Platonism, according to Cocchiarella, it is necessary to postulate the set of all subsets as given, and this gives rise to the well known (and obviously disagreeable) phenomenon of semantic incompleteness.

Hintikka, on the other hand, suggests that much of the history of mathematics should be examined in light of conceptual categories of standard and non-standard interpretation. According to Hintikka, these conceptions are present whenever we refer to totalities. If we admit arbitrary functions or sets into the mathematical argumentation, we accept a particular point of view. For those who like slogans, the word 'all' really means 'all' for standard conceptions, whereas from a non-standard point of view 'all' means everything that can be defined. Of course, an advocate of a constructivist position would seek to show that the idea of a really existing totality, defined in a non-predicative way, is only an illusion.

3. The emergence of first order language. A historical outline

This section examines aspects of the historical development of formal logic, and in particular what can be called the 'triumph' of first order logic. Since the 1930s, in fact, the only formal logic has been first order logic. However, it has been historically proven that no clear distinction was drawn between first order and higher order systems when logic first developed, and when the meta-theoretical properties of calculi and theories were not yet defined with absolute rigour. This was for essentially two reasons. First, formal logic was an entirely new science; it borrowed mathematical methods and philosophical traditions, creating a highly distinctive field of knowledge rather difficult to reduce to other well known

disciplines. Second, and more importantly, the first authors were unaware of the fundamental distinctions between semantics and syntax, and among language, calculus and theory.

The early logicians conceived formal systems as fragments of a mathematical language developed for a variety of purposes, from characterization of mathematical structures to definition of universal laws of rational discourse. Moreover, they used the concept of 'formal' with a meaning different from the contemporary one. The first systems of logic utilized a sort of intuitive semantics and were always applied to a certain domain of discourse. The subsequent sharp separation between syntax and semantics, and the birth of formal semantics, were among the most important advances in the history of logic. Since it is impossible to provide, even briefly, a history of formal systems, I shall concentrate on three historically relevant aspects that shed light on the relationship between first order and higher order logic.

(i) Fundamental trends at the origins of modern logic.

Many authors identify three currents of thought between the second half of the nineteenth century and the first twenty years of the twentieth.

The first tradition, which we can call algebraic, includes, among others, Boole, Schroeder, and Löwenheim. In this tradition, formal logic was very similar to abstract algebra. The purpose of this project was to construct a universal calculus which could be used in various areas of mathematics. Logical axioms were satisfied by numerous systems which were then rigorously organized. Axioms themselves formed a general abstract structure. The domain of reference changed constantly because different sets of objects could satisfy the same axiomatic structure.

The second tradition adopted the practices and methods of mathematics and comprised Dedekind, Hilbert, Peano, Huntington and Veblen. The aim of logic was to characterize mathematical structures. But, unlike the algebraic tradition, in this case there was no quest for generality. The logical system was not required to apply to all branches of mathematics; rather, axiomatic systems were constituted for each single mathematical theory. To be stressed is that pure logical investigation was not contemplated; rather, the aim was to rigorize mathematical theories.

The aim of the third tradition, that of the logicist approach, was to codify the underlying logic of all rational scientific discourse. This approach was taken by Frege, Russell, and perhaps the early Wittgenstein and Ramsey. According to Frege, logic was the science of "to be true", and since the concept of truth had to do with all human knowledge, it was evident that logic concerned the most general features of actual discourse. Logicism came rather close to Leibniz's idea of a *lingua characterica* and of a calculus ratiocinator powerful enough to codify and deduce every possible meaningful statement. Logicism logic, therefore, was a constantly interpreted formal system because it reflected these general and fundamental (analytical) features of the universe of discourse.

> I did not wish to represent an abstract logic by formulas, but to express a content […] in more exact and clear fashion [11]

These approaches, which differed in both their conceptions of logic and their technical developments, independently elaborated a theory of quantification. Of course, the same quantifiers were interpreted in different ways, but it was always possible to find two kinds of variables: individual variables and predicate variables. For example, Peirce's and Schroeder's systems included something similar to a comprehension principle for classes: that is, an axiom according to which every proposition (or propositional function) defines a class.

Frege's *Begriffsschrift* [10] is the first example of predicate calculus in the modern sense. It presented axioms and deduction rules; moreover, it drew a rudimentary distinction between semantic and syntax. The quantifiers in Frege's system referred to individual variables and conceptual variables (for Frege, concepts were denotations of predicates). It was thus logically possible to define relations of identity, equipollence and ancestral which played a fundamental role in defining the concept of natural number. Members of the mathematical school also introduced variables referring to classes (or sets). In common mathematical practice, in fact, claims are frequently and unproblematically made about expressions involving the existence of functions or sets. But if we want to translate these concepts into logic, we must adopt the expressive power of second order languages.

(ii) First order predicate calculus as a subsystem.

When was a distinction between first order and higher order systems first introduced? In 1915 Löwenheim [21] proved a fundamental result which was reconsidered and improved some years later by Skolem. This is known as the Löwenheim-Skolem theorem. Put very briefly, this theorem states that if a first order theory has an infinite model then it has models of every cardinality, and if a theory has models of every cardinality it also has a countable model. In light of these references to the concepts of model and the satisfiability relation, we can consider Löwenheim's 1915 work to be the origin of model theory and formal semantics. Here, however, we are interested in the partition between domains of predicate variables and domains of individual variables. Löwenheim first delimitated a certain region of logic and then examined certain features of the system obtained. His approach was highly meta-theoretical in that he investigated some properties of the logical system considered as an object.

> As far as we know, Löwenheim was the first in the history of logic to focus on first-order logic and to investigate some of its non trivial metalogical properties. [23]

Löwenheim's distinction was based on the notions of 'relational expression' and 'individual expression': formulas belonging to the latter category can have only individual variables. Of course, the system utilized by Löwenheim, namely

the relatives theory, is a higher order system: the proof is stated in that theory and has certain features of a well-defined subsystem. Skolem's contribution was a refinement of the proof which reduced meta-language to first order (using Skolem's functions).

Two years later (1917), Hilbert delivered a number of lectures in which he applied the axiomatic method to logic itself. He had successfully formalized geometry at the beginning of the century, and then moved to logic, defining a subsystem, named restricted functional calculus, which corresponded to current first order predicate calculus. However, Hilbert was doubtful whether it was possible to formalize the whole of mathematics with restricted functional calculus. In 1928, when *Grundzüge der theoretischen Logik* was published, first order logic was still considered to be a subsystem of logic *qua talis*. Nevertheless, Hilbert himself and his school set out to investigate some meta-theoretical properties of this part of formal logic.

(iii) Reasons for abandonment.

We have seen that complete awareness that the orders of formal systems are stratified came in the early years of the twentieth century. At that time, however, the ordinary conception of logic was broader than it is today. How is it possible, therefore, to explain the progressive abandonment of a part of logic considered fundamental only a few decades earlier?

There were various reasons for this abandonment: historical, theoretical, and others ascribable to the cultural climate. However, thorough analysis of the matter would be beyond the scope of this article. I shall instead focus on two factors which I believe to be crucial for explanation of the triumph of first order logic. In 1930, the young Austrian logician Gödel [14] proved the completeness of restricted functional calculus. This result was very much expected: Skolem had been very close to obtaining it, but it seems that his obstinately finitist views prevented him from doing so. According to completeness, for the subsystem considered, the notion of validity is reducible to the notion of demonstrability. It is clear that this feature was very important for the endeavour to formalize theories. Any consistent set of first order formulas has a model that satisfies it. Coherence implies existence, to use one of Hilbert's well-known slogans. A year later, Gödel proved the syntactical incompleteness of formal theories containing at least elementary arithmetic. The semantic incompleteness of higher order logic follows as a corollary of this theorem. Two years later, first order logic was no longer considered a subsystem but but rather as the only complete logic, while the system commonly considered to be the most suitable formal system became a fragment of mathematics (and not logic) by reason of its semantic incompleteness. Gödel himself considered logic to be only first order predicate calculus, but he never justified this position. Obviously, it is impossible to attribute strictly formalistic and finitist positions to him. Gödel probably considered first order logic to the highest expression of the formality and rigour necessary for justification of scientific knowledge.

The upshot of the foregoing brief historical and conceptual survey is that a fundamental factor in the development of logic was the introduction of a quantification theory, and that this theory was developed by schools with very different approaches. Quantification involves the idea that there exists an object domain across which it is possible to range the variables of language. Although different views are taken on logic's field of application, they all admit elementary order objects (individuals) and higher order objects (properties, relations, sets, classes, concepts). The new logic was essentially higher order. Hilbert and Löwenheim analysed a particular subsystem of logic – restricted functional calculus – which examined the quantifiers referring only to individual variables. This was first order calculus. Moreover, logical inquiry was conducted not *in* the formal system but *on* the formal system, so that its results on the soundness, completeness and satisfiability of systems were obtained metatheoretically.
In the end, a series of technical results and a certain philosophical approach made first order logic into the only 'logic' in the proper sense of that term.

4. Dedekind and categoricity

In *Was sind und was sollen die Zahlen?* [8] Dedekind defines a particular formal system similar to set theory and provides, on the basis of the notions introduced, a very rigorous reconstruction of the natural numbers theory. The primitive concepts are System (which is the equivalent to the concept of set), Abbildung (the analogous to concept of function) and a 'to be a part of' relation among sets and objects. To be noted is that this relation comprises both the nexus of membership ($\in$) that holds among elements and sets and the nexus of inclusion (proper or improper) that holds among sets. Dedekind's symbology is not used in the following incomplete reproduction of the system.
We shall examine the fundamental notions of 'chain' and 'chain regarding a function' Starting from these concepts it is possible to define the natural numbers sequence rigorously.

Let f be a map, a an object, and S a set. By $f(a)$ is meant the mapping of a, by $f(S)$ the set of all mappings of the element s of S. We may therefore state that set K is a chain if and only if when an element belongs to K also its mapping belongs to K. Formally:

K is a chain $\Leftrightarrow$ (om a) $(a \in K \rightarrow f(a) \in K)$

Hence, assuming that K is a chain, if $a \in K$ then also $f(a) \in K$; therefore, if $f(a) \in K$ also $f(f(a)) \in K$, and so on.

Definition 44 concerns the concept of chain in respect to a certain function. Let A be a part of S. Then A_0 is the intersection of all chains of which A is a part. That is:
$A_0 = \cap\{K|K \text{ is a chain and } A \subset K\}$

We now fix a particular element, the 1. According to the above definition, 1_0 is the intersection of all chains in respect to a certain function of which 1 is a part. If 1 belongs to the chain K, we have that $f(1) \in K$, $f(f(1)) \in K$, and so on. Hence let A, A', A'', A''' be all the chains of which 1 is part (and also containing $f(1)$, $f(f(1))$, and so on.); 1_0 is, say, the common part of all these chains. The fixed fundamental concepts in paragraph 72 state four conditions which must be satisfied by a system for it to be simply infinite. The interesting paradigmatic example is, for Dedekind, the series of natural numbers.
A system N is said to be simply infinite when there exists a mapping f with domain N and an element 1, such as:

(a) $f(N) \subset N$

(b) $N = 1_0$

(c) $1 \notin f(N)$

(d) is 1-1

(a) states that the mapping set of N is properly included in N. With f interpreted as a successor function, (a) means that every successor of a number is a number. There exists, however, an element belonging to N, but not to $f(N)$; that is, there exists an element which is not the successor of any number. Axiom (c) states that 1 is not a successor of any number while it is assumed that $1 \in N$. Dedekind, note, takes 1 (and not 0) to be the initial element of the number series. Axiom (b) defines the set of natural numbers as the intersection of all chains containing 1. Finally, (d) states that f is one-to-one; that is, $f(a) = f(b) \rightarrow a=b$: if two numbers have the same successor, they are identical.

5. Categoricity Theorem

In paragraph 132 Dedekind shows that:

> All simply infinite systems are similar to the number-series N and consequently […] to one another.

In contemporary terms, this means that second order arithmetic (PA_{II}) is categorical. A theory is categorical when it admits only structurally identical models. that is, given any two models $\Im_1$ and $\Im_2$, they are isomorphic. There therefore exists a mapping (isomorphism function) such that it preserves the structure. This mapping f relates elements of domain d_1 (the domain of $\Im_1$) to elements of domain d_2 (the domain of $\Im_2$) so that the two following isomorphism conditions are satisfied.

Let, then, $\Im_1$ and $\Im_2$ be two models of arithmetic, while d_1 and d_2 are the respective object domains, 0_1 and 0_2 are the elements which correspond to 0 in the respective models, and s_1 and s_2 are successor mappings. Thus:

•) $f(0_1)=0_2$

••) $f(s_1(x)) = s_2(f(x))$

(•) states that the mapping establishes a correspondence between a particular element of the first domain (the zero of d_1) and a particular element of the second domain (the zero of d_2). (••) does likewise with the successor of x in the first domain and the successor of the image of x in the second domain. It is evident that the mapping preserves the structure of models with respect to the zero and to the successor. This means that, once the existence and uniqueness of the function have been proven, it is possible to apply the induction on the entire domain and thus demonstrate that for every object in d_1 there is an object in d_2 and vice versa. By complete induction, it is hence possible to show that all models satisfying the conditions hitherto stated are structurally identical. The proof is structured through the following steps:

1. Introduction of Peano-relations

A Peano-relation is a particular relation between elements belonging to d_1 and elements belonging to d_2. Therefore, if Rxy is a Peano-relation, it follows that $x \in d_1$ and $y \in d_2$. There are two conditions which a Peano-relation must satisfy.

(i) $R(0_1,0_2)$

(ii) $Rxy \Rightarrow R(s_1(x)s_2(y))$

Generally, a Peano-relation is a set consisting of ordered couples which contains couple $<0_1,0_2>$ and is closed to the successor. This means that if it contains an element x, then it also contains the element s(x), i.e. the successor of x, the element s(s(x)), and so on. Note the closeness of the concept of Peano-relation to that of chain introduced above. If Peano-relations are sets of ordered couples which contain an initial couple (<0,0>), and if they are closed to the successor, a chain can be defined as an one-place Peano-relation containing an initial individual (the 1 or the 0) and closed with respect to a certain function.

2. Definition of the intersection of the Peano-relations (R_0)

The intersection of Peano-relations is defined as a particular set consisting of all couples belonging to all Peano-relations. R_0xy is equivalent to $\forall R(Rxy)$. In this case, too, one discerns an analogy with Dedekind's original notion of 'chain respect to f'. For Dedekind, A_0 is the intersection of all the chains which contain A.

3. Proof that the intersection of the Peano-relations is a mapping

This point is the core of the categoricity theorem. From a conceptual point of view, proving that an intersection (like the intersection of the Peano-relations) has the features of a mapping requires demonstration that a particular set possesses particular properties. At the basis of the proof, in fact, is the assumption that properties, relations and functions are conceived extensionally (nor could it be otherwise, at least in a context of classical mathematics). They are therefore sets.

Induction must be used to obtain the third step, as follows:

3.1) Basis. We must show that R_0 is a mapping with respect to the initial element 0. We must then state the existence and uniqueness of correspondence.

3.1.1) Existence. The existence of correspondence is not particularly problematic since it follows directly from the definition itself of the Peano-relation.

3.1.2) Uniqueness. The issue of uniqueness is more complex. We must show that R_0 relates the element 0_1 to a single element belonging to domain d_2, namely 0_2. The uniqueness is proved by introducing a particular relation R'_0 which is entirely similar to R_0 but denies the property of uniqueness to the mapping, and by showing that the introduction of this mapping gives rise to contradiction.

3.2) Step. The procedure is analogous for the step. We must show that R_0 is a mapping with respect to the successor of an arbitrary element x. Once again, we shall proceed by stating existence and uniqueness.

As we have seen, the principle of arithmetical induction is the cornerstone of the entire proof of Dedekind's theorem. We can formally translate this principle as:

$$(PI_{II}) \quad \forall X((X0 \wedge \forall x(Xx \rightarrow Xsx)) \rightarrow \forall x Xx)$$

The intuitive meaning is rather simple: if a property holds for 0 and if it holds for a certain number, then it also holds for its successor, so that the property holds for the entire domain. This axiom codifies the informal procedure of proof by induction on the natural numbers structure. It is proved that a property holds for an element of lowest complexity, and it is shown that it also holds for the successor. We thus obtain the basis and step of the induction. This principle obviously cannot be formalized in a first order language; Peano's first order arithmetic (PA_I) presents an axiom schema

$$(PI_I) \quad \alpha(0) \wedge \forall x(\alpha(x) \rightarrow \alpha(s(x))) \rightarrow \forall x \alpha(x))$$

The fundamental difference between the two formulations is that in the former case the universal quantifier covers the domain of properties, whilst in (PI_I) α is a meta-theoretical letter which is first order definable for any property. This means that the categoricity proof is not importable into PA_I. We shall see why. As in the second order case, it is possible to define the intersection of the Peano-relations, and we can prove both the basis and step of our induction. What is impossible to obtain is the complete induction on the entire domain (d_1 and d_2). Indeed, the interesting property is defined using the concept of 'all the Peano-relations', and this concept involves a higher order quantification.

In conclusion, the categoricity proof of elementary arithmetic manifests the expressive difference between first order language and higher order language.
I shall return to the both logical and conceptual importance of these results later. I now concentrate on the consequences of categoricity and incompleteness for the meta-theoretical properties of second order logic.

6. Incompleteness

In 1931, Gödel published probably the most influential article in the entire history of logic. He proved the syntactical incompleteness of mathematical non-trivial theories (containing, that is, at least the primitive recursive arithmetic). In every sufficiently powerful theory it is possible to construct (using the well known procedure of arithmetization) a true but not demonstrable sentence in that same theory. In particular (and this is the corollary, called Gödel's second theorem) the undecidable sentence is equivalent to affirmation of the theory's consistency. Gödel thus put paid to all efforts to prove the consistency of a theory by demonstrative means belonging to the theory itself. The implications of Gödel's theorem for logic and philosophy of mathematics were exceptional. Of interest here are two of its main consequences for second order languages.

Let PA_{II} be the conjunction of second order Peano axioms, and let G be Gödel's sentence. We start from the hypothesis (*ex absurdo*) of second order calculus completeness. We therefore have that $PA_{II} \Vdash G$ entails that $PA_{II} \vdash G$; but we know that $PA_{II} \nvdash G$ (because of Gödel's first theorem), hence, by *modus tollens*, we have that $PA \nVdash G$. This means, by definition of logical consequence, that there exists a model which makes PA_{II} true and does not make G true. But G is the sentence that states the consistency of the theory itself, and it is true in the standard model of PA_{II}. The model where G is false must therefore be non-standard. But this contradicts the categoricity theorem. We are therefore forced to conclude that second order calculus is not semantically complete.

The semantic incompleteness of higher order logic is the aspect on which authors have concentrated most closely, and they generally agree that the lack of a complete calculus is a flaw in second order logic. However, to cast negative light this incompleteness it is necessary to sharply define the positions taken up on such key concepts as logic, calculus, and so on. In short, a philosophy of logic is required. Incompleteness entails, trivially, that the satisfiability of the consistent sets theorem does not hold: that is, there are consistent sets of second order formulas which have no model. For example, let $PA_{II} \cup \{\neg G\}$ be the union of second order Peano axioms and the denial of Gödel's sentence. If Cons PA_{II} then Cons $PA_{II} \cup \{\neg G\}$ because $PA_{II} \nvdash G$ and $PA_{II} \nvdash \neg G$ according to Gödel's theorem. Hence, we have that Cons $PA_{II} \cup \{\neg G\}$. But if there existed a model which made $PA_{II} \cup \{\neg G\}$ true, it would also make $\neg G$ true. PA_{II} is categorical, and its models are structurally identical. Consequently, $\neg G$ is false in the standard model, and there is no model which makes $PA_{II} \cup \{\neg G\}$ true even if it is consistent. The incompleteness proof of second order logic is always obtainable from other results.

However, an absolute incompleteness proof does not exist: in the present case, I have used the categoricity of arithmetic and, of course, Gödel's theorem. It is also possible to provide a incompleteness proof which employs Church's result of indecidability.

The soundness and completeness meta-theoretical properties have to do with the power of syntax (that is, of calculus) to capture the contents expressed by semantics. The minimal soundness requirement concerns the reliability of our deduction procedures, while the completeness requirement means that the totality of true sentences must be captured by calculus. Accordingly, when proving soundness and completeness, we assume the semantics and investigate the resources of syntax. The problem at this point is this: in the second order case, where the calculus is not sufficiently powerful with respect to the standard interpretation of language, what is the value of semantics?

7. Logic and intended models

Let us summarize:

(a) Although first order predicate calculus is today considered to be 'the' logical system, it was long conceived as a subsystem of a wider, higher order formal language.

(b) The meta-theoretical properties of first order predicate calculus are well known: its results are sound and complete; every logical consequence is a derivable sequence and vice versa. The range of truth and the range of derivability are coextensive.

(c) However, it is sufficient to add the Peano axioms to first order calculus to have the syntactical incompleteness of formal theories.

(d) If, on the contrary, we formulate our theory in a more powerful language (higher order) we can derive categoricity, that is, the structural identity of models.

(e) But Gödel's result applies to a second order arithmetic as well; in this case syntactical incompleteness becomes the semantic incompleteness of calculus.

It is evident that the notion of model plays a crucial role in this kind of meta-theoretical reflection. In the following paragraph I shall focus on certain aspects of this concept and seek to show its fundamental importance for a philosophical approach to formal logic.

Every theory, formal or empirical (or also pertaining to human sciences), has a model, or a set of objects, which makes the sentences of the theory true. However, this very general definition may not be completely satisfactory for our purposes. Firstly, it is necessary to draw a careful distinction between the empirical

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sciences and the formal ones. In the former, the 'model' is, in a certain sense, already given: it is empirical reality analysed from a certain point of view and according to specific procedures and operations. Matters change if the theories under examination are the formal ones of algebra, analysis, or geometry. What are the objects that give truth, for example, to the axioms of Euclidean geometry? The answer defines the objects of a mathematical theory on the basis of axiomatic structures, and following Carnap, we can call it 'inner'. Only numbers obey the Peano axioms just as geometry's axioms define geometry's objects. The problem is the more or less real reference of the formal objects which we manipulate in mathematical theory. This confronts us with a full-blooded philosophical question. The concept of model (and the closely related questions of structure and interpretation) does not refer to the deep nature of objects. On affirming that the geometry has an algebraic model, what one wants to say is that the axioms of geometry can be satisfied also by objects that are not 'geometric' but belong to another theoretical field: number theory. This statement requires further comment. If it is true that model theory investigates the possible systems of objects on the basis of which we can interpret the axioms of formal theory without examining the nature of the structure itself, or its ontology, it is all the more true that every meta-logical and philosophical reflection on formal theories must make careful consideration of model theory. Investigating the semantics of a theory enables us to grasp the relationships between the language and the section of reality which the theory seeks to describe. The statements of a theory are meaningful only when they are interpreted correctly; and the interpretation consists in assigning every sign in the language to a certain object in the domain. Every ontological approach to the objects of formal theories always refers to the objects of models of formal theories. Of course, the interpretations given in contemporary model theory are not as intuitive as Dedekind's example; the very semantic is , in its turn, formalized and has a large amount of technical concepts.
In this regard, now informally presented is the Löwenheim-Skolem theorem, which is a fundamental result for model theory and can be an interesting starting-point for ontological discussion of the foundations of the formal theory and on the meaning of higher order logic. In his 1915 paper Löwenheim proves that:

(a) if a theory expressed in a first order language has a countable model, then it has models of any cardinality;

(b) if a theory expressed in a first order language has models of whatever cardinality, then it has also a countable model.

Recall that the cardinality of a model is the cardinality of its object support. A model for Peano axioms (that is, a model of arithmetic) must have at least countable cardinality, given that every numeral must denote an object in the structure. Part (a) is termed 'Upward', and part (b) is termed 'Downward'. As is evident, the Löwenheim-Skolem theorem (hereafter LS) states that every first order theory has a plurality of models which make the same axioms true. These models are of different cardinality, and for this reason any categoricity result is excluded.

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Dedekind's theorem states the opposite: numerically different models cannot be placed in a 1-1 correspondence as, on the contrary, it is the isomorphism mapping. The LS theorem is therefore a non-categoricity theorem.

But how is a model of a mathematical theory constructed? This question may appear naive, but the answer introduces notions important for our present purposes. An object support for the Peano theory, for example, is constituted by a set of object N = {0,1,2,3, …}, and so on to infinity. A 'less than' order relation is then defined on these objects. This idea of model is particularly intuitive and does not raise difficulties if we exclude the representation, which cannot be other than finite, of an infinite number of elements. What, then, is the structure of non-standard models? We add a constant c to the language L of arithmetic and thus obtain the language L+. Let D be the set of all propositions formulated in the language L true in the standard model; let D+ and D together be the following sentences:

0<c, 1<c, 2<c, 3<c, 4<c, 5<c, … and so on.

Now, let Γ be a finite set of sentences belonging to D+. Of course, Γ has a model. To see this, suffice it to consider the standard model of D and interpret the constant c on the greatest number of those in Γ. But according to the compactness theorem, D+ has a model $\Im'$. Since $\Im'$ includes $\Im$ (that is, the standard model) $\Im'$ makes all expressions of D true. But $\Im'$ is, say, richer because it contains a denotation of c which is greater than every number in the standard model $\Im$. Thus $\Im'$ is a model of D but it also has an infinite element. Such models are said to be non-standard.

It is therefore possible to construct alternative models of arithmetic that, even though they make all sentences of the theory true, nevertheless contain non-standard elements. We name m the non-standard element (so $\Im(c)=m$) of $\Im'$. Now, since $\Im'$ satisfies Peano's axioms, we can prove that the only element without a predecessor is the zero. Then m must have a predecessor, which we name p(m). Hence we can obtain a structure like

…, p(p(m)), p(m), m, s(m), s(s(m)), …

These particular substructures of the non-standard part of the model are said to be orbits. There are infinite orbits and each of them is isomorphic to the integer numbers order (it is sufficient, in fact, to interpret m on zero and define the order relation)

A more interesting feature is that every non-standard model contains a copy of the standard model as well as other elements. Moreover, Dedekind's concern was to characterize the system of numbers excluding, say, extraneous objects.

> «What, then, must we add to the facts above in order to cleanse our system S again of such alien intruders t as disturb every vestige of order and to restrict it to N?» [7]

In the terminology used here, 'alien intruders' are non-standard elements. The cardinality of the model just presented, however, is countable. The LS theorem states something more profound: every first order theory admitting countable models, like PA for example, has models of every cardinality. We thus find that the theory is unable to fix its references unambiguously: the object universe to whose axioms it refers is vague. Every model has an intuitive and natural section, the standard part, but it comprises (or, at least, it may comprise) a series of infinite elements that are its non-standard part.

What is the cause of this inadequacy? Why cannot the language characterize up to isomorphism the intuitive content of mathematical theories? There are two main approaches to the problem: (a) consider the language, or better, the relationship between language and reality to be indeterminate; this can be called semantic relativism; or (b) place the source of indeterminacy at the ontological level: mathematical structures are intrinsically vague objects.

Hence for semantic relativists what is indeterminate is the reference to the theory's objects. The denotation relation is defined in a theoretical context and it is impossible 'to go outside' and see how things are. For ontological relativists, by contrast, the vagueness of reference is a sign of the deep-lying ontological indeterminacy of structures.

Position (a) is acceptable to almost all philosophical approaches; indeed, the LS theorem itself forces us to consider the concept of model as non-absolute. The philosophical question is how to deduce a deeper-lying ontological indeterminacy from this referential vagueness. But if we accept this last point, we reject all realist positions, even minimal ones. A mind-independent mathematical domain cannot exist if it is not determined in its fundamental properties. Ontological relativity opens the door to the constructivist approach both in an intuitionist context and in a more formalist one where the ontology is established by the axiomatic structure chosen.

Position (a) is also assumed also by thinkers collocated on very different positions who maintain that the formal results discussed thus far simply reflect the inadequacy of formal language in describing mathematical domains. Basic notions like 'uncountable' are vague because first order predicates cannot fully characterize the intuitive mathematical concepts normally used. According to this position we may add the technical fact that the LS theorem does not apply if the theory is formulated in a second order language. For example, ZF_{II} (see Zermelo Fraenkel's second order axiomatic set theory) is quasi-categorical: that is, it is categorical up to an inaccessible cardinal k. This essentially means that the models of ZF_{II} are structurally identical and that they therefore have the same cardinality. The vagueness is thus a consequence of the expressive poverty of first order logic like the vagueness of the arithmetical theories analysed above. While the relativistic point of view, mainly in its ontological version, is committed to rejecting realism, in analogous manner the present approach suggests a realist point

of view. To admit the inability of language to describe structures is to accept the existence of a richer, mind-independent reality.

8. Two fundamental problems of formal discourse

Resuming some of the results obtained earlier, we define a formal system as a triple <L, D, S> constituted by a formal language, a set of deductive rules which enables passage from one expression to another without loss of truth, and a semantics – that is, a particular interpretation function which maps every sign of the language with an object of the domain. The meta-theoretical soundness and completeness theorems concern the relationships between S and D. The soundness theorem establishes that everything deducible is true; the completeness theorem states the opposite. Of course, since calculus and semantics are language sensitive, admitting certain linguistic structures influences the meta-theoretical results obtained.

The first fundamental problem of logic is how to demonstrate completeness. This does not mean that an incomplete logic cannot be called a logic. But, for the logical consequence relation to be finite, it is essential that the set of rules dominate the semantic field.

The second fundamental problem of logic is how to characterize mathematical structures. To show this requires us to go back some way in time. Historically, formal systems were first developed in a mathematical context; they borrowed their methodology from mathematics, and their purpose was to provide mathematics with more secure foundations. In this very close link with mathematical knowledge, the main requirement of a formalized mathematical theory was that it should characterize the structures to which it referred. As we have seen, it is intuitive to state that arithmetic is about natural numbers, analysis is about real numbers, and so on. Well-known non-categoricity results show that once a mathematical theory has been formalized in a first order language, although its objects are intuitively well defined, from a formal point of view we cannot eliminate the non-standard models which cause the vagueness of the theory objects. The univocal characterization of the object universe is the informal counterpart of the concept of categoricity analysed previously.

If completeness and categoricity are the two fundamental problems of formal systems, then discussion of second order logic casts light on that particular phenomenon which we can term the *complementary thesis*. According to this principle, if we determine the logical consequence relation effectively, the notion itself of model becomes indeterminate. If, conversely, we focus on object reference and fix the intended model, we are no longer able to characterize the logical consequence notion.

But the intrinsic limitation of formal systems does not have only negative consequences. The semantic incompleteness of higher order logic clearly shows that the syntax of a language powerful enough to characterize non-trivial mathematical theories categorically cannot cover its semantics. The model of a theory is therefore not reducible to something entirely describable linguistically:

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something is left out; some true sentences (that is, some logical consequences) are not reached by calculus.

If we accept these results, we can see that an antirealist option is at least counterintuitive in the light of the above reflections. In particular, it seems that the mathematical domain is not entirely dependent on the linguistic means that we use to describe it; and *a fortiori*, construction of the mathematical domain by language itself is ruled out. The intended model, therefore, is not only what we have in mind when we think about natural numbers (or what the community of mathematicians decides is the reference of arithmetical sentences); it is also an object that is intentional in the proper sense.

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Is Truth a Genuine Property?

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Abstract. I argue that the Tarski-like definition of truth in L suffices to establish truth as a genuine property. I defend the Tarski-like definition against (i) the modal objection (Soames 1984; Etchmendy 1988; Putnam 1994), (ii) the explanatory force objection (Field 1972), (iii) the truth-conditions objection (Bar-On et al. 2000), and (iv) the substantivity objection (Blackburne 1984).

1. Introduction: the status of the Tarski-like definition

The Tarski-like definition is a definition in the form of equivalence:

(*) (∀x)(x is true in L if and only if x is satisfied by all sequences of objects)

What kind of equivalence is it? Is it extensional or intensional?

Intensional equivalence is stronger than extensional equivalence. Contrary to mere extensional equivalence, intensional equivalence is necessary. If we spell out the notion of necessity in terms of possible worlds, then we say that an equivalence is intensional if and only if it holds in every possible world. As Künne (2003 p. 25-26) points out, there are different ways and strengths in which the intensional project can be attained. Where "F" and "G" are two predicates, the intensional equivalence might express that:

(i) "F" and "G" necessarily have the same extension;

(ii) it is known a priori that "F" and "G" necessarily have the same extension;

(iii) the knowledge that "F" and "G" necessarily have the same extension is self-evident;

(iv) "F" and "G" are synonymous.

Equivalences of type (i) might be discovered by empirical investigation, whereas equivalences of types (ii), (iii) and (iv) are conceptual. As Künne (2003 p. 27) stresses, there is a difference between equivalences of type (ii) and equivalences of types (iii) and (iv). The first might ask for a piece of work requiring great effort, as in the case of "x is a triangle if and only if x is a closed, plane, and rectilinear figure whose internal angles add up to 180°". Equivalences like (iii) and (iv) require only lexical competence to be established. The Tarski-like definition for a language L is not the product of empirical investigation. This seems to rule out (i).

We must distinguish two kinds of intensional definition: (a) constructive definitions and (b) analytic definitions. Frege (1969 pp. 219-270) introduced the distinction between constructive definitions and analytic definitions by saying that analytic definitions are more like axioms than definitions, while constructive definitions are stipulations used to introduce a new expression as an abbreviation for an old one or to bestow an old expression with a new meaning. The philosophical concern of any definition cannot reside in a stipulation for introducing an abbreviation. Moreover, the Tarski-like definition is accompanied by a criterion of material adequacy, the Convention T. If we want to introduce a new meaning for "true in L" by a stipulation, we will not be obliged to lay down any criterion of material adequacy. It seems reasonable to conclude that the Tarski-like definition is not a constructive definition. If it is an analytic definition, what is its strength? The conceptual analysis underlying the Tarski-like definition does not purport to capture the meaning of "true in L" by a synonymous expression: "true in L" and "satisfied in L by all sequences of objects" are not synonymous. Again, Convention T is a criterion of material adequacy. If the aim of the definition were to define the meaning of "true in L", its correctness could be tested by appealing to our lexical competence only. Nor would it be convincing to hold that they are self-evidently co-extensive. It should be concluded that it attempts to give the analytic definition of the property of truth in L. The suggestion is that the properties of truth in L and satisfaction in L by all sequences of objects are necessarily co-extensive, as are the properties of being a triangle and being a closed, two-dimensional and rectilinear figure, whose internal angles add up to 180°.

The interpretation of the Tarski-like definition as an analytic definition of the property of truth in L enables us to counter a first objection. It is the objection that the predicate "true in L" and its Tarski-like *definiens* are epistemically different (Soames 1999 pp. 243-4; Künne 2003 p. 224). For the sake of exposition, let's deal with a language L that contains only two sentences: "Die Erde bewegt sich" and "Der Mond ist rund". We can specify enumeratively what it is for each sentence to be true:

(D) $\forall$x(x is true in L $\leftrightarrow$ ((x = "Die Erde bewegt sich" and the earth moves) or (x = "Der Mond ist rund" and the moon is round))).

Now, compare the following two sentences:
(a) "Der Mond ist rund" is true in L if and only if the moon is round
and

(b) ((“Der Mond ist rund” = “Die Erde bewegt sich” and the earth moves) or (“Der Mond ist rund” = “Der Mond ist rund” and the moon is round)) if and only if the moon is round

(b) is obtained by substituting “’Der Mond ist rund’ is true in L” in (a) with the *definiens* we get from (D).

The objection is that (a) and (b) have different epistemic properties. The knowledge of (a) is sufficient to justify the belief that the sentence “Der Mond ist rund” does not mean that the earth moves. The knowledge of (b) does not provide any information on the meaning of that sentence. The conclusion is that the predicate “true in L” and its Tarski-like *definiens* are epistemically different, since the former can justify beliefs on the meaning of sentences, whereas the latter cannot. However, this is not a problem for the interpretation of the Tarski-like definition as an analytic definition of the *property* of truth in L. The fact that the predicate “true in L” and its Tarski-like *definiens* have different epistemic properties prevents us from treating the Tarski-like definition as a definition of the *meaning* of “true in L”, but it is harmless against the idea that it defines the *property* of truth in L. Meanings are what count for informativeness and inferential role. Two expressions might have different meanings but the same semantic content. The property of being awake when Phosphorus is visible is the same as the property of being awake when Hesperus is visible. However, the sentence “John is awake when Phosphorus is visible” can be used to justify the sentence “John is awake in the morning”, whereas the sentence “John is awake when Hesperus is visible” cannot.

In the following sections, I will argue that the Tarski-like definition suffices to establish the property of truth as a genuine property.

2. The modal objection

To construe the Tarski-like definition as an analytic account of the property of truth in L implies that the expressions “true in L” and “satisfied by all sequences of objects” have necessarily the same extension. Some philosophers (Etchmendy 1988; Soames 1984; Putnam 1994) claim that the *definiens* and the *definiendum* have different modal properties. The upshot of their objection is that the equivalence (*) is not necessary. We can shape the objection in the following way: if the language L contains only two sentences, the Tarski-like definition of truth in L is:

(a) $\forall$x(x is true in L $\leftrightarrow$ ((x = “Der mond ist blau” and the moon is blue) or (x=“Die Schnee ist weiss” and snow is white))).

If we consider the sentence “Die Schnee ist weiss”, from (a) we obtain:

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(b) “Die Schnee ist weiss” is true in L ↔ ((“Die Schnee ist weiss”=“Der Mond ist blau” and the moon is blue) or (“Die Schnee ist weiss”=“Die Schnee ist weiss” and snow is white)).

The right-hand side of (b) is logically equivalent to the sentence “snow is white”. Therefore, the sentence “Die Schnee ist weiss” is true in L in every possible world in which snow is white. The objection is that the sentence “Die Schnee ist weiss” is true in L even in those possible worlds in which it means that water is liquid and water is liquid and snow is not white; and it is not true in L in those possible worlds in which it means that the earth is cubical and the earth is not cubical and snow is white. The conclusion is that the Tarski-like definition would be a mere extensional definition. It would be an account of the extension of “true in L”, not of the property of truth in L.

This objection rests on a misunderstanding of the Tarski-like definition.[1] It overlooks that the Tarski-like definition gives an absolute definition of truth in L. One of the essential features of the Tarski-like definition is that the meanings of the expressions of L are taken as known. Convention T says explicitly that we must define truth in L in such a way that T-sentences meet a crucial condition: the sentences of the meta-language on the right-hand side of T-sentences must be the *translation* of the sentences of the object-language on the left-hand side. The notion of translation presupposes the notion of meaning. A translation is correct if and only if it is meaning-preserving. The possible worlds in which “Die Schnee ist weiss” means that water is liquid and water is liquid and snow is not white are not possible worlds in which “Die Schnee ist weiss” is true in L without being satisfied. Those are worlds in which speakers do not speak L, since they use the string of sounds and signs “Die Schnee ist weiss” with a different meaning than it has in L. But the Tarski-like definition does not give the account of truth for mere strings of sounds and signs. It gives the account of truth for sentences, that is, strings of sounds and signs endowed with their *meanings*. The semantic properties of the languages for which we want to provide the Tarski-like definition are essential to those languages. There are no possible worlds in which those languages lack those properties. If we appeal to meanings in order to identify languages, then the equivalences produced by the Tarski-like definitions are necessary. As a result of this, they can be interpreted as definitions of properties.

The assumption is that semantic properties are taken to be essential properties of linguistic expressions. In order to do so, we need to think of languages as abstract entities that have the same semantic properties in every possible world. This highlights a crucial lesson: the notion of meaning is indispensable for identifying languages as abstract entities. Languages are not syntactically identified as set of types of strings of sounds and signs. Languages are abstract entities formed by pairs of types of strings of sounds and signs coupled with meanings.

One might try to reject the commitment to the notion of meaning. He might concede that the construction of the Tarski-like definition presupposes the use of a

[1] See Künne (2003 p. 220-21), Patterson (2002; 2003), Carpintero (1996 p. 129), Davies (1981 p. 28).

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translation manual from the object-language into the meta-language; but he might also deny the uniqueness of the translation and take a further step from the indeterminacy of translation to the denial of meanings. There would be many properties of truth in L, one for each translation manual from L into meta-L. The analogy is with the property of having a certain position or a certain velocity in space.[2] There are no absolute position and velocity, because they are relative to a given coordinate system. Still, as Putnam (1985 p. 339) pointed out, there is a difference between the case of position and velocity in space and the case of the property of truth in L. Although there are no absolute position and velocity, relative position and relative velocity are defined: two impartial observers may agree on the position and velocity of a thing relative to a definite coordinate system, even though they use different coordinate systems. On the contrary, the relative property of truth in L is not defined: given a language L it cannot be said which relative property of truth in L applies to its sentences. It might seem that there is no such indeterminacy, since L-speakers might agree on using the homophonic translation from L into meta-L. To use Quine's (1969 p. 49) words, L-speakers might "acquiesce" in their own language. But it is easy to see that the relative property of truth in L can be defined in this way only for L-speakers. An impartial observer, who is not an L-speaker, cannot determine the relative property of truth in L that way, because he cannot adopt the homophonic translation. Therefore, two impartial observers, who are not L-speakers, might not come to an agreement on the relative property of truth that applies to the sentences of L.

Another important consequence is that if we accept the Tarski-like definition, we cannot accept to spell out the notion of meaning in terms of truth-conditions. It would be plainly circular to use the notion of meaning for constructing the analytic definition of truth in L, and at the same time to cash the notion of meaning in terms that require an account of the property of truth in L.[3]

Convention T uses the two-place meta-metalinguistic predicate "x is equivalent in meaning to y", where "x" ranges over the sentences of the object-language and "y" over the sentences of the meta-language. But how should we explain the relation of being equivalent in meaning? Any explanation according to which x is equivalent in meaning to y, if and only if x and y are true under the same conditions, would require that we already possess a truth predicate for the object-language and the meta-language in a meta-meta-language. But if we are able to define truth in L in a meta-meta-language, then we can already define it in a meta-language, since the meta-meta-language is a meta-language. Then the Tarski-like definition would be deprived of its philosophical importance. The analytic definition of a given notion protects its interest only if it does not presuppose the availability of another analytic definition of the same notion. In conclusion, not only the Tarski-like definition of truth in L requires the availability of the notion of meaning, but it also forces the rejection of the truth-conditional conception of meaning.

[2] Cfr. Quine (1969 p. 49).

[3] The following argument is presented in Patterson (2002 pp. 7-8).

3. The explanatory force objection

In this section, I will reply to Field's (1972) objection that the Tarski-like definition does not reduce truth in L to any physicalistically acceptable notion. Field claims that a physicalistically acceptable definition requires something stronger than mere extensional equivalence. What he is likely to mean is an equivalence that is to be discovered by empirical investigation, on the same model as the equivalence that water is H_2O. Clearly, any such definition is the product neither of a stipulation nor of conceptual analysis. Field says that a definition of this kind provides an explication of the *definiendum*. The explication consists in showing how the notion being defined is realised in the physical world. Differently from mere extensional equivalence, this kind of equivalence is meant to hold in all possible worlds with the same natural laws as the actual world. His qualms concern the definition of satisfaction for simple sentential functions, namely the basic clauses of the recursive definition of satisfaction. Field draws an analogy between the Tarski-like definition of satisfaction for simple sentential functions and the definition of the notion of chemical valence that one would obtain if he associated each chemical element with an integer. This definition of valence would be extensionally correct, but it could not provide an explication, since it does not reduce the notion of valence to physical properties of elements. By analogy, the Tarski-like definition of satisfaction for simple sentential functions does not explain how the property of being satisfied in L is realised in the physical world. So, the Tarski-like definition is not acceptable from a physicalistic point of view, since it does not ensure anything stronger than extensional equivalence. Is Field's argument well taken?

Field's criticism that the Tarski-like definition provides only the definition of the extension of "true in L" is based on the same misunderstanding as the modal objection. Field takes utterances, namely emissions of strings of sounds and signs, to be the primary bearers of semantic properties.[4] The utterance of the string of sounds/signs "Schnee ist weiss" is true in the actual world if and only if snow is white. Field says that we can imagine other possible worlds in which "Schnee ist weiss" is true if and only if water is liquid. These are the worlds in which speakers use the string of sounds/signs "Schnee ist weiss" to mean that water is liquid. On the contrary, according to Field, the Tarski-like definition implies that the string of sounds/signs "Schnee ist weiss" is true in every possible world in which snow is white irrespective of its use. Field claims also that an account of semantic properties that does not make them vary from one possible world to another in accordance with the change of speakers' linguistic behaviour, is bound to fail. But the demand that semantic properties vary from one possible world to another in accordance with speakers' linguistic behaviour, is sound only by assuming that they are primarily exemplified by emissions of strings of sounds and signs. Field's

[4] Cfr. Field (1972 p. 378).

point is that the proper account of the property of truth in L must answer the following question:[5]

(Q) In virtue of what facts regarding speakers' linguistic behaviour do the emissions of certain strings of sounds and signs have the property of truth in L?

The answer, Field says, is to be discovered by empirical investigation: equivalences stating necessary co-extensionality between the predicate "true in L" and some predicate describing speakers' linguistic behaviour. But the Tarski-like definition is not meant to answer question (Q). It is meant to answer:

(Q*) What are the semantic properties of the expressions of L?

As stated above, to answer this question we need to investigate speakers' linguistic behaviour. The investigation of speakers' linguistic behaviour comes into play in the identification of the language they speak and more precisely, in the identification of the meanings of the strings of sounds and signs they utter. Thus, empirical investigation comes into play to answer the question:

(Q**) In virtue of what facts regarding speakers' linguistic behaviour do certain strings of signs and sounds have certain meanings?

Contrary to Field's view, the Tarski-like definition offers the definition of the property of truth in L. To defend this stance we need to endorse the claim that the property of truth in L is an essential property of the language L to be investigated through conceptual analysis. To this end, we must think of languages as abstract entities that essentially possess their semantic properties. Then the question of the acceptance of languages, as abstract entities, becomes the question of (i) the acceptance of types of strings of sounds and signs and (ii) the acceptance of meanings. It seems to me that the question of the acceptance of types of strings of sounds and signs can be easily answered. Take a sentence for instance, and take it into consideration as a type: it is not an emission of sounds or signs, but a pattern of sounds and signs that can be repeated. Its existence is not jeopardised by the fact that there are no emissions of it. We might follow Quine's (1960 p. 195) suggestion and construct types of expressions as sequences of phonemes or characters. A sequence can be thought of as the set of n pairs $<a_1, 1>$, $<a_2, 2>$… $<a_n, n>$ and each phoneme or character a_i can be thought of as a set of emissions, since in this case there is no risk that there are no such emissions. It is much more difficult to answer whether or not we can accept meanings. I wish only to point out two considerations: the first is that we need to endorse a notion of meaning as use. As mentioned above, we need to make sense of the notion of meaning without resorting to any semantic notion. The second is that the explanatory force objection misses the target in the same way as the modal objection does. They are both

[5] I owe this interpretation to Soames (1984 p.411).

misdirected, since what is forgotten is that the Tarski-like definition presupposes the availability of the notion of meaning. It is true that any sound criticism of the notion of meaning as use would also undermine the Tarski-like definition. But neither the explanatory force objection nor the modal objection provides such a criticism.

4. The truth-conditions objection

In this section, I will discuss the argument that the meaning of a sentence must be shaped in terms of its truth-conditions. If the claim that the Tarski-like definition is incompatible with the truth-conditional conception of meaning is to be considered as true, then the argument directly threatens the Tarski-like definition: the Tarski-like definition is flawed since, on the one hand, it requires the notion of meaning, but on the other hand, it cannot have it. The argument I want to counter is the following.

It is commonly accepted that the meaning of a sentence determines, together with worldly circumstances, its truth-value. How can we explain this feature of meaning? An obvious response is that the meaning of a sentence is identified with its truth-condition, i.e. the worldly circumstance that decides whether the sentence is true or false. The argument has been formulated as follows (Bar-On et al. 2000 p. 4):

(1) the meaning of a sentence - taken together with a possible world - determines the sentence's truth-value in that world;

(2) the meaning of a sentence is a function from possible worlds to truth-values;

(3) Such a function is a truth-condition;

therefore:

(4) the meaning of a sentence is its truth-condition.

(2) is the crucial step. It identifies the meaning of a sentence with a function from possible worlds to truth-values, namely with its truth-condition. This identification rests entirely on (1), that is the "sufficiency claim": the claim, commonly accepted, that the meaning of a sentence is *sufficient* for determining its truth-value, given how things are in the world. Does the claim justify the identification of meanings with truth-conditions?

Actually, it seems that what we do understand from (1) is that each sentence has a truth-condition associated with it, and that the meaning of the sentence plays an essential role in determining which truth-condition it is. It is not clear why we should be forced to accept the truth-conditional conception of meaning while agreeing on that conclusion. The advocates of the conception of meaning as use might object that (1) is not strong enough to vindicate the truth-conditional

conception of meaning. They might agree that, once the meaning of a sentence is given, no more than a worldly circumstance is needed in order to decide whether the sentence is true or false. However, they might embrace some version of the conception of meaning as use and hold that meanings are individuated by means of constitutive patterns of use of linguistic expressions. These constitutive patterns of use would be traceable in the linguistic practice, namely in the inferential and referential abilities of speakers and no semantic vocabulary would be needed to describe them. They might defend the thesis that (implicit) knowledge of meaning is what makes people speakers. But what allows a person to be a speaker is the (implicit) knowledge of how to use linguistic expressions. For all of that, advocates of the conception of meaning as use might insist that linguistic expressions have semantic contents associated with them. They do not need to deny that linguistic expressions have semantic contents. What they wish to deny is that semantic contents are to be invoked to give the account of meaning. They should accept the Tarski-like definition: there is no mystery on how - given a worldly circumstance - the meaning of a sentence is sufficient to establish its truth-value. Given a sentence S, its meaning and morphological structure determine to which language it belongs. The language, to which it belongs, essentially possesses its semantic properties. So, a determinate truth-condition is associated with S and, given the worldly circumstances, a determinate truth-value is associated with it as well. We do not expect to be able to explain why the expressions of a given language have the semantic content they have, the same way as we are not able to explain why water is H_2O.[6] In both cases, the process of asking for an explanation ends with analysis, be it conceptual or empirical. In conclusion, we can accept the association of sentences with truth-conditions without compromising our conception of meaning as use. We can accept step (1) - the sufficiency claim - without moving on to step (2) - the truth-conditional conception of meaning.

Some philosophers (Bar-On et al. 2000 p. 16) have contrasted this line of defence of the conception of meaning as use. They argue that (1) highlights a fact that needs an explanation. The fact that the meaning of a sentence - together with worldly circumstances - determines its truth-value would be an undeniable fact that can be explained only by the identification of at least part of that sentence's meaning with its truth-condition. For, as this objection points out, if truth-conditions are not at least part of the meaning of sentences, it would be difficult to explain the way meanings play the role of determining their truth-values. This attack focuses on the idea that semantic properties call for an explanation other than that provided by conceptual analysis. This explanation should be the product of empirical investigation. The underlying view is the same as the one Field proposes: semantic properties need to be explained through a physicalistic reduction. The rationale for demanding a physicalistic reduction of semantic properties is to credit them with causal efficacy. But the rationale for crediting semantic properties with causal efficacy is the willingness to resort to them in

[6] Horwich stresses this point: the game of demanding explanations reaches an end with the analysis, be it conceptual or empirical. (1998 p. 66).

explaining linguistic competence and behaviour. Why do speakers use linguistic expressions that way? This is due to (the implicit knowledge of) their semantic contents. And why do linguistic expressions have those semantic contents? The physicalistic reduction is required to answer this question, basically in terms of naturalistic theories of reference. It turns out that the need for explicative theories of semantic properties stems from the demand of employing them for the explanation of linguistic competence and behaviour, i.e. from the demand of shaping the notion of meaning by means of them. In so doing, it seems that the claim that there is an undeniable fact in need of explanation, namely the fact that meanings together with worldly circumstances determines truth-values, starts with the assumption of the truth-conditional conception of meaning rather than giving force to it.

5. The substativity objection

In this section, I will face the objection that the attempt of the Tarski-like definition to establish the property of truth in L as a genuine property is bound to fail. The objection (Blackburne 1984 pp. 230-1) consists of two parts: the first one focuses on the fact that the Tarski-like definition does not capture the common denominator of all truths in L. It does not tell us what all true in L sentences have in common. The other focuses on the fact that the Tarski-like definition does not tell us how to extend it to new sentences. The target of the objection is the enumerative character of the Tarski-like definition. The property of truth in L is defined in terms of the property of being satisfied in L and the property of being satisfied in L is spelled out through a recursive definition. The recursive definition starts with basic clauses that define in an enumerative way what is needed for each simple sentential function to be satisfied in L. Therefore, what truth consists of varies from one sentence to another. The alleged conclusion goes that there is no genuine property that all true sentences share. The enumerative character also underlies the second part of the objection. Define truth for a language L. Then add a sentential function "Fx" to L. The objection is that we are not able to get the necessary information - needed to extend the definition to the new language obtained by the addition of "Fx" to L - from the Tarski-like definition of truth in L.

The first part of the objection rests on the assumption that if P is a genuine property, there must be something that the bearers of P have in common. According to this view, then, the nature of a property P requires the existence of a common denominator for all Ps. However, exceptions do occur and undermine the common denominator requirement. There may be properties such as the conditions under which they are exemplified vary when their bearers vary. The satisfaction conditions for those properties are a function of the objects that exemplify them. If P is one of these properties, then the project of specifying the common denominator of all Ps is unattainable. Take identity,[7] for instance: what constitutes

[7] I borrow the example of identity from Wright (1994a; 1994b).

identity changes when the objects concerned change. Everything that is part of my being identical to myself is essentially different from all those elements that constitute everybody else's being self-identical. This consideration is a starting point, and it would be a mistake to infer that there is no such a thing as self-identity. The idea is to take truth in L to be one of these properties.[8] The enumerative character of the Tarski-like definition reflects the peculiarity of the property of truth in L: there are as many things in which truth in L consists, as there are sentences of L. But this should not strike us as a surprise. So long as we want to reduce truth in L to semantic relations that hold between the expressions of L and the objects they speak about in virtue of their meaning, sentences of L expressing different meanings will have different truth-conditions associated with them.

The second part of the objection claims that the Tarski-like definition does not provide a general definition that enables us to extend the notion of truth to newly added sentences. Suppose we are given the Tarski-like definition of truth in L, and a new sentential function "Fx" is added; the claim is that we are not able to state the truth-conditions for the sentences we can form by using "Fx". This second part of the objection seems to underestimate the role of Convention T. Convention T provides the rule-governed pattern for constructing adequate truth definitions.[9] It says that an adequate definition must allow for the derivation of the T-sentences; in such a case, the sentences of the meta-language used on the right-hand side of the biconditionals are the translation of the sentences of the object language denoted on the left-hand side. The very same rule-governed pattern states how to extend a given definition of truth in L to newly added sentences. Given a new sentential function "Fx", we need to translate it into the meta-language and to construct the corresponding satisfaction condition. A fundamental lesson is then learned: the ability to extend the Tarski-like definition of truth in L to new cases presupposes the availability of the meaning of the new expressions added to L. Once again, the notion of meaning appears to be essential for defending the Tarski-like definition.

Besides, Convention T can be appealed to in order to reply to the criticism that the Tarski-like definition does not define the translinguistic property of truth. The Tarski-like definitions define the properties of truth in L_1, truth in L_2, truth in L_3…which are all different properties, since L_1, L_2, L_3 are different languages. Nevertheless, we can define translinguistic truth in the following way:

(D) $\forall x$(x is true if and only if $\exists L$(x is true in L))

The reason why we are able to understand this definition is not due to our general awareness of what all true sentences have in common independently of the language in which they are formulated. On the contrary, it is due to the fact that we know how to construct the Tarski-like definition for any language. The general character of the Tarski-like definition consists of the uniform nature of the form

[8] Sher (1999 p. 139) argues that truth in L is one of such properties.

[9] This point is stressed by Mou (2001 p. 103).

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taken by the different definitions for different languages. This uniform nature of their forms is guaranteed by Convention T.

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A Puzzle about Restricted Recombination in Modal Realism

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Abstract. This paper addresses a specific issue inherent to David Lewis' conception of possible worlds, namely whether or not they are liable to being limited in size. The paper purports to show that, if a certain argument against unlimited worlds' size is valid, then the way of countering it by means of positing an upper limit to size (as Lewis himself and John Divers have suggested) leads to a troublesome distortion of some modal phenomena, such as *de re* ascriptions of properties. Even if somewhat disagreeing with Lewis' favoured solution of the topic at issue, it must be remarked that what follows should not be intended neither as a defence nor as an attack on Lewis' overall modal ontology.

1. Lewisian Possible Worlds

Among the postulates of Lewis' modal realism, as first outlined in Lewis 1968 and subsequently developed in Lewis 1986 and Lewis 1991, there are the following:

(1) Individuals exist.

(2) Sets exist.

(3) Nothing else but sets or individuals exists.

Let us skip further details about sets for now, and concentrate on individuals.
According to Lewis, possible worlds are just individuals, even though of a peculiar kind. More in detail, an individual *i* (whereas 'individual' is to be taken as primitive) is a *world* iff (i) *i* is a spatiotemporal entity; (ii) *i* has spatiotemporal parts, (iii) all of *i*'s parts are with no exception spatiotemporally interconnected, (iv) the mereological sum of *i* with an individual that is not part of *i* is not a

spatiotemporal entity.[1] Therefore, worlds are individuals whose parts are all spatiotemporally connected among them.
Moreover, Lewis' ontology endorses the Principle of Unrestricted Mereological Composition (or Unrestricted Summation):

(4) For any n-tuple $x_1,\ldots, x_n$ of individuals existing in w, there exists in w an individual constituted by the mereological sum of $x_1,\ldots, x_n$.

In other words, (4) has it that, if Keith Richards, Rimet's Cup, Varenne, Ville Savoye are our supposedly basic individuals, then there exists the individual (call it Yoda) composed by Keith Richards + Rimet's Cup + Varenne + Ville Savoye, however scattered in space each of those basic individuals is, or might look to us. Thus, worlds are nothing over and above individuals of this kind, differing from Yoda only for the fact that they are much larger than it. But how much larger, exactly?
This question is one not so straightforward to address. At first instance, Lewis' answer is that possible worlds are to be seen as *maximally large*, so to speak: w is a world iff w is a spatiotemporal entity & the mereological sum of w with any further entity that is not part of w is *not* a spatiotemporal entity.[2] One could at first read such definition as covertly modal: w is a world if and only if it is not possible to add any spatiotemporal item x to it such that x is spatiotemporally connected to w. As a consequence, the definition should be read as asserting that, if w is "already" a world, then it is complete: any new item one could fancy about has to lie outside.[3] Thus, it seems that, even though Lewis' definition does not make use of modal terms it is nevertheless committed to a modal cloaked clause, *viz*. maximality. At bottom, this is perhaps no surprise: for as Ted Sider has for one recently stressed, if analysis is meant *identity*, then any reducing, non-modal term – such as Lewis' definition of 'possible world'- must willingly or not embed modal notions, albeit in a disguised form, on pain of failure of extensional equivalence.[4] And even though extensional equivalence is not sufficient for any analysis to count well, it is however certainly necessary. That is probably why even Lewisian realists should not be exaggeratedly worried with eventually surrendering to modal infiltration in their *definientia*.[5]

[1] Whereas 'being part of' is primitive, in standard mereology the sum of n objects (for $n \geq 2$, of course) is the smallest object of which the n objects are all part. Accordingly, a bucket is the mereological sum of a round container and a handle. Furthermore, for two or more objects to constitute a sum they have to be part of the same world; hence, according to Lewis, the sum of my laptop and a unicorn is not an object of the actual world since the two don't coexist in the actual world.

[2] Lewis 1986: 69. See also van Inwagen 1986: 186-187.

[3] 'Outside' is to be intended as elliptical for 'outside the logical space of w', even though Lewis' conception of possible worlds equates different logical spaces with spatiotemporal boundaries, since according to him worlds are totally disconnected spatiotemporal wholes.

[4] Sider 2003: 185.

[5] Lewis is not worried, by the way, for he himself defines worlds as maximal mereological sums: Lewis 1986: 73.

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2. The Principle of Unrestricted Recombination

Before addressing the problem of worlds' size, which is the main concern of this paper, we must understand how much Lewis' worlds are allowed to vary and differ from each other, since, as we shall soon uncover, the problems of size is closely related to matters of variety.

As to the assortment of different worlds to be admitted, Lewis endorses a principle granting that the modal space is logically *complete*. That the modal space is complete is tantamount to securing that no possibility whatsoever is left out, on pain of downright semantic failure of our modal talk and practices.[6] According to Lewis, semantic success is assured by the Principle of Plenitude (**PP**).[7] Broadly speaking, **PP** is a truthmaking principle: it amounts to the claim that, for any way a world *w* might be, there is a world *w*' that is just thus-and-so.[8] **PP** therefore grants that enough worlds do exist as truthmakers for every well-formed modal sentence. Accordingly, if unicorns don't exist in *w* but it is nonetheless possible that they do, it follows that there is a world *w*' where unicorns exist. Hence for every distinct possibility (with regard to a given world *w*), there always exist so many worlds as they are needed in order to make true any modal statement involved.

PP, though, has not to do with worlds only, that is to say, with spatiotemporal wholes. As the simple case from unicorns shows, plenitude is also related with parts of worlds, *viz*. with non-maximal individuals. More in detail, **PP** also rules over what can coexist with what. In other words (given Lewis' modal reductionism) **PP** rules over what does coexist with what in any possible world. In order to do this, **PP** gives rise to a further principle, the Principle of Recombination. This principle reads as follows:

(5) Any *duplicate* of any individual existing in a world *w* coexists in a different world w_1 with any duplicate of any other individual existing in w_2.

(5) amounts to what Lewis elsewhere calls 'the patchwork principle for possibility'.[9] It asserts that, whenever two distinct individuals are possible, then they are also compossible, that is, they may exist without overlapping within the same region, or spatiotemporal whole. To put it otherwise, (5) is the principle of Unrestricted Composition as applied to the modal space: whereas Unrestricted Composition has it that individuals may be freely recombined within the same world, (5) extends Unrestricted Composition across different worlds.

[6] According to the so-called Principle of Possibility, any modal statement is reducible to a non-modal one relativized to a set of worlds (even empty, in case of necessary falsehoods). Thus, stating that it is possible that dyads exist is logically equivalent to asserting the existence of a set *S* of possible worlds such that dyads exist in every world comprised in *S*.

[7] Lewis 1986: 86.

[8] It needs to be remarked that Lewis, strictly speaking, does rebut the foregoing statement of **PP** (Lewis 1986: 87) since in his opinion it is either void of meaning or set-theoretically flawed. I nevertheless think it is intuitively quite fit for conveying the requirement that there not be logical gaps among worlds.

[9] Lewis 1983: 77. See also Wasserman-Hawthorne-Scala 2004: 306.

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In order to fully grasp what is at stake in (5) we need ask what is a duplicate (of an individual). According to Lewis, duplicates are *not* counterparts.[10] While any counterpart x^* of x is an individual more or less resembling to x, resemblance being dependent upon some context-sensitive factors, duplicates are taken to be atom-by-atom *perfect* replicas of other-worldly individuals. The reason why **PP** calls for duplicates is the following: suppose that x exists at w_1 and y exists at w_2; not only, then, must **PP** grant the existence of a world w_3 where x's counterpart and y's counterpart coexist. A further world w_4 where x and y "in flesh and blood" coexist is also needed, on pain of violating the plenitude requirement. For while counterparts are qualitatively distinct individuals tied together by relations of (varying) *similarity*, duplicates are *copies* of other-worldly individuals; so while counterpart relations have to do with *extrinsic* similarities (these including - among other things - how their respective surroundings are), this is not the case with duplicates, which share every intrinsic property.[11] Hence, in order to fill in the logical space without gaps, we must take into account *copying* worlds: according to Lewis, a world copies a class of possible individuals iff «it contains non-overlapping duplicates of all the individuals in that class».[12]

Suppose now that tailless kangaroos exist in w_1 but not in w_2, that winged donkeys exist in w_2 but not in w_1. The Principle of Recombination asserts that there is a world w_3 such that tailless kangaroos and winged donkeys exist in w_3. This is not the end of the story about Recombination, though. As a matter of fact, when combined with Unrestricted Composition, Recombination affords much more than a case like w_3. It also yields the existence of worlds where parts of tailless kangaroos are linked to parts of winged donkeys, this for every part both of the former and of the latter, granted of course that tailless kangaroos and winged donkeys have parts (if they actually don't, take individuals which do have parts; divide them into parts and differently recombine those parts among them; repeat such step for every part involved and finally you will have truly experienced the scope of Recombination). Hence, the whole story about tailless kangaroos and winged donkeys has it that there exists a world w_4 composed by individuals which are the mereological sums of the duplicates of donkeys' wings and tailless kangaroos' rest of the body; a world w_5 composed by the above sum plus duplicates of winged donkeys, a world w_6 composed by the duplicates of kangaroos' tails and the duplicates of wingless donkeys, and so on. Generally speaking, then, Recombination is also *unrestricted*, that is, it is such that

[10] Lewis 1986: 88.

[11] In short, while counterparts do not comply with the indiscernibility of identicals, this is never the case with duplicates.

[12] Lewis 1986: 101. x in w and y in w' are counterparts iff there is a relation of similarity R in degree r between x and y. Hence, how much counterparts resemble each other is likely to depend on what is counted as salient feature. To the contrary, a duplicate of an individual x is defined as the individual x' sharing all the intrinsic properties of x, whereas a given property P is intrinsic iff it is such that it does not depend on the (non-) existence of some other things; otherwise P is extrinsic. Hence, 'having positive charge' is intrinsic, 'being larger than' is extrinsic (Lewis 1986: 61).

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(6) **UR**: For any set *O* of possible non-atomic individuals, there is a world containing duplicates of every part of every individual included in *O* and for any $S \subseteq$ O there is a world containing duplicates of elements of *S*.

(6) adds up to Lewis' claim that «[…] patching together parts of different possible worlds yields another possible world».[13]

3. An Argument against UR

(6) has been challenged, notably by Forrest–Armstrong 1984 and by Nolan 1996, because they have variously claimed that it leads to contradiction. In this section we shall expose such argument, as put forward by Daniel Nolan, who amends and shortens the previous argument by David Armstrong and Peter Forrest. In the sections to come we shall proceed first to discuss a reply to such argument, as advanced by John Divers on behalf of Lewisian realism, and second to counter in turn Divers' reply. Roughly, the rationale for resisting Divers' proposal is that it leads to "*de re* aberrations", *viz*. apparently inexplicable performances of a family of modal statements concerned with *de re* ascriptions of properties to individuals.
To begin with, we need to point out that according to **UR** it is the case that, for any individual *x* existing in *w* and for any part of *x*, there is a world *w*' such that the latter has among its parts any number of duplicates of (parts of) *x*. Thus suppose – say – that *x* has *n* duplicates in w_1, $n + 1$ duplicates in w_2, n + 2 duplicates in w_3,…, $n + m$ duplicates in w_n. Recombination is unrestricted, however, so that it yields the existence of a further and distinct world w^* ($w^* \notin \{w_1,\ldots, w_n\}$), such that w^* is composed by duplicates of all the individuals existing in *each* world in $\{w_1,\ldots, w_n\}$. Needless to say, the argument can be easily generalised so as to generate a possible world $w^{\#}$ that recombines all the individuals of all possible worlds. Hence, $w^{\#}$ will have as its parts every duplicate of every individual existing in some world *w*, in that it is one of Lewis' axioms that individuals can be constructed out of individuals with no limitation aside from spatiotemporal connectedness.[14] If the foregoing line of reasoning is correct (as it seems it is) it follows that $w^{\#}$ just contains (the duplicates of) all the possible individuals. It seems therefore that **PP** + **UR** do license the existence of $w^{\#}$.
As previously stated, instead, Nolan puts forth an argument to the effect that it is not possible that $w^{\#}$ exist:

> [S]uppose (for *reductio*) that there is a set of all possible objects [that is, $w^{\#}$]. This set must have a cardinality, as it is part of the definition of cardinality that all sets have it – call it C. But if it has a cardinality, then there must be a greater cardinality than it (e.g. the cardinality of its

[13] Lewis 1986: 88-89.

[14] Recall further that, since according to him worlds are totally disconnected entities, there are no transworld individuals, sums being possible only within a same world.

> power-set) [this follows from Cantor's Theorem: for any set x whatsoever, $| x | < |\wp x|$]. Call one such cardinality C*. From the principle of [unrestricted] recombination, for some object, there is a world that contains C* duplicates of that object. So there are at least C* objects to be found in worlds, so the set of all possible objects must have at least C* members. But C* is of course strictly larger than C – so the set of possible objects (with cardinality C) must be larger than itself. *Reductio*.[15]

Nolan is not embarrassed by such conclusion, for he suggests that the reference of 'All possible individuals' be not regarded as a set but as a proper class. Accordingly, *qua* proper class, $w^{\#}$ cannot be used in order to generate sets upon. In such a case, therefore, the above argument is simply a non-starter.

I do not want to pursue Nolan's route. Even though one might find it convincing, I want to address here a different issue, that is, Divers' preference for taking an alternative solution. In point of fact, Divers has urged that, since Nolan's proposal seems flawed with the dubious attempt to conflate mereology into proper class theory,[16] it would be much sensible to follow a proposal by Lewis 1986 consisting in setting a limit over the size of worlds.[17] More in detail, Divers invokes the following constraint upon **UR** (call it Restricted Recombination (**RR**)):

> There is some number N such that no spacetime is larger than N, and for all individuals x, y, and for all numbers n, m such that $(n + m) \leq$ N, there is a world in which there are at least n duplicates of x and at least m duplicates of y.[18]

Thus, **RR** states that, for some number N, it is *de dicto* impossible that worlds with cardinality greater than N exist (even though it is not metaphysician's task to discover which n really corresponds to N: what it matters here is only that such number exists, whichever it is). As a consequence, if any world w has cardinality equal to N and if N cannot be exceeded, **UR** is no more available, for it generates worlds whose cardinality is 2^N, while it is now contended that everything greater than N is not a possible world.

The Lewis – Divers solution must sustain two main charges. As I shall argue, while the first can be parried, the second seems apt to cause more trouble to **RR**. Let's assess them in turn.

[15] Nolan 1996: 246-247.

[16] Divers stresses indeed four separate drawbacks of Nolan's solution (Divers 2002: 103). The one mentioned above is perhaps the most telling.

[17] Lewis 1986: 101 ff.; Divers 2002: 102.

[18] Divers 2002: 102.

4. The Modal Status of N

The Principle of Recombination – says Lewis – holds «size […] permitting».[19] This means that N should be somehow limited. But how? Which is actually such N than which no greater number can count as world's size? As previously hinted at, the problem to be coped with here is not just finding it out, but also supplying a modal justification for it, whichever this number might be. Suppose that such number has cardinality α. How are we to justify α? The point here is not: Why α and not β (after all, there could be strong reasons in favour of α).[20] The point is rather the following: Which is the logical status of α? Or: How should we modally characterise it? It is apparent that we are not dealing with physical necessity, since according to Lewis physical necessity is to be interpreted quantificationally, so that a set of worlds models physical necessity when accessibility restrictions are imposed upon the set in order to narrow the range of the quantifiers. To this regard, as Lewis puts it, physical necessity is truth at all worlds satisfying a given restriction, namely sharing the same laws of nature.[21] It is therefore physical modalities to be modelled upon logical ones, not the other way round. Hence, Divers would not be allowed to back his proposal by resorting to physical necessity in order to warrant N. This won't do just because we are not dealing here with subsets of worlds (however large), but with the whole domain *W*.

N must then be governed by *logical* necessity. Thus, the only alternative seems this: assuming, for some α, that the maximum size equals α, it follows that it is logically necessary that such number equal α. This looks *prima facie* perplexing, though: how to impose a modal constraint upon the whole enterprise of modal talk? For, if we assert it is necessary that modal matters behave in a certain way, then we are committed to positing a further modal reality where such constraints governing the modal reality hold. But this requirement is in danger of pushing the genuine realist into troubled waters, for the same point is prone to be generalised in such a way as to start an infinite regress: metamodal reality will be

[19] Lewis 1986: 101.

[20] Peter van Inwagen has charged **RR** with culpable vagueness or, worse, plain inadequateness (see van Inwagen 1986: 198-199). If **PP** is as much secured by – say – 100 as by 2^{100} worlds, then why prefer the latter to the former? In other words, according to van Inwagen it would be equally plausible to set N very high along the line, or very low, for what only matters is that N not exceed the limit. But van Inwagen declares to be unable to entertain the belief that possible worlds are few in number. However, he confesses also that his personal preferences towards very large numbers are not safer, for any answer will be necessarily lacking of the required supporting evidence. And this suffices, according to him, to rebut modal genuine realism as incredible. It is not that clear, though, which van Inwagen's point is: why should six or twenty or one hundred be viable options? The actual world is presumably made up by far more than six or twenty or one hundred individuals. At any event, even granting that it is composed by just one hundred individuals, it follows at once that there are at least $2^{100} - 1$ possible worlds, and this is trivially greater than one hundred. In sum, since the actual world is just one among all the possible worlds, it seems that any not-sufficiently-great number will be ruled out by default just because (what we label) the actual world is, if anything, "sufficiently great". Hence, if van Inwagen's point against genuine realism is that it allows *any* value for worlds' size (or for N), his reply does not seem very forceful.

[21] Lewis 1973: 4-5.

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in need of modal reduction, and so on *ad infinitum*. Thus the genuine realist should be prevented from going metalinguistically at will if she wants (as she in fact does) to preserve reductionism over modality. To put it otherwise, the metalanguage too should be eventually cleared of any modal term, or else the extensionalist programme over modality is endangered.

In defence of genuine modal realism against the foregoing perplexities, Divers has developed an attractive rejoinder. Roughly, his argument is that, within genuine realism, discourse over modal reality cannot be treated as a case of *ordinary* modalisation: any time we talk about the properties or the logical behaviour of the whole *W*, we are confronted instead with *extraordinary* modalisation. The main difference between the two is that the latter does *not* license world-restrictions upon the quantifiers. In ordinary cases, in fact, modal statements are reduced to non-modal ones *via* suitable world-restrictions; thus, according to the genuine realist

(7) It is possible that elves exist

gets transformed in

(8) $\exists w \exists x$ (w is a world & x is part of w & x is an elf).

As is easily seen, quantification in (8) is bound to a given world. On the contrary, this is simply nonsensical if we wish to extraordinarily modalise, for there is no further modal reality working as basis of reduction, as it were, for the relevant modal statements. As a matter of fact, there is no hierarchy in Lewis' modal ontology, for all his primitives are non-modal (or assumed to be such). This implies that restricted quantification is not available in extraordinary cases. Suppose however that we are to evaluate the truth-value of some statements concerning modal reality as a whole, say:

(9) There exists no more than one possible world.

(9) cannot be expressed *via* restricted quantifiers simply because it does presuppose instead unrestricted quantification. That is to say, according to Divers, Lewis' realism commits to the following rendering of (9):

(10) $\sim\exists w_1$ ((w_1 is a world & $\forall w_n$ (w_n is a world $\supset w_n \neq w_1$)).

The crucial difference between (10) and (8) is that the former does not employ the notion of 'parthood'. Stated otherwise, (10) does not bind matters of predication (of existence) to specific worlds, as is commonly the case in possible-worlds semantics.

Therefore (10) treats quantifiers as not restricted to worlds. If so, then, what is the difference between (10) and a modal statement about modal reality such as

(11) It is possible that there exist no more than one possible world?

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None, according to Divers. As a matter of fact, there seems to exist no way to formalize (11) other than by

(12) It is possible that ($\sim\exists w_1$ (w_1 is a world & (w_n) (w_n is a world $\supset w_n \neq w_1$)).[22]

Unfortunately, though, the *de dicto* modality in (12) does not admit of sensible translation through non-modal terms, since

(13) $\exists w_1$ (w_1 is a world & w_1 is such that $\sim\exists w_2$ (& w_2 is a world & (w_n) (w_n is a world $\supset w_n \neq w_1$)))

is not well-formed. Moreover, even if it were, what (13) asserts would hardly be convincing: either the first quantifier is just redundant – in order to say that *x* does (not) exist we don't need to assert that something else exists or not - or it unjustifiably posits a hierarchy among different worlds, for it puts w_2 within the scope of the first quantifier, *viz*. the one binding w_1, as if how a world is should rely upon the existence or the nature of a different world. This account would constitute evidence for the claim that possible worlds are *not* in themselves, that is to say, that being a world should depend on whether some different individual is a world or not. But there is no evidence that Lewis does not regard worlds as in themselves.[23]

If this is so, then, (13) does not make sense: as such it does express nothing. This is why Divers maintains that statements about modal reality cannot in turn be modal. And since it seems plausible to urge that what holds of modal reality does so out of necessity, it follows that in extraordinary modalisation impossibility is just negation of a categorical statement, while possibility and necessity are both conflated into affirmative categorical statements.[24]

Armed with that, it seems that we have got the means to bypass the first objection. Thus, if the ontological inquiry should provide us with an upper limit to the number of "atoms" of any possible world, we should not fear the infinite regress because such achievement would not be in need of further warrant, since the semantic tools supplied by extraordinary modalisation techniques could afford a way out of the modal justification for N.

However, as we shall see in the following section, a more worrying objection might be raised against **RR**: at bottom, the status of N will be showed as equally taxing. Let's now turn to this second protest.

[22] We are referring here only to *de dicto* modality for the sake of simplicity, similar remarks holding for *de re* cases as well.

[23] Nolan 1996: 241. Cfr. with what Lewis says in Lewis 1986: 3: «[…] if worlds are causally isolated, nothing outside a world ever makes a world […]. Some of us even make assertions to the effect that other worlds exist. But none of these things we make are the worlds themselves». It seems further that (13) speaks against the principle (endorsed by Lewis) that there are no necessary connections between distinct existences: see Lewis 1986: 87.

[24] See Divers 2002: 47-50.

5. Maximal Worlds vs. Boundary Worlds

Roughly speaking, the reason why **RR** is (still) perplexing is that setting an upper limit like N is at odds with the Principle of Maximality. The Principle of Maximality has it that worlds are maximal entities: *complete* alternative histories, filled up to the tiniest detail. According to the Principle of Maximality, a Lewis-world is a maximal spatiotemporal sum of individuals. Granted all of this, let us now consider the following:

(14) For every w, if w is a world then N is its maximum size.

This is just rehearsing **RR**: nothing wrong with it so far. Thus (14) asserts that no w can be composed by more than N individuals: however little or great N is, it is Divers' nuclear tenet that N cannot be exceeded. As we saw above, this constraint is a case of extraordinary modalising, so that it is tantamount to asserting that no world exceeds N, period. If so, then it is not easy to deny that

(15) If w is formed by k individuals (k < N) individuals, then there is a world w_1 such that w_1 is formed by exactly N individuals.

Why should (15) be true? The reason is straightforward: **RR** once more. For according to it anything can coexist with anything else *up to N*. Hence, we can conclude that there are worlds featuring maximum size. No wonder, then, that (15) is true; at bottom, (15) just stems from Restricted Recombination. For while according to **UR**, there is no upper limit to the cardinality of any single possible world, according instead to **RR** no world is such as to recombine *ad libitum* duplicates from different worlds. Sooner or later Recombination has to stop. It should be apparent that (15) is an expected consequence from **RR**.

Upon establishing (15), we are allowed to derive that

(16) For any world w, if w is composed by k individuals, there exists a different world w_2 such that (i) w_2 contains every duplicate of the individuals which compose w; (ii) w_2 contains $k + j$ individuals, and (iii) $k + j$ = N.

When (16) is the case, let us say that the latter world *extends* the former. Moreover, w_2 is an interesting entity. First off, *qua* possible world, w_2 is maximal with respect to the number of its parts, just by the Principle of Maximality: whichever the number of individuals a world contains as its parts, it is the case that no further individual is part of that world. This is the common, Lewisian definition of 'possible world', according to which any world might be composed by many billions of parts, or just by a single atom: both worlds would be equally possible (recall that Unrestricted Composition does not imply that worlds must be exaggeratedly "populated", so to speak: a world where only one paramecium – and nothing else – exists is a perfectly legitimate possible world). What is most remarkable is that w_2 seems to be *doubly* maximal, for here 'maximal' means also that w_2 cannot be extended in turn: there is no further world beyond it. As is easily

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seen, this is a peculiar meaning of 'maximal'. Nonetheless, it sounds acceptable in light of the restraint imposed by N: w_2 cannot be augmented further on pain of exceeding N itself. This in turn means that it is not possible that w_2 be augmented, whence it immediately follows that there is no world copying w_2 yet extending w_2 by additional parts. Hence, w_2 is maximal also with respect to size.

Let us say, then, that w_2 is a *boundary* world in that it occupies the outer limit of the logical space. If boundary worlds exist, then they will behave quite oddly with respect to otherwise harmless *de re* statements; but this upshot is puzzling, to say the least. To anticipate a bit, the argument to be advanced below has it that worlds like w_2 are not apt to work as truthmakers for a good deal of *de re* ascriptions of properties to individuals. So, if we should have independent reason to think that those statements might be true nonetheless, then any such maximally-sized world would misrepresent modal facts because it in fact denies the truth of those statements. But correctly representing modal facts is just the duty possible worlds should faithfully serve.

More in detail, the argument is as follows. Let's suppose that (i) w_2 copies w, (ii) w_2 extends w. Therefore w_2 has exactly N individuals, $x_1, \ldots, x_N$. Moreover, assume that w is what we refer to as 'the actual world' (*our own* actual world, according to Lewis' indexical theory of actuality); therefore x_1 is a w_2-*duplicate* of an earthly human being of w: call this duplicate Oleg. If Oleg exists, unexpected consequences will immediately follow about him. Suppose that, being Oleg biologically behaved like earthly human beings, he is a freak: his hands are only eight-fingered. Because his fellow beings are ten-fingered, just like us, he complains over his misfortune. He regrets his condition for he does believe that he could have been ten-fingered, like his more fortunate fellows. But could he really, if **RR** obtains? Happily or not for Oleg, the answer is No. It is actually easy to see that, if the *de re* statement

(17) Oleg could be ten-fingered

is true, then there exists a world w_3 such that Oleg is ten-fingered therein.[25] But does this world exist? Well, in order to firmly grasp what is at stake here, we need first to realise that upon assuming that Oleg is biologically behaved like earthlings do, Oleg is *not* an atom. Stated otherwise, it is the case that he admits of being broken down into his constituting parts. Therefore, if Oleg can be ten-fingered, any

[25] Oleg's possibly being ten-fingered is apt to raise further problems concerned with material constitution. Roughly, if mereological essentialism (**ME**) is true, then ten-fingered Oleg cannot be a duplicate of eight-fingered Oleg, for according to **ME**, for any objects x and y, if x is part of y, then necessarily y exists only if x is part of y. Hence, no object could be a duplicate of any other if the former lacks some part the latter has, or *vice versa*. Hence Oleg cannot be differently constituted than he is in fact, while still being Oleg, whence it trivially follows that Oleg cannot be ten-fingered; that is, **ME** entails that (17) is necessarily false. Even though this objection scores a point, it should be rejected firstly because **ME** is far from being uncontroversially true; secondly because, even granting **ME**, if the truth-value of (17) does not depend anymore from there being an upper limit to size, then my argument to the effect that (17) is false in boundary worlds would be still valid, albeit trivially. Thus the argument has to be prefixed by the following proviso: unless **ME** is true, in which case the argument is redundant, it can be showed that some (allegedly harmless) *de re* statements are at odds with the existence of boundary worlds.

world where he is in fact ten-fingered is a world where the following individuals exist: (a) eight-fingered Oleg, (b) the first missing-in-w_2-finger, (c) the second missing-in-w_2-finger. This straightforwardly follows from Unrestricted Composition: any individual can be subdivided in any way whatsoever within a same world, provided that such individual not be a mereological atom, that is, an object with no proper part. Since it is very plausible to suppose that Oleg (whether eight- or ten-fingered) is not a mereological atom, it follows that his missing-in-w_2 fingers have to exist in some possible world different than w_2, if it is indeed possible that Oleg be ten-fingered.[26]

Take now a world w such that it is maximally sized and where Oleg is eight-fingered. We are already acquainted with one such world: it is w_2. This world does exist, yet – as previously remarked - it is a weird world. Why? Because w_2 is such that it is false in it that Oleg could be ten-fingered. As a matter of fact, there is no world that contains every duplicate of w_2 and where *both* eight-fingered Oleg *and* his two missing fingers exist, for w_2 is maximal with respect to N, that is, is a boundary world. As such, it disallows extensions: there is no world w_m that extends w_2. In other words, there is no world w_m whose cardinality is greater than w_2's, and is formed by all the duplicates of every individual existing in w_2. If this is so, then (17) is not possibly true in w_2.

We supposed at start, however, that (17) is a harmless ascription of a *de re* modal property to Oleg; in other words, we conceded that that there is nothing wrong in granting Oleg the modal property of possibly being ten-fingered. Quite to the contrary it now seems that N-maximality is backfiring damagingly, for it makes such *de re* ascription very problematic or downright nonsensical in w_2: Oleg cannot be ten-fingered. But why? As a matter of fact there seems to exist no sensible rationale for denying (17): what is modally wrong with Oleg's being ten- rather than eight-fingered? Apparently anything at all, for it is no contradiction to ascribe him the property 'being ten-fingered'. Yet this is what we are bound to conclude upon accepting limitations to size. Indeed, N ends up with positing worlds such that no possibility concerned with *de re* ascriptions of new items is allowed therein. But it is legitimate to ask why this should happen: why should eight-fingered Oleg be prevented from possessing a *de re* modal property so long as that property is not contradictory? After all, being ten-fingered does not logically collide with any non-trivially-different property, so why should it be nevertheless banned from the logical space? How are we to accommodate within boundary worlds any *de re* treatment of properties involved with further individuals? It seems that this modal deficiency makes them scarcely belong to the set W of possible worlds. But it was conceded at the outset that they were genuine possible worlds. Perhaps they don't.

[26] It could be also retorted that (17) cannot be true because it has nothing to do with duplicates but with counterparts. As a matter of fact, (17) deals with *de re* modality, thereby calling for counterparts. The case in question is instead related to duplicates, hence invoking counterparts is utterly inappropriate. The only way to reply to such an objection seems the following: if duplicates are perfect copies, then it follows that duplicates must also share every modal property of their matrixes, as long as modal properties are taken to be *intrinsic*. That is, the objection might stand only when supported by an argument to the effect that modal properties are extrinsic properties; as far as I know such argument has not been advanced yet.

Even worse, this predicament is apt to being easily (and worryingly) generalised. Proceed in fact as follows: take any world whose cardinality is lesser than N; build up the corresponding boundary world, filled in with every duplicate of all the individuals of the starting world; and finally you will be faced with a world stuffed to the breaking point, where things might not be different so long as their being otherwise should involve more individuals to be added. Thus, worlds behaving like w_2 would be such that fewer possibilities would be available in them. But this just contradicts the very structure of **S5**, which is apt to underlie Lewis' genuine realism.[27]

As a matter of fact **S5** should not have any upper bound, or outer limit; consider once more (17): it is apparent that nothing interferes with Oleg's being eight-fingered just contingently. Even though one might deny this claim right from the start, let's indeed grant for the sake of argument that it is the case that being eight-fingered is not essential to Oleg. Hence there are worlds where he is not eight-fingered. Let 'p' stand for 'Oleg is ten-fingered'. If Oleg is not eight-fingered essentially, then $\Diamond p$ is true. But if $\Diamond p$ is true, then it cannot be **S5**-true, since, according to **S5**, for any proposition p, if p is possibly true, then it is necessary that p is possibly true. This means in turn that p is possible with respect to any world w whatsoever. But this is false as soon as worlds are limited in size. For we have just seen that if no world exceeds N (for some N), then there exist boundary worlds, that is, worlds admitting of no extension. World w_2 above is one of them. Since $\Diamond p$ is false with respect to it, either we have to jettison **S5** as the correct logic for genuine realism, or it has to be explained why boundary worlds cannot exist.

7. Conclusion: Setting the Agenda for RR

It therefore seems that, even though (17) can be possibly true elsewhere (for instance in any eight-fingered-Oleg-world that is not maximal with respect to N), we are left with the odd-sounding result that for any non-maximal world there exists at least an N-maximal world whose modal behaviour is free-floating. Perhaps this conclusion is not to be counted as a *reductio* against the existence of such worlds. My claim is rather that, unless one shows why boundary worlds are

[27] That's not really true, for **S5** cannot be the correct framework for genuine realism. As a matter of fact, genuine realism calls for counterparts, and Lewis' Counterpart Theory (as stated in Lewis 1968) does not validate the characteristic **S5** axiom, namely $\Diamond p \supset \Box\Diamond p$. Even though the objection cannot be pursued here at greater length, there are some serviceable counters. In first place, the axiom is in fact validated if closed sentences *only* are allowed as substituends for the schematic p (see Lewis 1968: 122). Secondly, it is an open question whether certain modal inferences must be negotiated in order to conform to Counterpart Theory, or *vice versa*: it might be in fact that counterpart relations should be taken as equivalence relations in order to accommodate ordinary judgements of modal validity (this move is what Divers calls 'Deregulation' in Divers 2002: 139 ff.; for a treatment of counterpart relation as an equivalence relation see Hughes-Cresswell 1996: 353 ff., Hazen 1979: 332). Thirdly, Recombination (either restricted or not) is not concerned with counterparts but with duplicates, and there is no compelling argument to the effect that **S5** is unfit to deal with duplicates. Lastly (and more parenthetically) it seems that Counterpart Theory's validating a logic weaker than **S5** is at odds with quantification over non-actuals, for it is thereby to be explained how to satisfy **PP** in any logical space narrower than **S5**.

not genuine possible worlds, they seem to command a deep revision of the practice of significant *de re* ascriptions of properties, because such practice is no more captured by systems as strong as **S5**. For, if genuine modal realism is framed within **S5**, yet worlds' size is conferred an upper limit, then an argument can be devised according to which there are boundary worlds. But boundary worlds violate accessibility as an equivalence relation; hence if (but only if) genuine realism is committed to **S5**, then it has to jettison size constraints. For we have built up a proposition that it is logically possible everywhere but in boundary worlds. This fact contradicts the semantics for **S5**, hence Lewisian modal realism has to do away with either such semantics or with size constraints.

To sum up, then, it seems that the supporter of **RR** owes us a better explanation of such newly-acquired modal phenomena, for she will have to explain (i) Why **S5**, that is to say, an adequate semantic counterpart of Lewis' modal realism, should be abandoned; (ii) With what, in such a case, **S5** should be replaced in order to do justice to *de re* ascriptions in boundary worlds; or (iii) how to deny the existence of boundary worlds. Until the **RR**er provides us with plausible answers to (i) and (ii), or to (iii) it seems that, after all, taking worlds as proper classes is likely to sound as less exacting.

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Classical Realism and Aristotelian Essentialism

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Classical realism says first of all that observable objects exist even apart from being perceived, or imagined, or contemplated. This way of stating the point is perhaps slightly better than the common statement that observable objects exist independently of being observed, although this common statement is also good as far as it goes. For the claim of independence can be taken as something purely negative, but the point of saying observable things are real is to affirm their autonomy relative to observing subjects as a positive fact regarding those things. Thus, these things exist apart from awareness as opposed to being constructs or projections of the mind or senses. Instead, they exist within themselves. This is the positive fact that is conveyed from the other side by saying observable things exist apart from awareness. Insofar as real things are independent of awareness, their independence follows from such existence within oneself, as opposed to being what is primary.[1]

The true import of this point may be seen by considering a possible challenge. Contrary to Berkeley, classical realism can accommodate the claim that observable things are known by God and depend on being known by God in order to exist. Saint Thomas Aquinas was clearly a classical realist, and he affirmed exactly this kind of dependency on God. Classical realism is contrary to idealism, not because things are diminished by being known, but on the basis that observable things would be dissolved away if idealism were true. Such things would be dissolved away as having only ideal or intentional being. Therefore, Aquinas could (and did) say that things are real on the basis that God gives to these things their own natural

[1] Compare James K. Feibleman, "The Subjectivity of a Realist," *Southwestern Journal of Philosophy* 1 (Fall 1970): 7-19, especially 8-9. Feibleman points out that a legitimate realism must be able to define perceivable material objects in terms of the categories of natural science, apart from any reference to the relations these objects may have to knowing subjects.

being instead of merely contemplating these things as ideal constructs.[2] More properly, God contemplates some things that He does not make, but then these things are at most only possible or hypothetical. They do not exist in fact, and this is so exactly because God does not create those things. Divine creation is a free exercise of voluntary choice that goes beyond the awareness God enjoys within Himself.[3] Thus, God's knowledge is the cause of things, but only because His knowledge informs His will and not as though pantheism or reductive idealism were true.[4] Again, contrary to what Berkeley proposed in the Third Dialogue concerning the creation of observable things, God creates material bodies by giving to these bodies their own natural being instead of merely imposing experience on finite minds.[5] So then, the great concern is that observable things exist within themselves as having their own natural being.

Moreover, the relevant sense of existing within oneself so as to have one's own natural being entails or includes being actually existent as opposed to being merely possible or potential. Thus, phenomenalism fails. Insofar as observable objects are made up out of merely hypothetical experiences as well as actual experiences, these objects are merely possible or potential instead of actual. To be sure, phenomenalism succeeds in separating (at least partially) observable objects from the concrete activity of observing on the part of perceiving subjects (or would if it could be made to work out in detail). But it does so (or would do so) at the price of downgrading these objects from being actually existent. (This point is, of course, one of the standard objections to phenomenalism.[6])

Yet all this is only the beginning. Classical realism includes the claim that things exist apart from experience. But there is also the claim that these very things stand behind experience as well. Sensory experience corresponds to observable things and is also caused by observable things. Indeed, such experience lines up with what is observable exactly because it is derived from what is observable. To be sure, naïve realism fails, but then what this fact shows is only that the correct analysis of corresponding or lining up has to be appropriately sophisticated instead of naïve.

Of course, given all this, there will then be true subjunctive conditionals concerning hypothetical experiences. Phenomenalism is almost correct on this limited basis. But, so far from having subjunctive facts and hypothetical events constitute things, these things will instead be what supports subjunctives and hypotheticals. Subjunctive facts concerning hypothetical events will exist derivatively, as an outgrowth of what is actually existent in the sense of being

[2] See Saint Thomas Aquinas, *Summa Theologica*, pt. 1, qu. 14, art. 8; qu. 18, art. 4; qu. 19, art. 4; qu. 56, art. 2, ans. and rep. 3; idem, *Summa Contra Gentiles*, bk. 4, ch. 13, par. 10; idem, *Disputed Questions on Truth*, qu. 2, art. 14; qu. 4, art. 6, 8.

[3] See Aquinas, *ST*, pt. 1, qu. 14, art. 8, 9; qu. 19, art. 3, 4, 10; qu. 25, art. 5; idem, *SCG*, bk. 1, chh. 66, 81, 82, 83, 88; bk. 2, chh. 23, 24, 26, 27; idem, *DQT*, qu. 2, art. 8, 14.

[4] See Aquinas, *ST*, pt. 1, qu. 14, art. 8; idem, *SCG*, bk. 2, ch. 23, par. 4; idem, *DQT*, qu. 2, art. 14.

[5] See Aquinas, *ST*, pt. 1, qu. 55, art. 2; qu. 56, art. 2; qu. 57, art. 1; qu. 58, art. 6, 7.

[6] See, for example, A. C. Ewing, *Idealism: A Critical Survey*, 3rd ed. (London: Methuen, 1974; New York: Barnes & Noble Books, 1974), 294-99, 353-54.

categorical or occurrent. Phenomenalism is wrong as being one of those theories that mistake the symptom for the basis.

Beyond this, there is another important consideration. There might perhaps be ephemeral clusters, or even mere "bursts," of observable attributes as all that the real world contains to be perceived through the senses. Strictly speaking, there would presumably have to be some suitable spatiotemporal field in which these clusters or bursts are situated, and there would have to be enough causal structure in the world for causal influences from these clusters or bursts to be propagated to perceiving subjects. But the real world of observable things might perhaps be void of any depth or solidity or stability beyond this.

In point of fact, however, all this is clearly not what the real world of observable things turns out to be. It should be noted that this fact is both contingent and knowable only *a posteriori*. Moreover, this fact is contingent and *a posteriori* even given that there is some sort of real world of things perceivable by the senses such that these things stand behind experience. On the other side, this fact is necessary relative to having classical realism be true. At this point, it will be necessary to explore what turns out to be true. Only then will it be in order to examine the further implications that this fact has.

The real world of observable things that exist and stand behind experience in point of fact is made up largely of objects that cohere, continue or persist, and interact. For what is observed, so far from being mere bursts, turns out to be things such that a given thing can be observed through different senses, observed by numerically diverse subjects, observed again on different occasions, and examined by being subjected to further interactions.[7] Veridical experience is based on having the processes among these objects impinge on the perceiving subject. (In the present context, the living body of the perceiving subject must be considered as one such object among others.) On this basis, the real world of observable things turns out to embody a very large measure of structure and solidity. All this is included in classical realism.

In addition, what has to be noted here is that this structure is causal. This point is clearly obvious as regards interacting and impinging, which are causal notions. But this point holds good also for cohering and persisting. Of course, this claim about cohering and persisting depends on analyzing causation in terms of the truth of subjunctive conditionals instead of analyzing causation in terms of relations among events.[8] Given this subjunctive analysis, this claim is reasonably straightforward. For then one may speak of A as causing B in any case in which B is derived from A, or in which B exists or is present in virtue of A. Thus, in order for an object to cohere, its parts or pieces must stay together because they are made

[7] See L. Susan Stebbing, "Concerning Substance," *Proceedings of the Aristotelian Society*, n.s., 30 (1930): 285-308, especially 306-08. Stebbing argued that something like the idea of substance is needed in just this way in metaphysics for the correct analysis of perceptual judgements. Thus, an observable object cannot be an occurrent, nor a series of occurrents, nor a complex of occurrents (let alone of sense data).

[8] For this kind of subjunctive analysis, see J. L. Mackie, *The Cement of the Universe: A Study of Causation*, Clarendon Library of Logic and Philosophy, ed. L. Jonathan Cohen (Oxford: Clarendon Press, 1974), 29-58, 147-48, 153-54, 180-83, 189-92.

or caused to do so and not through coincidence or accident. Again, in order for something to persist, the thing as it exists later must be derived from, or exist in virtue of, having existed earlier.[9]

Then again, properly speaking, one should say that causation must be analyzed at least in terms of the truth of subjunctive conditionals. For there are other analyses that will also work, since these analyses also allow one to say that a thing holds together because it is made to do so and exists later in virtue of having existed earlier. Causation can be, and has been, analyzed in terms of necessity, of dependency, of activity, and perhaps in yet other terms as well. All these analyses will work. However, all these analyses will presumably include or entail having the appropriate set of subjunctive conditionals be true, whatever else may or may not be involved. Nothing more is needed in the present context.

Given that cohering and persisting are causal functions, a thing that coheres and persists must have the appropriate character within itself to exercise these functions. This character will be what Aristotle called the substantial nature of the thing. Moreover, this substantial nature is the whole basis for the thing to exist as the very thing that it is. Apart from this basis, there would be only the cluster of fragments and pieces, as well as the series of time slices, instead of the thing as a "genuine thing" that coheres and persists. There would be in the world only point instant events (or whatever the minimal elements turn out to be) unless there were also Aristotelian substantial natures in the world. Since this nature of a thing is what makes it be the very thing that it is, it is necessary to the thing. As Aristotle said, a thing's substantial nature is what is essential to it.[10]

On this basis, the character and status of a thing's essence within the thing is far less mysterious than is sometimes imagined. For a thing's essence is not a "pincushion" to which the thing's attributes are extrinsically attached. The essence is itself the substantial nature of the thing and not a "bare particular" with the thing's character wrapped around it. This whole idea of the essence as like a pincushion or an empty core comes from speaking of things as "having" this or that substantial nature. For this way of speaking to be literally correct would involve that the thing being referred to is already constituted as a thing, prior to its own substantial nature. But in fact, this idea is a fiction. To speak of a thing as "having" its substantial nature is correct only as a loose way of speaking.[11] Again, this substantial nature is what is essential to the thing. So far from being blank or neutral, the essential character is the basic causal principle within the thing.[12] This

[9] On persistence, compare Mackie, *Cement of the Universe*, 154-59.

[10] Aristotle *Metaphysics* 7.4.1029^{b}13-15.

[11] For the thesis that the unknowable substratum is a grammatical fiction, see Morris Lazerowitz, "Substratum," chap. in *The Structure of Metaphysics*, with a Foreword by John Wisdom, International Library of Psychology, Philosophy, and Scientific Method, ed. C. K. Ogden (London: Routledge & Kegan Paul, 1955), 144-62. See also C. B. Martin, "Substance Substantiated," *Australasian Journal of Philosophy* 58 (March 1980): 3-10, especially 5-7, 9-10. Martin argues that the idea of the substratum is that of the object as having or bearing its attributes, apart from any question of what those attributes happen to be. He points out that this idea is an abstraction from the concrete reality of the object with its attributes.

[12] On the substantial character of a thing as the basic causal principle within the thing, see Aristotle *Physics* 2.1; idem *Metaphysics* 5.4, 5.8.1017^{b}10-17, 7.17; idem *On the Soul* 2.4.415^{b}8-416^{a}18. See also

very character is what makes the thing be what it is as something unified that continues across time.

Indeed, given Aristotelian essentialism, the whole idea of some bare substratum to which attributes may be extrinsically attached is more than just false. It is an irrelevancy, a "fifth wheel." A substance is not a mere complex, nor is it a mere cluster of ingredients. Therefore, there is no question of including a bare substratum as the critical ingredient that makes something into a "real thing." For a real thing is a substance as opposed to being any kind of mere complex or cluster, with or without a bare substratum.[13] This is so because a "real thing" is something that coheres and persists. But then, a real thing has to be an integrated unit with its own internal structure, which structure includes both causal and (broadly) logical necessities.

Thus, the substantial nature of a thing, so far from having the thing's character merely wrapped around it, provides for the attributes of the thing as well as for the thing's continuing unified identity.[14] This function of providing for attributes follows almost trivially from the fact of providing for the thing's identity. In making a thing continue to be and to be unified, the substantial nature defines the basis for what belongs to the thing. For the substantial nature provides the basis for something both to be comprehended within the thing's unity as a coherent thing and to fall within the thing's history or continuity as a persisting thing. The successive states and qualities of a substance must be intrinsically united and causally grounded in the nature of the substance.[15] Of course, the other side of this fact is that the substantial nature also defines the basis for what does not belong to the thing. In these ways, then, the attributes as well as the identity of a thing are derived from its substantial nature.

From this analysis, it does not follow that any attributes of a thing are (or can be) accidental or contingent to the thing. A thing's whole character throughout its history, including even its relations, could be given together as an indivisible block. This is what Leibniz proposed. Of course, Leibniz did this along with reducing away relations to being merely ideal. He adopted monadism instead of classical realism.[16] But given classical realism, the relations and interactions of things will themselves be real. To be sure, it is both contingent and *a posteriori*

Aquinas, *ST*, pt. 1, qu. 77, art. 6. See also Jacques Maritain, *An Introduction to Philosophy*, newly designed ed., trans. E. I. Watkin (New York: Sheed and Ward, [1962]), 163-64. See also F. C. Copleston, *Aquinas,* Pelican Philosophy Series, ed. A. J. Ayer (Harmondsworth, Middlesex: Penguin Books, 1955), 82-84.

[13] For the point that traditional substance theories differ in this way from both bundle theories and substratum theories, see Michael J. Loux, *Substance and Attribute: A Study in Ontology* (Dordrecht, Holland: D. Reidel Publishing Company, 1978; Pallas Paperbacks, vol. 8, 1978), 164-65. See also Joshua Hoffman and Gary S. Rosenkrantz, *Substance among Other Categories*, Cambridge Studies in Philosophy, ed. Ernest Sosa (Cambridge: Cambridge University Press, 1994), 48, 51.

[14] Strictly speaking, perhaps one should say the substantial nature provides for the other attributes of a thing on the basis that the substantial nature is itself the central cluster of essential attributes.

[15] See Ewing, *Idealism*, 165-66.

[16] On the other side, given that relations are genuinely real instead of merely ideal, then having a thing's relations be part of its character in the same way as its qualities must end in monism instead of monadism. See Stebbing, "Concerning Substance," 285, 289-90, 301, 308. Stebbing's answer was to reject the traditional logic from which the assimilation of relations to qualities results.

that there are numerically diverse concrete particulars, that they stand in relations to each other instead of being isolated or incommensurable, and that they interact with each other. Then again, all these things seem to be so, and all these points are integral to classical realism. Since the facts concerning other things will be contingent relative to a given thing, and since interactions change at least the relations of things, a thing's relations will be contingent accidents of the thing.[17]

But what if determinism should turn out to be true? In that case, there is only causal necessity for what exists and what happens. Thus, what is in fact is what must be, right down to the smallest detail of the most trivial relation. No other alternatives are genuinely real. Given all this, one would have to say everything is totally locked into the system of the world as a whole. So, instead of monadism, there would be a kind of monism. There would, in fact, be exactly the "block universe" that William James was concerned to deny. Moreover, this would be so even given full classical realism.[18]

This question of determinism and the block universe is a serious concern in itself. However, as regards the question of Aristotelian essentialism, the answer is easy enough. In this context, to say that a thing's relations to other things are contingent accidents of the thing means only that they are contingent from the standpoint of the thing's substantial nature, taken simply and strictly as such. This kind of contingency remains even given that all of the thing's relations are wholly necessitated in concrete reality. For the causal necessities involved are not just there or given. Instead, these necessities are implemented by the concrete activity of having things work, which working proceeds in accordance with the things' substantial natures. (This point is the critical difference between determinism and fatalism.[19]) Therefore, even given determinism, Aristotelian essentialism will still be true, for a thing's substantial nature will still be prior (logically or metaphysically as opposed to temporally) to at least the relations in which it stands, and perhaps to other facts as well.[20]

The importance of these points can be seen clearly by contrast with one of the established ways of denying substantial natures. There has been proposed that concrete particulars are just interweavings of universals.

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[17] On these points concerning relations, compare Ewing, *Idealism*, 119-37.

[18] But compare W. Donald Oliver, "Essence, Accident, and Substance," *Journal of Philosophy* 51 (November 11, 1954): 719-30, especially 719, 720, 723, 727-28. Oliver argues that determinism is contrary to classical realism, as well as to essentialism, exactly because the separateness of discrete objects would be cancelled out.

[19] Compare Saint Augustine *On the City of God* 5.9, in which he distinguished Divine providence from fate on the ground that the order of Divine providence is implemented largely through finite wills.

[20] This strong priority of a thing's essential character to the concrete facts concerning the thing may well have to be explicated by invoking natural kinds, at least insofar as such attributes are repeatable across numerically diverse things. Thus, one may have to say, given that a thing exists as being something of such and such kind or species, then this or that concrete fact is a contingent accident of the thing. Broad reached this conclusion long ago regarding things under determinism, by more or less the same route followed here. Compare C. D. Broad, "The 'Nature' of a Continuant," in *Readings in Philosophical Analysis*, ed. Herbert Feigl and Wilfrid Sellars, The Century Philosophy Series, ed. Sterling P. Lamprecht (New York: Appleton-Century-Crofts, 1949), 472-81, especially 478-81.

One of the standard problems for this proposal is that universals are necessary and eternal. Thus, the basic character of concrete particulars as contingent beings that arise and perish cannot be accommodated. The answer has been that universals come to be contingently associated somehow. Therefore, the complex is a contingent being, even though the constituent universals are necessary and eternal. This is all that is needed.[21]

Whether this answer will work depends on whether there is or can be any such contingent relation of convergence among universals. The obvious problem is that this sort of interweaving or interplay seems repugnant to what universals are and how they work. Universals are related by compatibility, entailment, exclusion, subsumption, and so on, none of which allows for contingency. All the contingency pertaining to universals concerns only their exemplification or instantiation. But this goes beyond the relations that universals have to each other as universals. So then, the interweaving or interplay of universals is not enough to constitute concrete particulars, contrary to this theory.

Of course, given that universals are exotic objects, then there may well be some way for them to be associated contingently. But this view of universals is contrary to the basic idea of universals as the kinds or species to which objects belong, the qualities objects display, and so on.[22] Plato came to see this point, and Aristotle insisted on it.[23] Given all this, it seems clear that this theory must fail. For there is no way to have both universals as kinds and qualities, instead of as separate objects, and also to have contingent relations among universals apart from exemplification or instantiation. But this is what any contingent relation of convergence would require.[24]

Then again, one who follows this theory might try denying the contingency of concrete particulars. Perhaps, in actual reality, there is only the necessity of universals and their relations as universals. After all, there would be no contingency if determinism were true. So, perhaps there are only universals and constructs from universals, with no real contingency in the world.

However, this answer will not work, for the comparison with determinism fails. In both cases, there is the lack of real contingency in the world, but for very different reasons. Given only universals, there would straight metaphysical

[21] For this theory, this objection, and this answer, see Loux, *Substance and Attribute*, 97-98, 115-16, 120, note 18, 121-23.

[22] Of course, this criticism depends on the strong division in type or level or category between universals as applying generally and objects as singulars. Compare Stebbing, "Concerning Substance," 290-301. Stebbing was concerned to defend the idea of substance by opposing Whitehead's attempt to weaken the division between universals and objects. Then again, it is at least very questionable whether what Stebbing opposed here can be reconciled with classical realism.

[23] Plato *Parmenides* 130e-134e; idem *Philebus* 14e-15c. Aristotle *Topics* 2.7.113^{a}24-33, 6.6.143^{b}11-32, 6.10.148^{a}14-22; idem *On Sophistical Refutations* 22.178^{b}36-179^{a}10; idem *Metaphysics* 3.2.997^{a}34-^{b}12, 3.6.1003^{a}5-17, 7.13.1038^{b}1-15, 7.13.1038^{b}34-1039^{a}3, 10.10.

[24] Compare Hoffman and Rosekrantz, *Substance among Other Categories*, 80-81, who observe that this supposed contingent relation of convergence must be defined very differently from pointing to the way various attributes converge (or are "compresent") in a substance (whether an object that displays these attributes together or a subject who is aware of these attributes together), for that way of defining this relation depends on the idea of substance that was to be explained away. But this way is what people have tried.

necessity for all that is and all that happens. This kind of necessity is far deeper and more powerful than the simple causal necessity involved in determinism. To put the point in terms of what was said earlier, given only universals, the actual reality of the world would be very much as if full fatalism were true instead of mere determinism. For the necessity belonging to things would not be implemented through the concrete activity of having things work. Instead, this necessity would obtain automatically, just by having things exist and be what they are. Furthermore, this necessity would be total, and so there would not be any room left for mere causal necessity based on how things work.[25] Now, classical realism looks to the causal structures and interactions among things as the basis for what is observed. Therefore, determinism can be, and the theory of concrete particulars as interweavings of universals cannot be, reconciled with classical realism.

Along the same line, a substance need not be wholly independent, contrary to Descartes and Spinoza. All that is needed is that there be the substantial nature within the thing, whereby the thing coheres and persists. Given this, a thing may depend on other things in various ways. What independence it has may be very limited.

Once again, the analysis in terms of independence is the wrong approach. The right way is to think of autonomy as a positive fact instead of independence, which can be taken as something purely negative. Thus, a substance has a kind of relative autonomy as having its own positive functioning (and thus its own value and density).[26] As a further consequence of this fact, a substance is then free from the kinds of dependency on other things that its own relations, qualities, history, and parts have on the substance. For these kinds of dependency are what the substantial nature fills or provides for. In this way, and in this way only, a substance has a kind of relative independence exactly insofar as it has (the positive fact of) relative autonomy. But all this is fully compatible with having substances produced by processes of natural generation or even by Divine creation.[27]

To be sure, insofar as a thing is dependent instead of autonomous, its attributes and behavior are derived from whatever produces or supports it (the natural order as a whole, or God, or whatever). The answer is more or less the same as for determinism. A thing's attributes and behavior are derived from what

[25]In his essay "Monads," Strawson tries interpreting Leibniz's monads as complexes of general attributes or concepts. He points out there are only logical relations, and no physical interaction, among these monads. P. F. Strawson, *Individuals: An Essay in Descriptive Metaphysics* (London: Methuen & Co, 1959), ch. 4, especially 124-31.

[26]On the density of created beings, see Aquinas, *SCG*, bk. 2, ch. 30 (where he explains how created beings embody various necessities, even though their existence is ultimately contingent).

[27]All this is what the school of Aquinas means by saying a finite substance exists in itself (whereas qualities and relations exist only in other things) but not of itself (only God exists of Himself). See Aquinas, *ST*, pt. 1, qu. 45, art. 4; idem, *SCG*, bk. 2, ch. 18; ch. 52, par. 6, 8. Among his followers, see Robert J. Kreyche, *First Philosophy: An Introductory Text in Metaphysics* (New York: Henry Holt and Company, 1959), 292-95, 301-02, 303. See also John Wild, *Introduction to Realistic Philosophy* (New York: Harper & Brothers, 1948), 325. See also Maritain, *Introduction to Philosophy*, 173-75. See especially Etienne Gilson, *The Christian Philosophy of St. Thomas Aquinas,* trans. L. K. Shook, C.S.B., with a catalogue of St. Thomas's works by I. T. Eschmann, O.P. (New York: Random House, 1956), 30-31, 445, note 5. For another perspective, see Copleston, *Aquinas,* 82.

produces or supports it only as such agencies work in and through the thing that is produced or supported. It is not as though the thing's attributes and behavior arise apart from the dependent thing's functioning, directly from the other agencies.

Even given classical realism, it might be that all of a thing's monadic or intrinsic attributes are essential to the thing. This is exactly what some versions of mechanical atomism say. Such theories are not contrary to classical realism in anything like the same way as either monadism or monism. The only question is whether such theories can account for the observed facts, most notably the facts of mental life.

Can a real thing with contingent relations also change intrinsically? Perhaps so. Indeed, Aristotle said the most distinctive mark of substance is that it can admit contrary qualities while remaining numerically one and the same, by changing within itself.[28] Was he right to say this? Classical realism is neutral on this point. Classical realism could accommodate Aristotle's theory on intrinsic change, and it could also accommodate mechanical atomism instead. In each case, the only philosophic question for classical realism is whether the explanation can be made to work out in detail and to account for all the relevant facts. Given this, then the truth is whatever it turns out to be.

Yet this point raises a serious problem. The difference between relations and other attributes may seem reasonably clear. On the other hand, the differences among monadic attributes are much less clear. Thus, what is necessary and what is contingent to a given thing may seem to be problematic. The end of this line is to say that selected attributes are necessary (and thus essential) only relative to a given description of the thing.

This kind of linguistic relativism allows nothing to be specially privileged within the things themselves as they exist in themselves. But a thing's substantial nature is the primary character of the thing, and so it is strongly privileged as being the basis for the thing to be what it is and to exist as it does. Therefore, linguistic relativism must deny that there is any substantial nature as a separate or special character of a thing, over and above its other monadic attributes. As a result, linguistic relativism must also deny the distinctive functions of the substantial nature. This denial includes denying that things cohere and persist in themselves, apart from the descriptions people impose.

Strictly speaking, this kind of linguistic relativism is not full idealism. For one can still say the minimal elements exist in themselves with their own character. But classical realism would be false, for almost all of the apparent structure of the world would turn out to be some sort of fiction or illusion.[29] To be sure, the arrangement of minimal elements in spacetime might turn out to be real. Again,

[28] Aristotle *Categories* 5.4^{a}10-^{b}18.

[29] Quine was notorious for denying Aristotelian essentialism in favor of necessity based on descriptions only. Significantly, though, Quine said he saw only Hume's regularities in natural necessity. See W. V. Quine, "Necessary Truth," chap. in *The Ways of Paradox and Other Essays* (New York: Random House, 1966), 48-56. Given this denial of strong natural necessity, one must also deny the apparent causal structure of the world (in terms of cohering, persisting, and interacting) from being real in the strong traditional sense that the school of Aristotle believed in.

there might perhaps even be causal relations among minimal elements. But the system of objects that cohere, persist, and interact would exist only as a construct in the mind instead of existing in fact.

It does not follow that there would be chaos. There would be only the flux, but this flux might be lawful instead of random. This is what Heraclitus proposed. However, given that almost all of the apparent structure of the world is some sort of fiction or illusion, almost all of the basis for regular order in the world would be then external or extrinsic relative to the processes in the world. On the other hand, classical realism makes the basis for regular order internal or intrinsic to processes, since these processes are made up of the functioning and interactions among objects. Therefore, classical realism would still be false, even given this kind of lawful order.

In fact, this kind of opposition to Aristotelian essentialism seems to be based largely on an underlying reductive empiricism. Given a strong version of reductive empiricism, there would be very little if any basis to find one attribute any more or less necessary (or contingent) to an observable object than any other attribute, apart from being necessary (or contingent) relative to this or that description of the object. Even a milder version of reductive empiricism, so long as it remains reductive, must deny at least Aristotelian essentialism, even if it could also avoid linguistic relativism. For reductive empiricism cannot accommodate or accept the idea of cohering and persisting as causal functions of a thing, and so it cannot accept or allow for the Aristotelian idea of a thing's substantial nature.

This point is clearly correct as far as it goes. All that can be said in reply is that reductive empiricism cannot accommodate classical realism either. For there are really just two alternatives that reductive empiricism can pursue concerning observable objects. To construct such objects from actual experiences alone (whether on the part of God, or of man, or whatever) is straight idealism of exactly the kind that is directly contrary to classical realism. On the other hand, to construct these objects from actual experiences together with hypothetical experiences is phenomenalism, which has already been shown to fail given classical realism. Only these two alternatives are available to reductive empiricism as having even any initial plausibility, and neither one will work.

Of course, there may be good empirical reasons to refer to or quantify over things that happen not to be perceived at the given moment, and to do so in the strong sense of saying these things exist fully and properly in spite of being presently unperceived. On this basis, classical realism can be reconciled with empiricism after all. Yes, indeed so, but then this kind of empiricism is not reductive. In order to justify such empirical judgements, one must speak (explicitly or implicitly) of structures adequate to support subjunctive conditionals.[30] An empiricism sophisticated enough to allow for all this will also be sophisticated enough to accommodate the idea of cohering and persisting as causal functions of a thing. So, this kind of empiricism can allow for the Aristotelian idea of a thing's

[30] The need for such structures is at least part of what Strawson established in his essay "Sounds." Strawson, *Individuals*, ch. 2.

substantial nature as well. Therefore, neither reductive nor sophisticated empiricism counts against the basic claim here that Aristotelian essentialism follows from classical realism.

It should be noted that reductive empiricism cannot really accommodate mechanical atomism any more than things that change intrinsically. This remains so even though mechanical atomism does not involve the same temptation to linguistic relativism as there is with things that change intrinsically. For, once again, the division between relations (which are all that change here) and intrinsic attributes is comparatively clear. For reductive empiricism, however, the main problem is not about change but things. Reductive empiricism cannot accommodate the idea of a thing's substantial nature as the primary basis for the thing to be what it is and to do what it does, and so it cannot accommodate any Aristotelian essentialism, with or without intrinsic change.

Thus, an empiricist may complain that there are only clusters of attributes presented to the senses, and the causal structures underlying these clusters are just whatever they turn out to be. There seems to be no particular necessity involved beyond what is entailed by a given description. There is only what shows up in experience, and this seems to be thoroughly contingent.[31] At most, there are such comparatively trivial necessities as that being colored presupposes being extended.

In one way, all this has merely to be conceded. As regards clusters of attributes presented to the senses, taken simply and strictly as such, there is no serious necessity involved. There is only the contingent fact of what shows up in experience. The serious basis of necessity comes with the observed facts of cohering and persisting. For these facts are causal, and so there is modal or subjunctive import. The problem for empiricism is that the way these facts are observed goes far beyond just having clusters of attributes be presented to the senses. Only some highly sophisticated version of empiricism can accommodate what is involved.

Along the same line, it is easy enough to answer the concern that there is in fact only what is actually real, with no further quality of necessity involved.[32] Yes, of course, but then no claim to the contrary is involved on the part of Aristotelian essentialism. Necessity, so far from being additional to what is actually real, is inherent in the actually real. The point is not that there is some magical quality of being "ironclad" attaching to privileged portions of what is actually existent in concrete reality. That would be absurd. But rather, what is actually real turns out to be good enough to support subjunctive conditionals, and this fact pertains to the things themselves as opposed to merely the descriptions people choose to impose. Moreover, this kind of "goodness" is included in being good enough to be actually real in the strong sense of existing within oneself as having one's own natural being. No quality or privilege is involved beyond that of being actually real.

[31] See Alan Sidelle, *Necessity, Essence, and Individuation: A Defense of Conventionalism* (Ithaca and London: Cornell University Press, 1989), 105-11. See also A. J. Ayer, *Philosophy in the Twentieth Century* (New York: Random House, 1982; Vintage Books, 1984), 268-70.

[32] See Sidelle, *Necessity, Essence, and Individuation*, 1-2, 21-22, 114-20,125-26.

At this point, the basic explication of classical realism comes back into play. Actual reality entails or includes independence of awareness in the manner explained. This remains so even though independence (considered as something negative) is more or less a corollary or side effect instead of being the main concern. Now, independence involves subjunctive import. For a thing to be independent is for it to be such that it would stand even if there were not experience of it.[33] Therefore, to be actually real is already to support subjunctive conditionals concerning hypothetical events or situations. The question, then, is how far this subjunctive import extends. The right answer is that there is whatever there turns out to be. What seems to turn out is that this subjunctive import may be good enough to support holding or standing through changes in relations and perhaps other changes as well. That is all. So, once again, one must conclude that reductive empiricism cannot accommodate Aristotelian essentialism largely because it cannot accommodate classical realism either.

The import of these points may be seen more fully by contrasting the weakest version of stable objects with the strongest denial of any such stability as intrinsic to things. Thus, one who denies essential attributes in things may try to argue that there are stable objects, although of a weaker kind than common sense imagines. He may say that there is a lesser version of stability in things instead of full coherence and persistence. One who says this may then go on to say that stable objects of some such lesser or weaker sort are enough to accommodate or explain the observed structure in the world.

This proposal breaks down at once. Any kind of stability in objects that is adequate to provide for the observed structure will also be strong enough to require something very much like essential attributes. For what is meant by speaking of objects as stable? When a thing is stable, it does not hold together only through chance coincidence. A stable thing has some basis within itself that makes it be unified and continue to be unified. Since this is made to be so, the basis is causal. Thus, there is some basis within a stable thing that makes it exist and keep existing as a single unit. This basis will be necessary to the thing in the same way that the substantial nature of a "true object" is necessary to the object. Therefore, even the weakest version of stability brings in something at least strongly analogous to Aristotelian essentialism.

On the other side, there could well be more than just the flux with the regular order of lawful patterns, even apart from anything like Aristotelian essentialism. One can at least imagine a world full of things that are almost like “true objects” but that are void of any stability. There could be things that are bundles or clusters of attributes (tropes or abstract particulars). Individual identity among clusters would be based on geometrical attributes only. All the other (monadic) attributes of a thing would exist as “attached to” (or inhering in) the thing’s bodily form or figure, its size and shape. There would be no coherence beyond the fact that the

[33] Strictly speaking, something might be independent of experience, and yet whether it would stand if there were not experience of it may be problematic. For the thing might be stochastic or probabilistic. The answer is that the thing would have the same basis to stand even if there were not experience of it.

bodily form exists as actually real. There would be no persistence beyond continuity across spacetime. (Even such persistence would have to be understood as the unbrokenness or freedom from gaps of the "path" across spacetime.) A thing could then change its attributes and still be the same thing so long as the bodily form or figure remained. Moreover, even this form might be distorted, and yet the thing remains, so long as the thing is unbroken (in the sense of not having any part or piece cut off). Again, there could be a fully developed system of kinematic facts, along with provisions for collisions of things with each other.[34]

Supposing some such world to be established, would this world then be real? By hypothesis, it exists in fact, as opposed to being any kind of fiction or illusion. Yet even so, the intuitive impulse is to say that this is a "toy" world or a "fake" world instead of being "genuinely" real. The explanation is that the things of this proposed world are void of stability because they are void of any intrinsic basis to be what they are and to do what they do. Whatever basis such things may have is wholly extrinsic to them. They are mere constructs at best. Thus, these proposed things are real in that they are unlike the things in dreams or delusions. But these proposed things are unreal in that they are like shadows or reflections. Such things exist in fact, but they are "empty images." So, their claim to be truly or genuinely real is weak, for their claim to exist within themselves is weak.

The importance of these points comes out clearly in the history of philosophy. According to Plato, observable objects are in fact shadows or reflections in (almost) exactly this way.[35] All the basis for stability in the world is up in heaven, above the things subject to multiplicity and change. Then again, Plato's theory is sometimes spoken of as "Platonic idealism." Given the points developed here, the claim of this theory to be called idealism is more than just that the Ideas are exalted. For they are exalted at the expense of concrete particulars. Thus, in one way, this theory is very much like reductive idealism, even though material things are acknowledged to exist in fact and even to interact with each other.

Quite clearly, it is of course bizarre to say Plato was almost like a reductive idealist. Indeed, if the question as to whether Plato was a classical realist had to be answered yes or no, then one would have to say yes. But all this is so only because Plato lived and worked before the critical questions had been fully developed. What can be observed by looking back at Plato with these questions in mind is that, even at best, Aristotelian essentialism can be denied only at a high price. A thing void of substantial nature can be actually real in some way strong enough to be significant. But this kind of actual reality is still something very much truncated, as opposed to the full version of natural being affirmed by classical realism.

[34] In fact, it is at best misleading to speak of such complexes as "bundles" or "clusters" of tropes or abstract particulars. Such a complex is really an integrated unit with its own internal structure, which structure includes both causal and (broadly) logical necessities, exactly like an Aristotelian substance, save only that it is empty of substantial nature. Aquinas believed that God, as an act of supernatural intervention, could displace the substantial nature from a thing and then maintain its other attributes as just this kind of empty complex. See Aquinas, *ST*, pt. 3, qu. 77, art. 1, 2, 3, 7.

[35] See Plato *Timaeus* 48e-52e, 53c-57d. One may note that, like Aquinas, Plato had the ongoing activity streaming down from heaven to maintain the empty shells of objects. Unlike Aquinas, Plato imagined this to be the normal way observable objects exist.

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Paolo Valore (ed.) *Topics on General and Formal Ontology*

Kantian, Analytic, and neo-Thomistic philosophy: Three moments in the history of existential predication

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The question of existential predication has played a significant role in bringing various philosophical schools to the field of ontology. The present paper offers a summary account of three moments in the history of the ontological issue of existential predication. The first moment occurs in Kant's attempt to bring clarity to philosophy as Newton did to the natural sciences. The second moment is selected from the analytical school's attempt to clarify the logic of existential propositions. The third moment occurs when the neo-Thomistic school introduces the metaphysical reasoning of Aquinas to bear upon the debate.

I approach this history of existential predication in a way slightly different from the majority of commentators. Usually the question of existential predication is addressed within the wider context concerning the validity of the ontological proof for God's existence. Norman Malcolm, for example, accuses Anslem of "defining things into existence."[1] According to Malcolm, Anselm considered existence a predicate. The truth is that existential predication, as understood in the Kantian sense, was of no immediate interest to Anslem. To critique Anselm's argument using the tools of Kantian philosophy is one thing, but proceeding with the critique as if Anslem himself had emplyed the same tools is another. Alvin Plantinga argues that "if this were Anselm's procedure – if he had simply added existence to a concept that has application contingently at all – then his argument

[1] Norman Malcolm, "Anselm's Ontological Arguments," *The Philosophical Review* 69 (January, 1960): 41-62

would be subject to Kantian criticism."[2] However, as Plantinga and others conclude,[3] Anselm constructed his argument upon a concept of the divine being that was presumed in the Christian intellectual milieu of the day.

I select the three moments mentioned above not because they focus on the ontological argument *in se,* but rather because they penetrate the essential ontological and epistemological questions underlying the question of predication. Although Kant raises the question of existential predication during a refutation of the ontological argument, his main concerns are ontological, epistemological, and logical rather than theological. Just as Anslem's ontological proof needs to be taken on its own terms, so must Kant be taken on his own terms. Kant himself was not so much preoccupied with Anselm's proof as he was with the attempt of Christian Wolff to reconstruct metaphysics based on a proper science of ontology. What I hope to offer the reader is a cursory glance at the direction the debate on existential predication has taken since Christian Wolff's identification of the *scienta entis in genere, seu quatenus ens est.*

1. Kant and existential propositions

The question of predication was adumbrated at the beginning of ontology's history as a particular science. Johannes Clauberg's definition of "ontology" as the "science which does not deal with this and that being … but with being in general" already suggested the issue of existential predication without addressing it directly.[4]

In the work of Christian Wolff, predication explicitly enters the field of ontology understood as the study of the most abstract predicates, namely being and the common affections of being. Wolff distinguished this area of study from special metaphysics, which he classified into four sub-branches: ontology, cosmology, psychology, and theology. Both general metaphysics (i.e., ontology) and special metaphysics were generally defined as the "science of being, of the world in general, and of spirits."[5]

It was against this background that Immanuel Kant, who, like Wolff, had expressed a desire to reestablish a coherent unity to metaphysics, composed his monumental *Kritik der reinen Vernunft.* It is in Kant that we see for the first time a direct critique of the question of predication. In the First Critique, we find a famous passage in which Kant rejects existence as a "real" predicate. His claim is that existence, when predicated of a subject, does nothing more than (1) "posit" the

[2] Alvin Plantinga, *God, Freedom, and Evil* (Grand Rapids: Eerdmans, 1977), 97-98.

[3] See for example Gerald B. Phelan, *The Wisdom of St. Anslem* (Latrobe: Benedictine Press, 1960).

[4] Johann Clauberg, *Elementa philosophiae sive ontologiae* (1647). José Merrater Mora notes that it was Rudolf Goclenius, not Clauberg, who first used the term "ontology". See "On the Early History of 'Ontology'," *Philosophy and Phenomenological Research* 24 (September 1963): 36-47.

[5] Christian Wolff, *Logica, Discursus Praeliminaris,* III, 79.

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subject in itself, and (2) "posit" the subject as an object standing in relation to my concept of it.[6]

Kant's most explicit formulation of his position appears during a refutation of the ontological proof for the existence of God.[7] His analysis of existential propositions leads him to conclude that "the real contains no more than the merely possible. A hundred real thalers do not contain the least coin more than a hundred possible thalers."[8] Kant raises some serious issues in regard not only to the validity of the ontological proof, but the nature of *all* propositions in which the verb "to be" serves as the predicate. According to Kant's definition, whatever kind of a predicate "is" happens to be, it cannot be a "real predicate". Kant defines a real predicate as one "which determines a thing" or "which is added to a concept of the subject and enlarges it."[9] Kant's claim is that a real predicate adds conceptual content to the subject of which it is predicated. It enhances and further determines the concept to which it corresponds. To take but a simple example, the proposition "the car is red" contains only two concepts: "car" and "redness". The subject-concept "car" is made conceptually richer by means of the predicate "red". The word "is", according to Kant, merely performs the logical function of a copula. In the proposition "the car is," however, the predicate "is" does nothing to enlarge or enhance the concept "car". Kant's fundamental claim is that "is" serves to "posit" the thing in itself.

A satisfactory understanding of Kant's distinction between real and non-real predicates is not possible without a grasp of the underlying prior distinction between synthetic and analytic judgments made in the *Critique of Pure Reason.* A careful scrutiny of this distinction, upon which all of the Kantian project stands, reveals that it too is primarily a distinction between types of propositions. Indeed, at times is seems as if Kant uses the terms *Urteils* ("judgment") and *Satz* ("proposition") synonymously. As we shall see, this opens Kant to the accusation of self-contradiction regarding the nature of existential judgments.

Although already quite well known, a brief review of terms is in order here. An analytic judgment "is one in which the predicate is covertly contained in the concept of the subject."[10] The stock example given in philosophy textbooks is "all bachelors are unmarried." A synthetic proposition, on the other hand, is "one in

[6] Immanuel Kant, *Critique of Pure Reason,* trans. N. Kemp Smith (Toronto: Macmillan, 1965), 506 (B 627).

[7] "Being is obviously not a real predicate; that is, it is not a concept of something which could be added to the concept of a thing. It is merely the positing of a thing, or of certain determinations, as existing in themselves. Logically, it is merely the copula of a judgment. The proposition, 'God is omnipotent', contains two concepts, each of which has its object – God and omnipotence. The small word 'is' adds no new predicate, but only serves to posit the predicate in its relation to my concept."

[8] Ibid.

[9] Immanuel Kant, *Critique of Pure Reason,* trans. N. Kemp Smith (Toronto: Macmillan, 1965), 504 (B 626c-627). Although "enlarges" may be the best English translation of "vergrößert", "enlarges" might suggest that a wider class of things falls under the concept of "car" when it is determined by the predicate "redness", which is obviously false. Strictly speaking, the concept "car" is actually narrowed or restricted by "redness" in this sense.

[10] Kant, *Critique of Pure Reason,* 48 (B 10).

which the predicate amplifies or adds something to the concept of the subject."[11] Examples of synthetic propositions include "some bodies are heavy" and "a green light indicates 'go'". Kant also distinguishes between analytic and synthetic propositions by means of their respective truth value. A statement is an analytic truth if and only if the concept of the predicate is included in the concept of the subject; otherwise, if the statement is true, it is a synthetic truth.

Several difficulties arise when we try to apply the analytic/synthetic distinction to existential propositions. From the First Critique, it is clear that existential judgments cannot be analytic because it is possible to have a complete concept of a thing apart from its real existence. Indeed, the fact that existential propositions are not analytic is the key to Kant's refutation of the ontological argument. On the other hand, we are hard pressed to classify existential propositions as synthetic. A synthetic proposition expands or adds something to the concept, but, as Kant argues in the First Critique, no amplification occurs when existence is predicated of a subject.

The preceding dilemma led J. Michael Young to conclude that existential judgments are actually no judgments at all.[12] Young argues that Kant allowed for two types of judgments and two types only: analytic and synthetic. Consequently, the distinction between analytic and synthetic judgments only applies to judgments that contain *real* predicates. Thus Kant relegates existential propositions to an entirely different category, which he calls "modal concepts". The distinguishing mark of modal concepts is that "in determining an object, they do not in the least enlarge the concept to which they are attached as predicates."[13] Instead, "modal concepts only express the relation of the concept to the faculty of knowledge." Kant, according to Young, seems to be saying here that the modal categories -- possibility, actuality, and necessity -- cannot serve as real predicates. Young proceeds to demonstrate that the attempt to equate existence with the modal category of actuality would end in failure.[14]

In addition to the logical issues, Kant's analysis of existential predication also raises several epistemological issues. Kant correctly assumes that, according to his definitions, if existence is a predicate, it would not be possible to have an "adequate" conception of anything. If a thing in the real world contains the predicate of "existence", it includes (at least) one more predicate than the thing does as an object of knowledge. But this would be absurd, for our concept would then not be of the thing, but of something else. In order to have a complete concept of the thing, the concept itself would have to contain the same additional predicate as the object. In short, any knowledge presupposes a strict identity between the thing-as-existing-in-reality and the thing-as-object-of-knowledge. J. William Forgie calls this the "Doctrine of Isomorphism".[15]

[11] Ibid.

[12] J. Michael Young, "Kant on Existence," *Ratio* 18 (December, 1979), 93.

[13] Kant, *Critique of Pure Reason,* 239 (B 266).

[14] Michael Young, "Kant on Existence," *Ratio* 18 (Dec. 1979), 96-99.

[15] "This (Doctrine of Isomorphism) is the view that there is a one-to-one correspondence between: (a) the predicates that could be included in a concept of N; (b) the predicates that could be included in a concept of

Although the passage from the *Critique of Pure Reason* we have considered thus far is Kant's most famous critique of existence and real predication, Kant had addressed the same question in an earlier essay entitled *Der einzig mögliche Beweisgrund zu einer Demonstration des Daseins Gottes* (*Beweisgrund* hereafter).[16] In this passage, Kant takes recourse to the idea of possible things existing in the mind of God. The highest being, argues Kant, has concepts of millions of things which do not exist. God knows them as "merely possible" things. Such a concept in the mind of God is a complete determination of the thing, but existence is not one of its predicates. If the possible thing were to become a real thing, existence would not be a predicate of it, for then the concept of that object would differ from the thing itself.

When we compare the corresponding passages from the *Critique of Pure Reason* and the *Beweisgrund,* two basic arguments emerge, both arriving at the same conclusion. First, Kant recognizes that it is possible to have a complete concept of an object even if the object does not exist in reality. We have seen that in the passage from the *Critique of Pure Reason* he illustrates this by the example of a hundred thalers. In the *Beweisgrund,* he turns to the notion of divine concepts to further elucidate his position. Both of the arguments share the major premise that it is possible to have a complete concept of a merely possible being. To this major premise we may add one of two minor premises. In the first argument, which is more clearly developed in the passage from the *Critique of Pure Reason,* we add the premise that the concept of a merely possible being cannot include existence. Therefore, existence cannot be a predicate if that potential object really were to exist. The second argument has as its major premise the same as the first. To this we add the minor premise that the concept of a possible thing would have every predicate contained in the concept itself *if* that object were an actual being (since the concept itself is already complete). The same conclusion follows from this argument as from the first: if the object were to exist in reality, existence would not be one of its predicates.

As we consider Kant's arguments, we must carefully acknowledge what Kant does *not* mean by "real predicate". Kant does not mean by this term that there is no difference between a hundred thalers I imagine in my pocket and a hundred thalers that actually exist in my pocket. Obviously, it makes all the difference in the world whether the money is really there or not. Rather, Kant is trying to tease out something radically unique about the predication of existence. He does not deny that existence is a predicate, but a *real* predicate. More importantly, Kant's

N (when N is actual, or at a time when N is merely possible; and (c) the predicates that could be possessed by the object of a concept of N when that object is a merely possible being." J. William Forgie, "Kant and the Question 'Is Existence a Predicate?'" *Canadian Journal of Philosophy* 5 (December 1975): 571.

[16] "Existence is not a predicate or determination of anything whatsoever. Take any subject you like, for example Julius Ceasar. Combine in it all its conceivable predicates (not excepting those of time and place). You will then see that, with all these determinations, it may or may not exist. The being which gave existence to the world and to this hero was able to recognize all these predicates – not a single one excluded – and could still regard him as a merely possible thing which, save his decree, did not exist." Immanuel Kant, *Der einzige mögliche Beweisgrund zu einer Demonstration des Daseins Gottes,* in *Werke,* ed. E. Cassirer (Berlin, 1922), vol. 2, 76, as trans. by J. William Forgie, 563.

analysis reveals that in delineating the nature of existential propositions, the question as to how existence is known cannot be avoided. In both of the arguments summarized above, we see a movement from the concept in the direction of the object as a really existing thing. It is a fundamentally Cartesian movement.

We can, however, take Kant on his own terms. This is precisely what Young tries to do. Young believes that Kant's claim to classify existential propositions as synthetic judgments contradicts Kant's definition of synthetic judgments. However, it could be argued that when Kant defines synthetic propositions, he does not claim that the predicate of such a proposition, although adding to or amplifying the subject, must add or amplify the subject with *conceptual* content. As George R. Vick says, Kant "left the door open, then, to existence being a non-conceptual predicate, that is, something which when affirmed of an object, would not enter into its concept."[17] In Kant's epistemological analysis of subject and object, he does not claim that real existence is simply constituted by an *a priori* categorical determination. Kant insisted that we are made conscious of existence only through sense-intuition. Because of this, the knower is "given" existence when confronted by an object.[18]

For these reasons, Vick argues that Kant believed there to be some kind of "content" to existence. Vick notes that Kant applied the term "concept" to "designate anything of which we are intuitively conscious in an *a posteriori* or empirical way."[19] It is according to this understanding of content, claims Vick, that Kant is able to classify existential judgments as synthetic.

Another way of stating the matter is to consider the relationship between subject and object and the placement of existence. Kant has made it clear in the two passages already seen that it is possible to have a concept of the object with all of its conceptual determinations without conceding that the object exists as a thing in reality. When this is considered in the light of Forgie's Doctrine of Isomorphism, it would be tempting to conclude that Kant is primarily making a claim about the really existing thing rather than about the concept itself. If this were true, the primary claim would be that existence is a "thing" not to be found in the really existing object at all. However, Vick argues that Kant is actually making a primary claim about the nature of the concept. That is, in a concept, there is no trace of the thing's real existence to be found. Rather, the real existence of a thing is given by the thing itself; it is not an *a priori* category of thinking. Therefore, even if we are to reject existence as a "real" predicate according to Kant's definition, we are able to accept the claim that there is some content to existence according to Kant's own use of the term "content". Let it be noted, however, that Kant does not make it clear whether the "content" of existence is evident in propositions in which "is" serves as a logical copula.

[17] George R. Vick, "Existence was a Predicate for Kant," *Kantstudien* 61 (June 1970), 360.

[18] "It follows, then, that in this view any mental activity that we might posit on our own initiative, which is to say, and act of 'thinking', could in no way suffice to supply our consciousness with content; hence through it alone we could not be conscious of anything existent." George R. Vick, "Existence was a Predicate for Kant," *Kantstudien* 61 (June 1970), 361.

[19] Ibid., 363.

The preceding analysis leaves us with two alternatives. If we accept the arguments of Young, we are left in search of a unique category for existential propositions. Young presents a plausible case that Kant contradicted himself when he claimed that existential judgments were judgments at all. They can be neither synthetic nor analytic if we conceive of existence as having absolutely no "content". This conclusion demonstrates the radical difference between the conceptual determinations of a thing and a determination which merely posits the thing in itself. Alternatively, we can accept Vick's thesis that Kant in fact made a valid conclusion in his classification of existential judgments as synthetic. Kant was aware that existence is not a conceivable determination but yet it could be predicated by virtue of its non-conceptual content.

In the section that follows, we will consider the *status quaestionis* as formulated in the analytic tradition represented by Willam Kneale and G. E. Moore. Both Kneale and Moore try to circumvent the issue of content, be it conceptual or non-conceptual, by applying a strictly logical analysis to the propositions themselves. Through a close scrutiny of meaning in ordinary language, both Kneale and Moore believe the question can be restated in a way that makes it easier to deny that existence is a predicate.

2. Kneale, Moore, and logical analysis

William Kneale and G. E. Moore's positions were elaborated in the wake of a symposium sponsored by the Aristotelian Society in 1935.[20] Kneale maintained that existence is not a logical predicate. He argues that the advancement of a revised logic in the forty years preceding the symposium had shed new light on the nature of existential propositions. He identifies the revised logic as a closer scrutiny of "logical form". "I am not sure," he writes, "that a definition of (logical form) is possible. But I can indicate that what is meant by saying that 'this is red' has the same logical form as 'this is green' and a different logical form from either 'this is near that' or 'if Hannibal had marched on Rome, he would have taken it."[21] "Logical form" is an analytic method of evaluating propositions apart from their truth or falsity, and solely on the grammatical structure of the proposition. According to Kneale, the development of this revised logic enables philosophers to show that former rejections of Kant's ontological argument were inadequate.

Even before an analysis of the proposition itself, however, the method of logical form rests on certain metaphysical presuppositions. Kneale uses Descartes' formulation of the ontological argument to contextualize the debate over the predication of existence. Recall that Descartes believed that the existence of God could be proved in a way similar to, but with greater facility than, the geometrical proof that the sum of the interior angles of a triangle is equal to two right angles. A

[20] William Kneale and G. E. Moore, "Symposium: Is Existence a Predicate?" in *Proceedings of the Aristotelian Society,* Supplement 15 (1936): 154-188.

[21] Ibid., 157.

perfect being, according to Descartes and other proponents of the ontological argument, must be said to have the property of existing. This property flows from the essence of God in the same way that the property of the sum of the interior angles in a triangle flows from the essence of a triangle. Kneale claims that even to embark on such a proof for the existence of a perfect being whose essence involves existence, one must first assert that such a being is *possible.* This leads Kneale to conclude that "there must be some sense of 'being' which is logically prior to existence and applicable to the possible as well as to the actual."[22] The distinction between the possible and actual is fundamentally different from the distinction made by Kant. Recall that Kant employed this distinction within the framework of the prior epistemological distinction between subject and object. The distinction made by Kneale pertains only to the proposition itself. The term "subject" according to logical form applies to the subject of a proposition (i.e., the subject of predication). When I assert that "the car exists", I am saying something about a subject which is already there in some way. Stephen Read suggests that a logical analysis of this type "is a way out of Plato's beard, the supposition that to say anything of an object, even that it does not exist, requires the existence of a logical subject of predication, in particular, the predication of non-existence."[23]

Let us turn to take a closer look at Kneale's analysis of logical form as applied to propositions. The first task is to define "predicate" with some precision so as to compare different types of propositions. "Predicate" as used in this context signifies "logical predicate". Kneale states that in a complex proposition, that is, "a proposition about other propositions," there may in fact not be a predicate where there appears to be one. Again, a predicate is an attribute of a subject. Kneale offers the example of a complex proposition in the form of "p is true". In a statement of this form, "is true" is not an attribute of a subject "p" but is rather the "truth value" of the proposition "p".[24] On the surface, it appears that Kneale's definition of "predicate" corresponds to Kant's definition of "real predicate". However, "predicate" in the sense that Kneale uses the term simply means that that which is stated of the subject is something directly attributable to the subject. As we have seen in the previous section, Kant's use of the term applies not to grammatical attribution, but to a logical relationship between subject and predicate based on the analytic/synthetic distinction. To avoid confusion surrounding the term "predicate", Kneale decides to call the elements of simple propositions "constituents and components".[25]

Essentially, Kneale's technique involves the transposition of the grammatical components of subject and predicate. When I assert that "the car exists" I am using an improper grammatical form that is more properly expressed by the proposition "there is a car". By expressing the thought in this way, we classify the proposition according to a category other than "existence". In regard to the original Cartesian

[22] Ibid., 155.

[23] Stephen Read, "Exists is a Predicate," *Mind* 89 (July 1980), 412.

[24] William Kneale and G. E. Moore, "Symposium: Is Existence a Predicate?" in *Proceedings of the Aristotelian Society,* Supplement 15 (1936), 158.

[25] Ibid.

confusion, Kneale has this to say: "If when Descartes talks of a class of reality he means substances or attributes or relations, existence belongs to no class of reality. The word 'existence' is not a symbol for anything which can be either a constituent or a component of a single proposition. It is only a logical auxiliary symbol." Consequently, the ontological argument is merely a "play on grammatical form."[26] Using Kneale's own example, the proposition "tame tigers exist" is logically identical to the propositions "there are tame tigers", "some tigers are tame", and "something is a tame tiger".

Close scrutiny of Kneale's analysis reveals that the metaphysical presuppositions underlying his use of logical form are crucial to his denial of existence as a logical predicate. "The fundamental thesis," he writes, "of those who believe existence to be a predicate is that there is a sense of 'being' logically prior to existence and applicable to the possible as well as to the actual."[27] According to Kneale, in order to speak of the possible, we must posit a subject as actual before we can proceed to add predicates to it.[28] At this point Kneale's method diverges sharply from Kant's in that he does not allow for the possibility that the knower possesses a complete conceptual determination of the thing prior to any reference to actual existence. As we saw in the *Beweisgrund,* Kant maintains that we are able to combine concepts in innumerable combinations without any reference whatsoever to their real existence. Kneale conversely maintains that actual existence must be posited, albeit not as a predicate, before we can combine concepts at all.

G. E. Moore offers a critique of Kneale's use of logical-form analysis. He begins by clarifying the original question. To ask, "Is existence a predicate?" is not to make direct reference to the subject "existence", but rather to finite forms of the verb "to exist". According to Moore, it is clear that Kneale rejects existence as a logical predicate, but it is less clear whether he rejects existence as a grammatical predicate. Nevertheless, it is the former claim that is the more important of the two. That being the case, the fundamental distinction participants in the debate strive to clarify is that between the proposition "the car is red" and "the car exists". The task confronting each participant is to offer a more precise explanation as to *why* these two propositions differ. For Kant, the difference lies in the fact that the predicate "exists" does not in any way amplify or enlarge the concept.

Moore understands Kneale to mean that in the propositions "tame tigers scratch" and "tame tigers growl", both "scratch" and "growl" can be understood as attributes, whereas "exists" in the proposition "tame tigers exist" cannot. Moore then furthers the discussion initiated by Kant and elaborated by Kneale through an examination of propositions different in grammatical form but logically identical.

Three propositions can be derived from a general proposition of attribution such as "tame tigers growl". These three propositions – "all tame tigers growl",

[26] Ibid., 164.

[27] Ibid., 170.

[28] "The word 'possible' cannot stand alone as a description of anything. In order to give it a sense we must say that it is short for 'possible fact.'" Ibid., 171.

"most tame tigers growl", and "some tame tigers growl" – all have distinct meanings. However, Moore claims that the proposition "tame tigers exist" is always logically identical to "some tame tigers exist".[29] That is to say that the proposition "tame tigers exist" never means "all tame tigers exist" or "most tame tigers exist".

Moore's next step is to compare the negative formulations of the propositions "some tigers do not growl" and "some tigers do not exist". Moore agrees with Kneale that the positive existential proposition "some tame tigers exist" simply means "there are some tame tigers". However, the same conversion cannot be performed on the negative statement "some tame tigers do not exist". The conversion would result in the proposition "there are some tame tigers which do not exist". Moore suggests that if there is any meaning to be found in this proposition, "exist" must be used in way different from the positive statement. If, as Kneale argues, "exist" is used in the same way in both statements, and if there is no meaning in the statement "there are some tame tigers which do not exist," there is also no meaning in the statement "some tame tigers exist", which itself is logically identical to the statement "tame tigers exist."[30]

This leads Moore to conclude that existence is not a logical predicate, but a grammatical predicate. Although he does not explicitly make the same metaphysical claims as Kneale prior to the logical analysis, the same presuppositions underlying Kneale's arguments also apply to Moore's argument. Namely, if the focus of the analysis in on the sentence itself, there must be some type of "being" prior to existence. As Kneale argued, there must be some type of being both prior and common to both possible existence and existence in reality. The proposition "there are some tame tigers that do not exist" is meaningless because of the contradictions between the components "there *are*" and "do *not exist"*. The statement simultaneously affirms existence and denies it. Because, Kneale argues, there is no third mode of existence, such a statement has no meaning whatsoever. The sentence "there are some tame tigers", on the other hand, simply identifies what it is that the "there" stands for, somewhat akin to the "positing" Kant claimed was the true role of existence as a predicate.

We can summarize the logical technique used by Kneale and Moore as an inversion of subject and predicate in existential statements. As Read points out, we can "transform an existential statement so that the grammatical subject appears as a predicate; one removes thereby the grammatical predicate 'exists'."[31] Such an inversion places primary emphasis on the subject of predication, such that the existential component of the proposition becomes merely a reference to the grammatical subject. Moore illustrates this referential property of existence by inviting the reader to envision himself pointing a finger towards the thing while simultaneously enunciating the existential proposition. The actions of enunciating

[29] Ibid., 178.

[30] Kneale notes that there could be some meaning to the sentence "there are some tame tigers which do not exist" if it is taken to mean the same as "some tame tigers are imaginary" or "some tame tigers are not real tigers". Ibid., 181.

[31] Stephen Read, "Exists is a Predicate," *Mind* 89 (July 1980), 412.

and pointing together make reference to the car which then becomes the grammatical subject.

The question arises: is an existential proposition no more than a reformulated grammatical expression of a referential act towards a really existing thing? If we accept that existence is not an attributable property of an object, and if existence is not a logical predicate, then the answer seems to be in the affirmative. In *Principia Mathematica,* Alfred Whitehead and Bertrand Russell offer further evidence for this conclusion as they reduce simple mathematical statements into formal logic, leading them to classify the predicate of existence as an "existential quantifier".[32]

I now turn to review two tendencies emerging from the logical analysis of existential propositions exemplified in the preceding examples. Both tendencies are concomitant to a reflection upon the relationship between a positive statement of a thing's existence and the other essential characteristics of a thing's nature. Metaphysicians generally refer to this as the question of the "priority" of being, which I will turn to in the next section. The first tendency is to provide a full account of the predictability of a grammatical object based on an absolute priority of a subject's existence. This tendency is apparent in the work of Kneale and Moore. Before we can say that the car has an attribute of "redness" in the sentence "the car is red", we must first be able to say "there is a car". Recall that, according to Kneale, this proposition is equivalent to "the car exists", though the latter is less clear than the former. This tendency assumes the priority in question to be logical in nature. David Braine explains: "From the point of view of the logician, the existence of a thing is not a predicate but stands always as a presupposition in the logician's sense of subject-predicate propositions about the thing; a pre-condition equally of the truth of the proposition and the truth of its negation."[33]

The second tendency places the existential quantifier of a thing at the mercy of the predicates determining it. This tendency, exemplified in the analysis of Russell and Whitehead, denies any priority of a thing's being in the order of logic. Formal logicians justify this denial through the necessary non-emptiness of the domain over which the quantification is made. There must be at least one predicate signifying an attribute of the thing before we can attempt to predicate existence to the thing through reference. If I say "there is a car", I can say with certainty that there is at least one other true attributive proposition, such as "the car is red" or "the car is large". This is to say that there is a priority of the proposition "the car is red" to the statement "the car exists". We find in this tendency a dependency of the referential proposition on the attributes of a thing.

In the next moment of the debate, we will listen to the voice of neo-Thomism in the twentieth century. Etienne Gilson is taken as an example of the attempt to confront the question of existential predication against a background of Aquinas' philosophy. Gilson's position is particularly interesting as he, like Kneale and Moore, takes the Kantian critique seriously on its own terms. Kant's philosophy

[32] Bertrand Russell and Alfred North Whitehead, *Principia Mathematica* (Cambridge: University Press, 1910).

[33] David Braine, *The Reality of Time and the Existence of God* (Oxford: Clarendon Press, 1988), 123

serves as the field upon which a dialogue between Thomistic and modern philosophy takes place.

3. Neo-Thomistic metaphysics and predication

The neo-Thomistic school of philosophy, whose most prominent members include Jacques Maritain and Etienne Gilson, was convinced that scholastic philosophy could engage in serious discussion with other philosophical schools, including the analytic tradition. For the Thomist, the heart of the question regarding the predication of existence is the distinction between essence and existence. Gilson writes, "if we tell them that existence is a predicate, they will understand that, according to Thomas Aquinas, actual existence, or *esse,* can be predicated of its essence as one more essential determination."[34]

Gilson's formulation of the question highlights the dilemma lurking underneath the question of existential predication. On the one hand, if we concede that existence is a predicate, it seems we have to admit that existence determines a thing essentially. On the other hand, if we deny the possibility of its real or logical predication, existence risks being eviscerated of meaning.

Throughout the debate on the predication of existence, an ineluctable epistemological question has emerged. Just how are essence and existence known? Kant has convincingly demonstrated that the concept of existence is the most vexing in the attempt to classify existential propositions. Gilson approached the question from the aspect of Thomistic epistemology, in which we find a basic distinction between two operations of the intellect.[35] These operations, simple apprehension and judgment, pertain to essence and existence respectively. Through the operation of simple apprehension, I know what something is. Through the act of judging, I know that it is. The first is called "simple" because it does not involve the basic operations of dividing and uniting which are subsequently expressed through propositions. Judgment, on the other hand, is a complex operation through which we actively unite two concepts in the mind. "Car" is a concept resulting from simple apprehension, "the car is red" is a proposition resulting through an act of judgment.

Joseph Owens resorts to the technique of grammatical transposition to illustrate the distinction between simple apprehension and judgment.[36] He begins with an epistemological presupposition. I can construct the concept of a unicorn in my mind. The concept, as Kant argues, will have all of the necessary and sufficient determinations that render it a clear and distinct idea. If I were to find a really existing unicorn, however, it seems that there would be more *meaning* to its determinations than when it was merely an imaginary being in my mind. This

[34] Etienne Gilson, *Being and Some Philosophers* (Toronto: Pontifical Institute of Mediaeval Studies), 224.

[35] Ibid., 48-71.

[36] Joseph Owens, *An Interpretation of Existence* (Houston: Center for Thomistic Studies, 1968), 14-43.

suggests that the mind is capable of other operations beyond that of conceptual knowledge.

Against this background we turn once again to a linguistic analysis. There is a difference between the statements "the unicorn is white" and "the unicorn is". But we can transform the statement into another proposition by placing "is" on the side of the subject. This results in the component "the existent unicorn", which, according to Thomistic epistemology, is a product of simple apprehension. Adding the predicate of the first statement so as to form yet another proposition, we have "the existent unicorn is white". Now I have the concept of a really existing unicorn that happens to be white. However, such a proposition still does not tell me whether the unicorn really exists or not. I am thinking about a really existing white unicorn , but I am not led to conclude that the unicorn really exists. If we take the transformation a step further and say "the existent white unicorn exists", the verb is rendered superfluous. It adds not new conceptual content to the sentence.

This analysis leads Joseph Owens to conclude that we must always refer compound propositions about really existing things to the things themselves. We must turn our gaze to the things themselves to ascertain whether a proposition is true or false.[37] Simply comparing one of the terms such as "unicorn" or "white" or "exists" will not help us determine whether the proposition is true as a whole. Therefore, a proposition expresses a type of knowledge that the intellect is able to attain of its objects, but a knowledge that cannot be expressed by a single world. It is a knowledge that can only be gained through a synthetic act and expressed verbally through a sentence. To be grasped by the mind, it is a knowledge that must be compositional in its very operation.

Against the same Thomistic background, Gilson begins his own analysis by emphatically agreeing with Kant's critique of existential propositions.[38] Gilson states that "actual existence cannot be represented by, nor in, a concept."[39] Based on this assumption, Gilson distinguishes between two types of propositions which he calls two-termed propositions and one-term propositions.[40] "The car is red" is an example of a two-termed proposition. In two-termed propositions, "is" performs the role of a copula linking subject and predicate. Gilson claims that in the one-term proposition "the car is", there is only one concept -- the car. He then asks the question, "if all propositions entail either a composition or a division of concepts, how can there be a proposition in which there is only one concept?" Gilson then proceeds to outline some possible solutions to this problem.

One possible solution involves the conversion of all one-term propositions to two-term propositions. We can, for instance, convert the proposition "Peter runs" into "Peter is running". Can a similar conversion be made in the case of existential propositions? Gilson converts the proposition "I am" to "I am being" to test the

[37] Ibid.

[38] "Being … is obviously not a real predicate, or a concept of something that can be added to the concept of a thing… By whatever and by however many predicates I may think a thing (even in completely determining it) nothing is really added to it, if I add that the thing exists." Ibid, 4.

[39] Ibid.

[40] Ibid., 191.

hypothesis. Although this conversion is possible "verbally speaking", Gilson notes that we are not left with a typical two-term proposition. Rather that the usual three parts of predication, we have four: (1) the subject, "I"; (2) the predicate, "being"; (3) the copula, "is", which, in turn, means once again (4) "being".[41] Thus, the conversion of existential propositions still leaves us with one term and a verb. It is here, notes Gilson, that "the metaphysical truth that existence is not a predicate is finding its logical verification."[42]

Louis-Marie Regis offers an immediate critique of Gilson's reasoning based on his own understanding of Aquinas. Regis finds no basis for the position that Aquinas considered only nouns to be concepts. It is the joining of concepts that is proper to the operation of judgment, not the knowledge of existence *in se*. There are thus two ways in which we can identify the predicate of the proposition "the car is red". According to Gilson, this is a two-termed proposition in which the second term is "red". The "is" merely acts as a copula. However, Regis points out that Aquinas would just as well have considered "is red" to be the predicate. Verbs, argues Regis, can be the result of knowledge gained through simple apprehension. If we excise the word "is" from the sentence, we must ask ourselves what "is" signifies. As Gilson's example illustrates, we know what "is running" signifies in the proposition "Peter is running". Psychologists call the thing signified a "judgment of action".[43] However, "is" may also be taken, according to Aquinas, as a verb *simpliciter.*[44] Taken in this way, "it is neither capable of expressing truth nor of constituting the enunciation and hence does not belong to the second operation of the mind (i.e., judgment)."[45]

In short, Regis concludes that "is" can be a predicate in the case of one-term propositions. Knowledge of existence can be had not only through the intellective operation of judgment, but through the concept of a verb. It is not necessarily acquired through abstraction, nor is it a quidditive concept. Rather, it is the concept of an "act"; indeed, as Thomas holds, the "act of all acts".[46]

The discussion between Gilson and Regis reveals two distinct uses of the word "predicate". According to Regis, to predicate is simply to say. Anything that can be said of a subject, even the act of existing, may be predicated of the subject. Gilson, however, notes that this is not the use of the term "predicate" in the modern sense. The term "predicate", particularly in the analytic philosophy we have already considered, refers to the proposition itself and the relation between the two

[41] Ibid., 192.

[42] Ibid., 193.

[43] Ibid., 191.

[44] "The verb 'is' is sometimes predicated by itself as an enunciation, as when it is said 'Socrates is'. By this we do not intend to mean anything other than that Socrates exists in reality." *Sententia super Peri hermenias II,* lect. 2, no. 2. Aquinas summarizes three distinct ways of predicating *esse.* The first is *ipsa quidditas vel natura rei,* the second is *ipse actus essentiae.* The third is understood as the copula signifying the composition or division that takes place in the act of judgment. See Thomas Aquinas, *In II Sententiis,,* 34, 1, 1.

[45] Etienne Gilson, *Being and Some Philosophers* (Toronto: Pontifical Institute of Mediaeval Studies), 218.

[46] Thomas Aquinas, *De Potentia,* ques. 7, art. 2. See also *Summa Theologiae* I, ques. 4, art. 3.

distinct entities of which it is composed. This is different from the Aristotelian sense according to which we simply assert that something *exists in a certain way.* When we say "the car is red", we assert that the car is actualized in a certain way. Even though the color of the car is accidental to the car itself, the redness is a certain formal actualization of the car.

According to Aquinas' understanding of judgment as an operation of uniting and dividing, *esse* does not signify a composition or division in the proposition itself, but rather in the composition and division performed by the mind in the very act of judging. Strictly speaking then, the act in itself does not result in the "real" predication of existence. Rather, it signifies the intellectual operation of composing and dividing.

Gilson's position can be summarized thusly. The proposition "the car is", in contrast to "the car is red", does not assert an act of composing and dividing performed by the mind. Even though existence is originally apprehended in the intellectual act of composing and dividing, no composition or division is immediately apparent in such a proposition. What it does signify is the existential act of the car. This "act" of the car is a concept that can be reasoned to through an examination of the intellective operation through which we originally grasp its existence. When conceptualized as an act, it makes sense to predicate the verb "is" to the subject "car" in the proposition "the car is". In such a proposition, Aquinas considered "is" to be a predicate.

4. Résumé

This brief review of three moments in the history of a particular ontological question is not meant to be exhaustive. Rather, it is meant to trace a specific way in which epistemology, metaphysics, and logic have had, and will continue to have, a major influence on how we understand ontology. An answer to the question "Is existence a predicate?" depends on one's understanding of what is meant by the question, which itself presupposes the epistemology with which one approaches the question. In Kant's case, the distinction between analytic and synthetic judgments sets the stage for the absolute uniqueness of existence as a predicate. Once he had denied that existence is a real predicate, philosophers turned to a closer linguistic examination of existential propositions to ascertain their logical meaning apart from their truth or falsity. This analytical approach led to a reappraisal of the question based on the basic distinction between simple apprehension and judgment in the neo-Thomistic tradition. The ground is fertile for continued dialogue on the more basic questions implied by the existential-predication question, and the field is wide open for the participation of phenomenology and other post-modern currents in philosophy.[47]

[47] The author wishes to express his deep gratitude to Bishop Patrick R. Cooney, to whom he dedicates this essay.

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